

**NEW**  
**Part-III 3-Tier**  
**2017**  
**MATHEMATICS**  
**PAPER—IV**  
**(General)**

*Full Marks : 90*

*Time : 3 Hours*

*The figures in the right-hand margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

**Group—A**

**(Compulsory)**

**(Elements of Computer Science)**

**[Marks : 45]**

1. Answer any one question : 1×15

(a) (i) What is Boolean algebra ?

In a Boolean algebra B, prove that  $x + x = x$  and  $x \cdot x = x$  for  $x \in B$ . 5

*(Turn Over)*

(ii) Explain different types of IF statements used in FORTRAN with flowchart. 5

(iii) Draw a flowchart to find the sum of the infinite series of  $\sin x$ . 5

(b) (i) Show that the power set  $P(S)$ , of a set  $S$  is a Boolean algebra under set operations, where  $0$  and  $1$  are respectively the null set and the universal set. 5

(ii) Represent the logic circuit of the following logic gates in terms of three basic gates including algebraic function and truth tables : NOR, NAND and XOR. 5

(iii) Draw a flowchart to find the value of  $n!$  5

2. Answer any *two* questions : 2×8

(a) (i) What do you mean by source program and object program ? How a source program is converted to an object program ? 2+2

(ii) Write short notes on :

(i) READ statement.

(ii) WRITE statement. 2+2

- (b) (i) What is an array ? Explain the features of an array and its uses. 4

(ii) Write short notes on DO loop in FORTRAN. 4

- (c) (i) Prove that the De Morgan's laws in a Boolean algebra  $(B, +, \cdot, 0, 1)$ . 4

(ii) Express  $(x_0 + y)(x + y + z')(y + z)$  as a function in conjunctive normal form. 4

3. Answer any *three* questions : 3×4

- (a) Express the following algebraic expressions into their equivalent FORTRAN expressions :

(i)  $\frac{x^3}{\sin(x+y)} + \sqrt{x^3 - y^3} + e^{(x-|x|)}$

(ii)  $A = \log_e \sqrt{a^2 + b^2} \sin \left[ \cos^{-1} \left( \frac{b}{a} \right) \right]$ . 2+2

- (b) Write the printed form of the output for the following statements :

PRINT 9, I, M, A, B  
9 FORMAT (5x, I<sub>2</sub>, 2x, I<sub>4</sub> // F<sub>6.2</sub>, 5x, E<sub>15.5</sub>)  
where I = -4, M = 2, A = -4.1, B = -0.00142.

4

- (c) Write a FORTRAN program segment to evaluate and print values of  $f(x)$  for  $x = 0.1$  to  $1.0$  with spacing  $h = 0.1$ , where

$$f(x) = \begin{cases} \sqrt{4x^2 + 1}, & \text{for } 0 \leq x < 0.5 \\ \sqrt{4x^2 - 1}, & \text{for } 0.5 \leq x \leq 1.0 \end{cases} \quad 4$$

- (d) Find the followings :

(i)  $(1101)_2 \times (10.1)_2$

(ii)  $(111011)_2 - (10011.11)_2 \quad 2+2$

- (e) Write a FORTRAN program to compute Greatest Common Divisor (GCD) of two given integers. 4

4. Answer any one question : 1×2

- (a) State the principle of duality in Boolean algebra. 2

- (b) What do you mean by the term "Non-executable" statement. 2

**Group—B**

*(Probability and Statistics)*

[Marks : 45]

**5. Answer any one question :**

1×15

(a) (i) State and prove Bayes' theorem.

5

(ii) Give the classical definition of probability of an event and criticize the main drawbacks of the classical theory of probability.

5

(iii) There are two identical urns containing 4 white, 3 red balls and 3 white, 7 red balls respectively. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball drawn is white. If the ball drawn is white, what is the probability that it was drawn from the first urn?

5

(b) (i) Determine the value of  $k$  such that  $f(x)$  defined by

$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function. Find the corresponding distribution function.

5

(ii) Show that poisson distribution is a limit of the Binomial distribution under certain condition to be stated by you. 5

(iii) Define "Skewness" and "Kurtosis". Explain their significance with diagram. 5

6. Answer any *three* questions : 3~8

(a) The pdf of a random variable  $x$  is given by :

$$f(x) = \frac{e^2}{\sqrt{\pi}} e^{-(x^2+2x+3)}, \quad -\infty < x < \infty.$$

Find the value of the expectation and variance of the distribution. 4+4

(b) Given  $x = 4y + 5$  and  $y = kx + 4$  are the regression lines of  $x$  on  $y$  and  $y$  on  $x$  respectively.

Show that  $0 \leq k \leq \frac{1}{4}$ .

If  $k = \frac{1}{16}$ , find the means of the variables and the correlation coefficient between them. 4+4

- (c) The expenditure of 1000 families is given below :

Expenditure (Rs.) :	40-59	60-79	80-99	100-119	120-139
Frequency :	50	?	500	?	50

The mean and median for the distribution are Rs. 87.50 (same). Calculate the missing frequencies.

4+4

- (d) If  $\theta$  be the angle between two regression lines, then

$$\tan \theta = \pm \frac{1-r^2}{r} \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}, \text{ where } r \text{ is the correlation}$$

coefficient of variable  $x$  and  $y$ . Interpret the cases when  $r = 0$  and  $r = \pm 1$ .

6+2

- (e) The runs scored by cricketers  $X$  and  $Y$  during 8 consecutive innings are follows :

X :	32	28	47	63	71	39	10	60
Y :	29	31	48	53	67	90	10	62

Find which of the batsman is more consistent in obtaining runs ?

8

7. Answer any *two* questions :

2×3

(a) Define compound event and mutually exclusive event.

3

(b) If  $f(x, y) = e^{-(x+y)}$  ;  $x \geq 0, y \geq 0$   
 $= 0$  , elsewhere

is a joint probability density function of random variables X and Y, find  $P(X < 1)$ .

3

(c) Show that the standard deviation is independent of origin but dependent on scale of measurement.

3