2017

MATHEMATICS

[Honours]

PAPER – VII

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[OLD SYLLABUS]

GROUP – A

(*Mathematical Probability*)

[*Marks* : 36]

1. Answer any *one* question : 15 × 1

(*a*) (*i*) Let A, B, C be mutually independent events. Then prove that A and $B \cup C$ are

(*Turn Over*)

independent and that A^C, B^C and C^C are mutually independent. 5

(*ii*) Show that the probability distribution function of a random variable is continuous from the right but has a jump discontinuity on the left at every point. 5

(*iii*) State Bernoulli's theorem and hence prove that if f be the frequency ratio of successes in a Bernoullian sequence of n trials with probability of success p, then $f \xrightarrow[\text{in } p]{} $ as $n \to \infty$. 5

(*b*) (*i*) Three concentric cycles of radii $1/\sqrt{3}$, 1 and $\sqrt{3}$ feet are drawn on a target board. If a shot fall within the innermost circle, 3 points are scored; if it falls within the next two rings the score is respectively 2 and 1 and the score is 0 if the shot is outside the outmost circle. If the probability density of the distance of the hit from the centre of the target be $\frac{2}{\pi}\frac{1}{1+r^2}$, find the probability distribution of the score. 4

(*ii*) Determine the value of the constant K which makes

$$f(x, y) = Kxy,\ 0 < x < 1,\ 0 < y < x$$

a joint probability density function. Calculate the marginal density functions and show that the variables are dependent. 6

(*iii*) Find the mean and variance of the pascal distribution : 5

$$x_i = i,\ \ i = 0, 1, 2, \ldots$$

$$f_i = \frac{1}{1+\mu}\left(\frac{\mu}{1+\mu}\right)^i,\ \ \mu > 0$$

2. Answer any *two* questions : 8 × 2

(*a*) (*i*) There are n urns each containing N balls of which N_1 are white and N_2 black $(N = N_1 + N_2)$. One ball is transferred from the first to the second urn, then one ball is transferred from the second to the third urn and so on, finally one ball is drawn from the nth urn. Find the probability that the ball drawn is white. 4

(ii) A point P is chosen at random on a line segment AB whose middle point is O. Find the probability that AP, PB and AO can form the sides of a triangle. 4

(b) (i) If p and q be independent variates each uniformly distributed over the interval $(-1,1)$, then find the probability that the equation $x^2+2px+q=0$ has real roots. 4

(ii) If $f(x, y)=x+y$ $(0<x<1, 0<y<1)$ be the joint probability density function of the random variables X and Y, then find the distribution of $X+Y$. 4

(c) (i) Prove the Schwartz's inequality for expectations that $[E(XY)]^2 \leq E(X^2)\,E(Y^2)$ and hence deduce that $-1 \leq \rho(X, Y) \leq 1$. 5

(*ii*) If X be a non-negative random variable having mean m, then prove that $P(X > \tau m) < 1/\tau$ for any $\tau > 0$. 3

(*d*) (*i*) If the mutually independent random variables $X_1, X_2, \ldots X_n$ all have the same distribution and their sum $X_1 + \ldots + X_n$ be normally distributed then show that each of them is normally distributed. 5

(*ii*) Prove that the linear combinations $aX + bY$ and $cX + dY$ of the random variables X, Y are uncorrelated it $ac\sigma_x^2 + (ad + bc)\rho\sigma_x\sigma_y + bd\sigma_y^2 = 0$. 3

3. Answer any *one* quesation : 3 × 1

(*a*) Prove that two events A and B having non-zero probabilities cannot be simultaneously mutually exclusive and independent. 3

(*b*) What are advantages of characteristic function over moment generating function of a random variable ? 3

4. Answer any *one* question : 2 × 1

(*a*) Write down the probability density function for bivariate normal distribution. 2

(*b*) State Tchebycheff's theorem. 2

GROUP – B

(*Statistics*)

[*Marks* : 27]

5. Answer any *one* question : 15 × 1

(*a*) (*i*) Explain the terms : population and sample. What do you mean by distribution of the sample ? 4 + 2

(*ii*) Show that the statistic $\chi^2 = \frac{nS^2}{\sigma^2}$ has a χ^2-distribution with $v = n - 1$ degrees of freedom and the sample mean $\bar{X}$ and sample variance S^2 are independent variates. 9

(*b*) (*i*) Describe briefly the method of maximum likelihood estimation. 5

(ii) Find the maximum likelihood estimate of the parameter μ in the Pascal distribution

$$x_i = i, \ \ i = 0, 1, 2, \ldots$$

$$f_i = \frac{1}{1+\mu}\left(\frac{\mu}{1+\mu}\right)^i, \ \ \mu > 0$$

Show that the estimate is unbiased and consistent. 5 + 2 + 3

6. Answer any *one* question : 8 × 1

(a) (i) Independent samples of sizes 30 and 55 from two normal populations having a common variance 17·6 were found to have means 23·0 and 21·9 respectively. Test at 1% significance level whether the population also have the same mean (Given that $P(z > 2{\cdot}58) = 0{\cdot}05$). 3

