UG/III/STAT/H/VI/17(Old)

2017

STATISTICS

[Honours]

PAPER – VI

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[OLD SYLLABUS]

GROUP – A

(*Statistical Inference*)

[*Marks* : 54]

Answer Q. No. 1 *Or* 2 and Q. No. 3 *Or* 4 and Q. No. 5

(*Turn Over*)

1. (*a*) Define an unbiased estimator, an estimable parametric function and an asymptotically unbiased estimator.

   (*b*) Suppose $(X_1, X_2, \cdots, X_n)$ is a random sample of size n from $U(0, \theta)$ distribution. Show that $X_{(n)} = \max_{1 \le i \le n} \{X_i\}$ is not an unbiased estimator for θ but $X_{(n)}$ is an asymptotically unbiased estimator of θ.

   (*c*) Suppose $(X_1, X_2, \cdots, X_n)$ is a random sample of size n from $N(0, \sigma^2)$ distribution. Find the constant λ for which

$$T = \lambda \sum_{i=1}^{n} |X_i|$$

   is an unbiased estimator of σ. 15

2. (*a*) What do you mean by a maximum likelihood (ML) estimator ?

   (*b*) State the important properties of maximum likelihood estimator.

(c) Suppose $(X_1, X_2, \cdots, X_n)$ is a random sample of size n from the population having p.d.f. :

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-\frac{(x-\alpha)}{\beta}} ; & \alpha \le x \le \infty \\ 0 ; & \text{otherwise,} \end{cases}$$

where $-\infty < \alpha < \infty$, $\beta > 0$. Find ML estimator of (α, β). 15

3. (a) State and prove Neyman-Pearson fundamental lemma.

(b) Suppose $(X_1, X_2, \cdots, X_n)$ is a random sample of size n from a population having p.d.f. :

$$f(x) = \begin{cases} \theta e^{-\theta x} ; & 0 < x < \infty \\ 0 ; & \text{otherwise,} \end{cases}$$

where $0 < \theta < \infty$.
Derive a UMP critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$.
Show that the test can be done using a chi-square statistic. 15

4. (a) Discuss the general method of construction of a likelihood ratio (LR) test.

(*b*) Describe the LR test procedure based on a random sample of size n drawn from $N(\mu, \sigma^2)$-distribution, where μ and σ^2 are both unknown, for testing $H_0 : \mu = \mu_0$ against all possible alternatives. 15

5. Answer any *three* of the following questions : 8×3

(*a*) Under the usual regularity assumptions, to be stated by you, state and prove Rao-Cramer inequality.

(*b*) Derive the shortest confidence interval for the mean of a normal population with unknown variance.

(*c*) Define a sufficient statistic. Show whether $X_1 + 2X_2$ is sufficient for θ if X_1 and X_2 constitute a random sample from Bernoulli (θ) distribution.

(*d*) Derive the large sample distribution of the sample moment-measure of skewness for the sample of size n drawn from $N(\mu, \sigma^2)$ -distribution.

(*e*) Show that the likelihood ratio test for testing the equality of variances of two normal distributions is an F-test.

GROUP – B

(*Theory of Sample Survey*)

[*Marks* : 36]

6. Answer any *three* of the following questions : 8 × 3

(*a*) Prove that, with usual notations, for estimating the population mean of a stratified population under simple random without replacement (SRSWOR) within each stratum.

$$V_{\text{opt}} \le V_{\text{prop}} \le V_{\text{rand}}$$

(*b*) Describe the method of linear systematic sampling. Show that variance of the unbiased estimator of population mean may be expressed as

$$\frac{(N-1)S^2}{nN}[1+(n-1)\rho],$$

where S^2 is population variance with divisor $(N-1)$ and ρ is the intra-class correlation coefficient between the values of the pairs of units. Hence show that systematic sampling is better than SRSWOR if

$$\rho \leq -\frac{1}{N-1}.$$

(c) For an SRSWOR of size n drawn from a population of size N, consider a class of estimators of population mean $(\bar{y})$,

$$\bar{y}' = \sum_{r=1}^{n} a_r y_r,$$

where a_r is the constant depending on rth draw, y_r is the value of study variable y on the unit selected at the rth draw. Show that $\bar{y}'$ is unbiased for $\bar{y}$ iff

$$\sum_{r=1}^{n} a_r = 1.$$

Show also that, under this condition,

$$V(\bar{y}') = S^2\left[\sum_{r=1}^{n} a_r^2 - \frac{1}{N}\right],$$

where S^2 is population variance with divisor $(N-1)$. Hence show that $V(\bar{y}')$ is minimized subject to the condition

$$\sum_{r=1}^{n} a_r = 1 \text{ iff } a_r = \frac{1}{n}; r = 1,2,..,n.$$

(*d*) Distinguish between a two-stage sampling and a stratified sampling. Indicate the cases when systematic sampling is equivalent to

(*i*) Cluster sampling,

(*ii*) Stratified random sampling.

(*e*) In SRSWOR, show that the value of the sample size n which minimizes $Q(n) = V(n) + C(n)$ is given by

$$n = \sqrt{\frac{S^2}{C_1}},$$

where the cost function $C(n)$ is given by $C_0 + C_1 n$ and the $V(n)$ is the variance of the sample mean.

7. Answer any *two* of the following questions : 6 × 2

(*a*) Mention the basic principles of sample survey. Discuss the non-sampling errors that may arise in a large-scale sample survey.

(*b*) Derive the standard error of the sample mean in case of simple random sampling drawn with replacement from a finite population.

(*c*) Obtain a ratio estimator of population mean under double sampling and also obtain its MSE.

(*d*) Give an unbiased estimator of population total under two-stage sampling with SRSWOR at both the stages. Show that the variance of the estimator is greater than the variance of the corresponding estimator in the single-stage sampling.

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