2017

MATHEMATICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[OLD SYLLABUS]

GROUP – A

(*Analytic Statics*)

[*Marks* : 36]

1. Answer any *one* : 12 × 1

   (*a*) (*i*) Show that every given system of forces acting on a rigid body can be reduced to a wrench. Hence define Poisson's central axis.

(*ii*) A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8\pi-4}$ from the centre of the circle of radius $a/2$. 12

(*b*) Define stable and unstable equilibrium. If there be more than one position of equilibrium then what will happen ? A perfectly rough heavy body rests in equilibrium on a fixed body at the highest point, common normal at the point of contact is vertical and c.g. of the upper body is at a height h above the point of contact, if ρ_1, ρ_2 be the radii of curvature at the point of contact, then show that

$$\frac{1}{\rho_1}+\frac{1}{\rho_2}<\frac{1}{h}$$

is the condition of stable equilibrium. 2+2+8

2. Answer any *two* questions : 8×2

(*a*) Two forces P and Q act along the straight

lines whose equations are $y = x\tan\alpha$, $z = c$ and $y = -x\tan\alpha$, $z = -c$ respectively. Show that their central axis lies on the straight line

$$y = x\frac{P-Q}{P+Q}\tan\alpha \text{ and } \frac{z}{c} = \frac{P^2-Q^2}{P^2+PQ\cos 2\alpha+Q^2}.$$

For all values of P and Q, prove that this line is a generator of the surface

$$(x^2+y^2)z\sin 2\alpha = 2cxy. \qquad 8$$

(*b*) Prove that any system of forces acting on a rigid body can be reduced to a single force and a couple whose axis lies along the line of action of the force. Identify the Poinsot's central axis. 7+1

(*c*) Two forces act, one along the line $y = 0$, $z = 0$ and the other one along the line $x = 0$, $z = c$. As forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2+y^2)z = cy^2$. 8

(*d*) Define sliding and rolling frictions. A semicircular disc rests in a vertical plane with curved edge on a horizontal and an equally

rough vertical plane, the coefficient of friction being μ; find the greatest angle that the bounding diameter can make with the horizontal plane. 2+6

3. Answer any *two* questions : 4×2

(*a*) Find the C.G. of a circular arc making an angle 2α at the center. 4

(*b*) Define limiting friction and angle of friction. Why frictional forces are non-conservative ? 4

(*c*) A heavy rod AB length $2b$ rests upon a fixed smooth peg at C and with its end B upon a smooth curve. It rests in all positions, find the polar equation of the curve with C as origin. 4

GROUP – B

(*Hydrostatics*)

[*Marks* : 27]

4. Answer any *one* : 9×1

(*a*) If the absolute temperature T diminishes

upwards in the atmosphere according to the law $T = T_0/(1 + \alpha z)$, where α is a constant, then show that the pressure at a height z is given by

$$p = p_0 \exp\left\{ - \left(\frac{z}{H} + \frac{\alpha z^2}{2H} \right) \right\},$$

where p_0 is the pressure and T_0 is the temperature at $z = 0$, and H is the height of the homogeneous atmosphere. 9

(*b*) Obtain the necessary and sufficient conditions for the equilibrium of a mass of heterogeneous liquid under a given system of forces whose components per unit mass at a point (x, y, z) parallel to co-ordinate axes are F_x, F_y and F_z. 6+3

5. Answer any *two* questions : 6×2

(*a*) A circular tube is half-full of liquid and is made to revolve round a vertical tangent line with angular velocity w ; if 'r' be the radius of the tube , then prove that the diameter

passing through the free surfaces of the liquid is inclined at an angle $\tan^{-1}\left(\frac{w^2 r}{g}\right)$ to the horizon, 'g' being the acceleration due to gravity. 6

(*b*) When the depth of the liquid is increased by *a*, the depth of C.P. of a plane lamina is found to increase by *y* and when instead, the depth of the liquid is increased by *b*, that of the C.P. is found to increase by *z* ; show that the depth of the c.g. in the original state of the liquid is $ab(b-a+y-z)/(az-by)$. 6

(*c*) A given volume *V* of a liquid is acted upon by forces

$$-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}.$$

Find the equation to the free surface. 6

(*d*) A semicircular tube has its bounding diameter horizontal and contains equal volumes of *n* liquids of densities successively equal to

$e, 2e, 3e, \ldots$ arranged in this order. Show that if each fluid subtends an angle 2α at the centre and the tube just hold all of them, then

$$\tan n\alpha = (2n + 1)\tan\alpha.$$ 6

6. Answer any *two* questions : 3×2

(*a*) Explain the texm 'perfect fluid' and pressure at a point in the fluid. 3

(*b*) Define the surface of flotation and surface of buoyancy. 3

(*c*) Find the work done in compressing a gas for adiabatic change. 3

GROUP – C

(*Rigid Dynamics*)

[*Marks* : 27]

7. Answer any *one* question : 9×1

(*a*) State D' Alembert's principal and deduce general equations of motion of a rigid body.

Prove that the motion of a body about its center of inertia were fixed and the same forces acted on the body. 1+4+4

(*b*) A wire is in the form of a semi-circle of radius '*a*'. Show that, at an end of its diameter, the principal axes in its plane are inclined to the diameter at angles

$$\frac{1}{2}\tan^{-1}\left(\frac{4}{\pi}\right) \text{ and } \frac{\pi}{2}+\frac{1}{2}\tan^{-1}\left(\frac{4}{\pi}\right). \quad 9$$

8. Answer any *two* questions : 6×2

(*a*) If the principal axis of a uniform semi-circular lamina at an extremity of its bounding diameter makes an angle θ with the diameter, then prove that

$$8\cot 2\theta = 3\pi. \quad 6$$

(*b*) A uniform rod is held at an angle α to the horizon with one end in contact with a horizontal table whose coefficient of friction

is μ. If it be then released show that it will commence to slide if

$$\mu < \frac{3\tan\alpha}{1+4\tan^2\alpha}.$$ 6

(*c*) Define compound pendulum. If a bent lever of arms of lengths $2a$ and $2b$, angle between them being 30°, makes small oscillation in its own plane about the fulcrum, then find the length of the corresponding simple equivalent pendulum. 1 + 5

(*d*) If a rigid body moves under the action of a system of conservative forces, show that the sum of its kinetic and potential energies remain constant throughout its motion. 6

9. Answer any *two* questions : 3 × 2

(*a*) Find the moment of inertia of a solid sphere about a diameter. 3

(*b*) Deduce the general equations of motion in two dimensions in case of varying mass of the rigid body. 3

(*c*) Define equimomental bodies. Under what conditions two bodies are said to be equimomental ? 3