UG/II/PHS/H/III/17(New)

2017

PHYSICS

[Honours]

PAPER – III

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

[NEW SYLLABUS]

GROUP – A

Answer any **two** questions :

1. (*a*) What does the statement : $f(z)$ is analytic at a point $z = a$ mean ? What is Cauchy-Riemann condition ? Test the following functions for analyticity :
$f(z) = z^2$ and $f(z) = |z|^2$. $2 + 2 + (2 + 2)$

(*b*) State Dirichlet's conditions and obtain the Fourier series for $f(x) = |x|$, $-\pi/2 < x < \pi/2$. 1 + 6

2. A series combination of an inductance L and a resistance R is connected in parallel with a lossless capacitor of capacity C. An alternating e.m.f. $V = V_0 e^{jwt}$ is applied across the combination. Find. 6 + 2 + 2 + 2 + 2 + 1

(*a*) The complex impedance at w.

(*b*) The resonant frequency w_0 and the impedance at resonance.

(*c*) The Q-value for the ckt.

(*d*) The power factor of the ckt at any w and w_0.

3. (*a*) Calculate the de Broglie wavelength of an electron passing through a potential difference of 1 Giga Volt. 2

(*b*) Due to Compton effect a 12 KeV photon is scattered at an of 60°. Find the energy of the recoil electron. (Deduce the formula you use, mass of the electron 0·5 MeV). 5

(*c*) Using uncertanity principle find out the ground state energy of a linear harmonic oscillator. 4

(*d*) Find out the eigen functions and eigen values of the operator $x + \frac{d}{dx}$. $2 + 2$

4. (*a*) What are the speed, direction of propagation and polarisation of an e.m. wave whose electric field is given by

$$E_x = 4\,E_0 \cos(3x + 4y - 500t)$$
$$E_y = 3\,E_0 \cos(3x + 4y - 500t + \pi)$$
$$E_z = 0. \qquad 2 + 1\frac{1}{2} + 1\frac{1}{2}$$

(*b*) State and establish Poynting's theorem. Show that this theorem can be expressed in the form

$$\frac{du}{dt} + \overline{\nabla} \cdot \overline{S} = 0,$$

where $S = \frac{1}{\mu_0}(\overline{E} \times \overline{B})$ and u is the total energy density. Give an interpretation of $\overline{S}$ by compairing the above equation with the equation of continuity. 6

(*c*) Calculate the following quantities for a plane wave travelling in vacuum and having an electric field amplitude $E_0 = 50\ \mu V/m$.

(*i*) Average energy density in the wave.

(*ii*) Average value of the Poynting vector. $2\frac{1}{2} + 1\frac{1}{2}$

GROUP – B

Answer any **five** questions :

5. (*a*) Show that

$$H_n'(x) = 2nH_{n-1}(x).$$

$H_n(x)$ are Hermite polynomial. 4

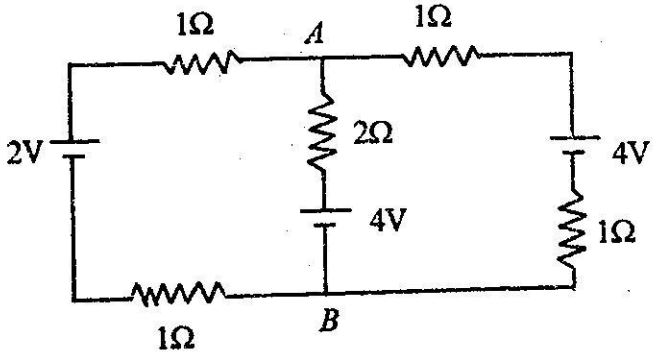
(*b*) Prove that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n(x) H_m(x) dx = 0, \quad m \neq n. \qquad 4$$

6. (*a*) Show that four dimensional volume element $dxdydzdt$ is invariant under Lorentz Transformation. 4

(*b*) A certain particle has life time 10^{-6} sec, when measured at rest. How far does it go before decaying if it's speed is 0·99C, when it is created ? 4

