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UG/II/STAT/H/III/17 (New)

2017

STATISTICS

[Honours]

PAPER – III

Full Marks : 100

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Notations and symbols used bear their usual significance

[NEW SYLLABUS]

GROUP – A

1. Answer any *five* questions : 5 × 5

(*a*) Let X be such that its variance exists.

(*Turn Over*)

If $E(X) = \mu$ and var $(X) = \sigma^2$, prove that, for any $t > 0$,

$$P[|X - \mu| \geq t\sigma] \leq 1/t^2.$$

(*b*) Prove that

$$E(|XY|) \leq \sqrt{E(X^2)E(Y^2)},$$

provided the expectations exist.

(*c*) If $X_n \xrightarrow{P} X$ and g is a continuous function, then show that

$$g(X_n) \xrightarrow{P} g(X).$$

(*d*) If X is distributed in the rectangular form with pdf

$$f_X(x) = \begin{cases} \frac{1}{\theta}, & 0 < x < \theta, \\ 0, & \text{otherwise} \end{cases}$$

show that $-2 \log_e (X/\theta)$ follows a chi-square distribution with two degrees of freedom.

(*e*) Let X and Y be two independent random variables, each distributed in the $N(0, 1)$ form. Show that $Z = X / |Y|$ has a Cauchy distribution.

(*f*) Show that the sampling distribution of the largest observation Y of a sample of size n from the rectangular distribution $R(0, \theta)$ has the pdf given by $n y^{n-1}/\theta^n$ on $(0, \theta)$ and 0, otherwise.

(*g*) State the relation between the lower α-point and the upper α-point of (*i*) a t_n-distribution and (*ii*) an $F_{m,n}$-distribution.

2. Answer any *one* question : 10 × 1

(*a*) Obtain the expectation and the standard error of sample mean for a random sample of size n drawn from a finite population of size N (*i*) with replacement, (*ii*) without replacement.

(*b*) Show that $\bar{x}$ and s'^2 for random sample of size n from a normal (μ, σ^2) population are distributed independently of each other.

GROUP – B

3. Answer any *four* questions : 5×4

(*a*) Describe the maximum-likelihood method of estimation. What are the properties of a maximum-likelihood estimator ?

(*b*) If X_i $(i = 1, 2, ..., n)$ be a random sample from a rectangular distribution with p.d.f.

$$f_\theta(x) = \begin{cases} \frac{1}{\theta} & ; \text{if } 0 < x < \theta \\ 0 & ; \text{otherwise,} \end{cases}$$

where $0 < \theta < \infty$, show that $X_{(n)}$ is a sufficient statistic for θ.

(*c*) Let $(X_1, X_2, ..., X_n)$ be a random sample of size n from a normal distribution with known mean μ and unknown variance σ^2. Obtain the MVB estimator of σ^2.

(*d*) Define the following terms : Null hypothesis, alternative hypothesis, level of significance and power of a test.

(*e*) For a normal distribution with unknown mean μ and unknown variance σ^2, obtain a size-α critical region for testing $H_0 : \sigma^2 = \sigma_0^2$ versus $H_1 : \sigma^2 < \sigma_0^2$, where σ_0^2 is specified real value.

(*f*) For a random sample of size *n* drawn from a $N(\mu, \sigma^2)$-population, μ and σ^2 being unknown, derive a $100\,(1-\alpha)\%$ confidence interval for σ^2.

4. Answer any *one* question : 10 × 1

(*a*) Let $(X_1, X_2, ..., X_n)$ be a random sample from a gamma distribution with pdf

$$f_\theta(x) = \begin{cases} \dfrac{\alpha^p}{\Gamma\alpha} e^{-\alpha x}\, x^{p-1} & ; \text{if } 0 < x < \infty \\ 0 & , \text{otherwise}, \end{cases}$$

where $\alpha > 0$, $p > 0$ and $\theta = (\alpha, p)$.

Show that

(*i*) $\sum_i X_i$ and $\prod_i X_i$ are jointly sufficient for θ, and (*ii*) if p is known, $\sum_i X_i$ is sufficient for α.

(*b*) When a simple regression equation of the form $y = \alpha + \beta x$ is fitted to bivariate data, how would you test hypotheses concerning α and β ?

GROUP – C

5. Answer any *three* questions : 5 × 3

(*a*) Find the value of $r_{1 \cdot 23 \ldots p}$ if the independent variables are pairwise uncorrelated.

(*b*) Show that the multiple correlation coefficient $r_{1 \cdot 23 \ldots p}$ is the highest possible value of the simple correlation coefficient between x_1 and a linear function of $x_2, x_3, \ldots, x_p$.

(*c*) For $\underset{\sim}{X}_{p \times 1}$ following $N_p\left(\underset{\sim}{\mu}, \underline{\Sigma}\right)$, show that

$$\left(\underline{X} - \underline{\mu}\right)' \Sigma^{-1} \left(\underline{X} - \mu\right)$$

follows a chi-square distribution with p degrees of freedom.

(*d*) State the p.d.f. of a multivariate normal distribution and state its important properties.

(*e*) Write a short note on multinomial distribution.

6. Answer any *one* question : 10 × 1

(*a*) Prove the relation

$$1-r^2_{1\cdot23\ldots p}=\left(1-r^2_{12}\right)\left(1-r^2_{13\cdot2}\right)\cdots\cdots\left(1-r^2_{1p\cdot23\ldots(p-1)}\right).$$

(*b*) Show that a sufficient condition for the joint distribution of $X_1, X_2, \ldots, X_p$ to be multivariate normal is that every linear combination of $X_1, X_2, \ldots, X_p$ is distributed normally.

[*Internal Assessment* : 10 Marks]

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