2017

STATISTICS

[Honours]

PAPER – II

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

1. Answer any *five* questions : 5 × 5

(*a*) For any two events A and B show that

$$\max\{0, P(A)+P(B)-1\} \le P(A \cap B) \le \min\{P(A), P(B)\} \le \max\{P(A), P(B)\} \le P(A \cup B) \le \min.\{P(A)+P(B), 1\}.$$

(Turn Over)

(*b*) Each coefficient in the equation

$$ax^2 + bx + c = 0$$

is determined by throwing an ordinary die. Find the probability that the equation will have real roots.

(*c*) Obtain the moment generating function of a geometric distribution. Hence obtain its mean.

(*d*) Find the factorial moment of order r for binomial distribution.

(*e*) Suppose that the joint probability density function of two random variables X and Y is

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

are X and Y independent ?

(*f*) Is a distribution function always right continuous ? Justify your answer.

(*g*) Let the events $A_1, A_2, \ldots, A_n$ be mutually independent. If $P(A_i) = p_i$, $i = 1(1)n$. Find the probability that none of the events occur. Also show that this probability cannot exceed

$$\exp\left(-\sum_{i=1}^{n} p_i\right)$$

(*h*) Show that the first two moments, μ_1 and μ_2, are the same for Poisson distribution.

(*i*) If X follows $N(0, 1)$ distribution, find out $E(|X|)$.

2. Answer any *two* questions : 10 × 2

(*a*) (*i*) Define conditional probability. Show that it satisfies Kolmogorov's axioms of probability. 5

(*ii*) Two events A and B have non-zero probability of occurence. If A and B mutually exclusive, will they be independent ? If A and B are independent, will they be mutually exclusive ? 5

(*b*) (*i*) Give an example of a continuous distribution having 'lack of memory' property and establish this property for this distribution. 5

(*ii*) Show that for negative binomial distribution

$$\mu_{r+1} = q\left[\frac{\partial \mu_r}{\partial q} + \frac{rk}{p^2}\mu_{r-1}\right],$$

where p is the probability of success of a Bernoulli variable. 5

(*c*) Show that the probability $P_{[m]}$ that exactly m among the N events $A_1, A_2, ..., A_N$ occur simultaneously is given by

$$P_{[m]} = S_m - \binom{m+1}{1}S_{M+1} + \binom{m+2}{2}S_{M+2} - ..... \pm \binom{N}{m}S_N,$$

where $S_r = \sum P(Ai_1 Ai_2 .....Ai_r)$, $1 \le i_1 \le i_2 < ...$
$< i_r \le N$ 10

(*d*) (*i*) Show that for any random variable for which β_1 and β_2 exist $\beta_2 \ge 1 + \beta_1$. 5

(*ii*) Prove that the normal distribution is always mesokurtic. 5

GROUP – B

3. Answer any *two* questions : 5 × 2

(*a*) Derive Lagrange's interpolation formula.

(*b*) Suppose $\phi(x)$ is a polynomial in x of degree n. Show that

$$\Delta^m \phi(x) = \text{a polynomial in } x \text{ of degree } (n-m) \text{ when } m < n$$
$$= \text{constant when } m = n$$
$$= 0 \quad \text{when} \quad m > n$$

(*c*) Prove that divided difference are symmetric fuctions of their arguments.

(*d*) Establish Simpson's $\frac{1}{3}$ rule for numerical integration.

4. Answer any *one* question : 10 × 1

(*a*) (*i*) Obtain the convergence criterion of iteration method to obtain numerical solutions of equation. 5

(*ii*) Use the method of separation of symbols to prove that

$$U_{2n} - \binom{n}{1} 2u_{2n-1} + \binom{n}{2} 2^2 u_{2n-2} - \binom{n}{3} 2^3 u_{2n-3} + \ldots\ldots + (-2)^n u_n = (-1)^n (c - 2ax),$$

where $u_x = ax^2 + bx + c$ 5

(*b*) Describe Newton-Raphson method for solving an equation involving a single unknown with its geometric significance. 10

GROUP – C

5. Answer any *three* questions : 5 × 3

(*a*) Mention the main functions of the central statistical office (CSO) of the Government of India.

(*b*) Show that Marshall Edgeworth index number lies between Laspeyre's and Paasche's index number.

(*c*) Explain the uses of price index numbers.

(*d*) What is time-reversal test in the context of formulating an index number ? Extend this idea to circular test of a formula.

(*e*) Describe the ratio of moving average method for determining seasonal indices for a monthly time series data.

(*f*) Distinguish between 'seasonal variation' and 'cyclical variation' of a time series.

6. Answer any *one* question : 10 × 1

(*a*) What do you mean by consumer price index number ? Describe the major steps in the construction of consumer price index number. 2 + 6 + 2 (*b*) What are the different methods for determining the trend of time series ? Describe how an exponential trend can be fitted to a time series data. 2 + 8