7. (*a*) Find the currents in the three branches and p.d. across AB. 4 + 1



(*b*) Prove that for two coils of self inductance L_1 and L_2 the mutual inductance $M \leq \sqrt{L_1 L_2}$. 3

8. (*a*) Given

$A = \begin{bmatrix} 1 & -2 \\ -2 & -2 \end{bmatrix}$. Determine the eigenvalues and eigenvectors of A. Find a similarity transformation T which diagonalises A. 1 + 2 + 3

(*b*) Show eigen value of a Hermitian matrix are all real. 2

9. (*a*) State Residue theorem. 2

(*b*) Evaluate

$$\int_C \tan z \quad \text{where } C : |z| = 2.$$ 6

10. Define charge sensitivity and current sensitivity of a moving coil ballistic glavanometer. Derive an expression for charge sensitivity.

$\left[\text{where } q = \frac{T}{2\pi} \frac{C}{BNA} \phi_0\right]$. What is the relation between charge and current sensitivity. 2 + 2 + 3 + 1

(*Continued*)

11. (*a*) An EM wave is passing from one linear dielectric medium to another region where there is no free charge or current. Write down the boundary conditions for the electric and magnetic field. 3

(*b*) If the wave is incident normally at the interface between two dielectric, find the amplitudes of the reflected and transmitted waves in terms of the amplitude of the electric field of the incident wave. 5

12. (*a*) Write down the Schrödinger equation for a particle in one dimensional infinite potential well of length L. 2

(*b*) Find out wave functions and energy eigenvalues. 3

(*c*) If

$$\psi(x) = \frac{1}{\sqrt{5}}\left[|\psi_1\rangle + 2|\psi_1\rangle\right]$$

where $|\psi_1\rangle$ and $|\psi_2\rangle$ denote the ground state and the first excited state wave functions. Find the expectation of '$\hat{x}$' in this state. 3

GROUP – C

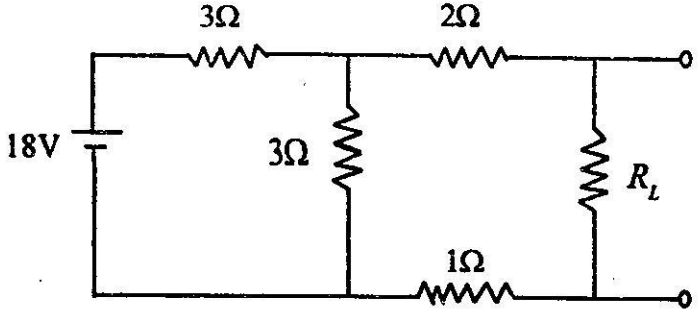
Answer any **five** questions :

13. Two men of rest masses 60 kg and 70kg respectively approach each other with speeds relative to the earth 0·8C. What mass does one find for the other ? 4

14. (*a*) Why is the rising and setting sun coloured crimson red ?

    (*b*) 'If the earth has no atmosphere, the sky would appear as black in day time" –Explain. 2 + 2

15. Find the value of R_L such that maximum power is delivered to it. Also find the value of maximum power. 4



(*Continued*)

16. Which property of an e.m. wave remains unchanged after reflection and refraction ? Justify your answer mathematically. 4

17. (*a*) A charged capacitor *C* is suddenly connected across a resistor *R*. Show that the energy which was stored in the capacitor is now entirely dissipated in *R*. 2

(*b*) What are normal and anomalous dispersion ? 2

18. What is normal distribution ? Give its standard form. Find the second moment of this distribution. 1 + 3

19. (*a*) Established the relation between phase velocity and group velocity.

(*b*) Show that the phase velocity of de Broglie waves is greater the velocity of light. 2 + 2

20. What is a mixed tensor of second rank ? Prove σ_p^q is a mixed tensor of the second rank. Define a metric or fundamental tensor. 1 + 2 + 1