(ii) Fit a straight line $y = c_0 + c_1 x$ and a parabola $y = d_0 + d_1 x^2$ to the following data and compare their goodness of fit : 5

x	3·5	8·4	16·8	23·9	27·1	28·8
y	4·4	9·2	20·6	31·1	35·0	37·7

(*b*) (*i*) The rainfall of a rainy season of a particular region in India is measured for few consecutive days and the measurements (in mm) are 9·4, 8·8, 10·6, 12·2, 11·8, 9·9, 10·8, 12·1, 11·7. Compute 99% confidence interval for mean and standard deviation of the population, assuming the population of measurement of rainfall is normal. Given that for 9 degrees of freedom $P(t > 3{\cdot}25) = 0{\cdot}005$, $P(\chi^2 > 1{\cdot}734926) = 0{\cdot}995$ and $P(\chi^2 > 23{\cdot}5893)$ 0·005. 6

(*ii*) Give advantages of interval estimation over point estimation. 2

7. Answer any *one* question : 4 × 1

(*a*) What do you mean by best critical region ? State Neyman-Pearson theorem in determining the best critical region. 2 + 2

(*b*) 400 mangoes are selected at random from a large stock, 53 were found to be bad. Test at 1% significance level the hypothesis that on the average 10% of the mangoes were bad. 4

GROUP – C

[Optional Paper - I]

(*Discrete Mathematics*)

[*Marks* : 27]

8. Answer any *one* question : 3 × 1

(*a*) In group of seven people, is it possible for each person to shake hand with exactly 3 other people ? Justify your answer. 3

(*b*) Define tautology. Prove that

$$(P \to Q) \to (\sim Q \to \sim P)$$ 3

9. Answer any *two* questions : 12 × 2

(*a*) (*i*) Find the number of integral solutions of

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 19$$ 5

(*ii*) How many 6-digit numbers without repetition of digits are three such that the digits are all non zero and 1 and 2 do not appear conscutively in either order ? 3

(*iii*) Prove that in the algebraic system $B = (A, \vee, \wedge)$ defined by a lattice $(A, \leq)$, the operations are associative. 4

(*b*) (*i*) Given sufficient number of stamps of two different values Rs.3 and Rs. 5. Show that any postage can be made up. 4

(*ii*) If n, e and r respectively be the number of vertices, edges and regions of a connected plannar graph, then prove that

$$n - e + r = 2.$$ 4

(*iii*) Prove that a directed complete graph always has a hamiltonian path. 4

(*c*) (*i*) For any finite set A, show that

$$|P(A)| = 2^{|A|},$$

using mathematical induction. 4

(*ii*) For a tree with p vertices and q edges prove that $p = q + 1$. 4

(*iii*) Let $(P, \leq)$ be a partially ordered set. Suppose the length of the longest chain in P is n. Prove that the elements in P can be partitioned into n disjoint antichains. 4

GROUP – C

[Optional Paper - II]

(*Mathematical Modelling*)

[*Marks* : 27]

8. Answer any *one* question : 15 × 1

(*a*) (*i*) Discuss Malthus single species exponential growth model. 10

(*ii*) Derive the relation between car velocity (u) and traffic density (ρ) on a high way. Find speed limitation (u max) on highway. 5

(*b*) (*i*) In a hypothetical population birth rate $= \left(\frac{1}{2} - \frac{P(t)}{800}\right)P(t)$ and death rate $= \left(\frac{1}{4} + \frac{P(t)}{200}\right)P(t)$, where $P(t)$ is the population size at time t with $P(0) = 2$. Assuming that the population to be closed to the outside find $P(t)$ and also find the population at the point of inflexion. 8

(*ii*) Discuss BLL (Bernardelli, Lewis and Leslie) model. 7

9. Answer any *one* question : 8 × 1

(*a*) The following system of equations governs a prey-predator model :

$$\frac{dx}{dt} = x(a - by - cx)$$

$$\frac{dy}{dt} = y(px - q)$$

Determine the critical points of the system and discuss their nature and stability. 8

(*b*) Show that the model represented by

$$\frac{dx}{dt} = x(15 - 5x - 3y),\ \frac{dy}{dt} = y(4 - x - y),$$

$$x \geq 0,\ \ y \geq 0$$

has a position of equilibrium, this position is unstable, only one species will survive and which species survive depends on initial consitions. 8

10. Answer any *one* question : 4 × 1

(*a*) In a population of birds, the proportional birth rate and death rate are both constant, being 0·45 per year and 0·65 per year respectively. Immigration occurs at a constant rate of 2000 birds and emigration at a constant rate of 1000 birds per year. Use this assumption to formulate a model of the population. Solve the model and describe the long term behaviour of the population in the two cases when initial population is 3000 to 8000. 4

(*b*) Determine the critical point of the system of the differential equations

$$\frac{dx}{dt} = f(x, y)$$
$$\frac{dy}{dt} = g(x, y)$$

when the critical point is said to be stable and asymptotically stable ? 4

GROUP – C

(Optional Paper - III)

(*Application of Mathematics in Finance and Insurance*)

[*Marks* : 27]

8. Answer any *one* question : 15 × 1

(*a*) From the following particulars calculate NPV and IRR of a project :

	Net Profit earned(Rs.)
Year 1	20,000
Year 2	22,000
Year 3	18,000
Year 4	24,000
Year 5	25,000

The project is having an initial outlay of Rs.50,000. Applicable tax rate is 25%. Prevailing inflation rate is 10%. 15

(*b*) What are the objectives of financial management ? Differentiate among risk, speculation and gambling. 8 + 7

9. Answer any *one* question : 8 × 1

(*a*) Describe in brief the arbitrage process.

(*b*) Write a note on costs and benefits of insurance to the society.

10. Answer any *one* question : 4 × 1

(*a*) Write a note on put option and call option.

(*b*) Write a note on 'Market Risk' associated with a corporate security.