NEW

2017

Part-II 3-Tier

MATHEMATICS

(General)

PAPER—III

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(*Linear Programming Problem*)

[Marks : 35]

1. Answer any *one* question : 15×1

(a) (i) At a cattle breeding firm it is prescribed that the food ration for one animal must contain at least 14, 22 and 11 units of nutrients A, B and C respectively. Two different kinds of fodder are available. Each unit weight of these two contains the following amounts of the three nutrients.

(Turn Over)

Nutrient	Fodder–I	Fodder–II
A	2	1
B	2	3
C	1	1

It is given that the cost of unit quantity of fodder-I and II are 3 and 2 monetory units respectively. Formulate this as a linear programming problem.

5

(ii) For the set of equations $2x_1 + x_2 + 4x_3 = 11$, $3x_1 + x_2 + 5x_3 = 14$, show that $x_1 = 2, x_2 = 3, x_3 = 1$ is a solution of the above set of equations. Is the above solution basic ? Find another basic feasible solutions.

(iii) Prove that the set of all feasible solutions of a linear programming problem is a convex set. 5

(b) (i) Solve the following L.P.P graphically :

Minimize $Z = 20x_1 + 10x_2$

Subject to
$$x_1 + 2x_2 \leq 40$$
$$3x_1 + x_2 \geq 30$$
$$4x_1 + 3x_2 \geq 60$$
$$x_1, x_2 \geq 0$$
5

(ii) Prove that the objective function has its optinal value at an extreme point of the convex polyhedron generated by the set of feasible solutions of the LPP.

5

(iii) Examine whether the set $S = \{(x, y) : x + 2y \leq 5\}$ is convex or not. Find the extreme points, if any, of the following set :

$$T = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 1,\ x_1 \geq 0,\ x_2 \geq 0\}$$

3+2

2. Answer any *two* questions : 8×2

(a) Find the optimal solution of the following transportation problem : 8

	D_1	D_2	D_3	D_4	a_i
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
b_j	4	3	4	4	

(b) A company has four machines on which to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table. What are the job assignment which will minimize the cost ? 8

MACHINES

		W	X	Y	Z
	A	18	24	28	32
JOBS	B	8	13	17	19
	C	10	15	19	22

(c) Find the optional solution of the following LPP by solving its dual :

$$\text{Minimize} \quad Z = 3x_1 + 2x_2$$

$$\text{Subject to} \quad 7x_1 + 2x_2 \geq 30$$

$$5x_1 + 4x_2 \geq 20$$

$$2x_1 + 8x_2 \geq 16$$

$$\text{and} \quad x_1, x_2 \geq 0$$

8

3. Answer any *one* question : 4×1

(a) Show that any point of a convex polyhedron can be expressed as a convex combination of its extreme points. 4

(b) Find the basis of E^3 that contains the vectors (1, 2, 0) and (2, 0, 1). 4

Group—B

(*Numerical Analysis*)

[Marks : 20]

4. Answer any *one* question : 8×1

(a) Explain the method of fixed point iteration for numerical solution of the equation of the form $x = \phi(x)$. Derive the condition of convergence. Write down the advantages and disadvantages of the method. 6+2

(b) By integrating the Lagrangian three point formula when the abscissa are at equal specing h with the origin taken at the central point prove that

$$\int_{-h}^{x} f(x)dx =$$

$$\frac{5h^3 - 3hx^2 + 2x^3}{12h^2} f(-h) + \frac{2h^3 + 3h^2x - x^3}{3h^2} f(0)$$

$$-\frac{h^3 - 2hx^2 - 2x^3}{12h^2} f(h) + E(x) \text{ where}$$

$$E(x) = \int_{-h}^{x} x\left(x^2 - h^2\right) f[-h,\ 0,\ h,\ x]\, dx .$$ 8

5. Answer any *three* questions : 4×3

(a) Solve the system of following linear equations : $x_1 + x_2 + 4x_3 = 9$, $8x_1 - 3x_2 + 2x_3 = 20$, $4x_1 + 11x_2 - x_3 = 33$, upto three significant figures by Gauss-Seidal iteration method. 4

(b) (i) Prove that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$.

(ii) Evaluate $\left(\frac{\Delta^2}{E}\right)x^4$, spacing being one. 2+2

(c) Using Newton's divided difference formula compute $f'(6)$ and $f''(6)$ from the following data :

x :	4	5	7	10	11
y :	48	100	294	900	1210

4

(d) Obtain the Simpson's $\frac{1}{3}$ rule from Newton Cotes' numerical integration formula. 4

(e) Evaluate $\int_0^1 \frac{dx}{1+x}$ by trapezoidal rule with $h = 0.25$. Also, calculate the absolute error. 4

Group—C

(Analytical Dynamics)

[Marks : 35]

6. Answer any *one* question : 15×1

(a) (i) A particle moves from rest in a straight line under an attractive force $\mu \times (\text{distance})^{-2}$ per unit mass to a fixed point on the line. Show that if the initial distance from the centre of force be '2a', then the distance will be 'a' after a time

$$\left(\frac{\pi}{2}+1\right)\left(\frac{a^3}{\mu}\right)^{\frac{1}{2}}.$$

7

(ii) A particle starts from the origin in the direction of the initial line with velocity $\frac{f}{w}$ and moves with constant angular velocity w about the origin under a constant negative radial acceleration –f. Show that the rate of growth of the radial velocity is never positive but tends to a limit zero and prove that the equation of the path is $w^2 r = f(1 - e^{-\theta})$. 8

(b) (i) A particle of mass m is falling under the influence of gravity through a medium whose resistance is equal to μ times the velocity. If the particle was released from rest, show that the distance fallen through in time t is $\frac{gm^2}{\mu^2}\left[e^{-\frac{\mu t}{m}} + \frac{\mu t}{m} - 1\right]$. 7

(ii) A particle moving in a plane has besides the central acceleration P, an acceleration T perpendicular to P, show that with usual notations, the equation to the path is given by $\dfrac{\dfrac{P}{u^2} - \dfrac{T}{u^3}\dfrac{du}{d\theta}}{\dfrac{d^2u}{d\theta^2} + u} = \int \dfrac{2T}{u^3}\,d\theta + C$, for arbitrary constant C. 8

7. Answer any *two* questions : 8×2

(a) A particle describes the catenary $y = c \cosh \frac{x}{c}$ under a force which is always parallel to the positive direction of y axis. Find the law of force and the velocity at any point of the path. 8

(b) Three perfectly elastic balls of masses m_1, m_2, m_3 are placed in a straight line. The first impinges directly on the second with a velocity u and then the second impinges the third. Show that the third ball will move after impact with velocity u if $(m_1 + m_2)(m_2 + m_3) = 4m_1m_2$.

(c) If a particle describes the cycloid $s = 4a\sin\psi$ with uniform speed v. Find its acceleration at any point.

8. Answer any *one* question : 1×4

(a) If the position of a moving point is given by $x = 2t^2 + 1$ and $y = 3t$, find its path, velocity and acceleration. 4

(b) A bullet of mass m moving with a velocity v, strikes a block of mass M, which is free to move in the direction of motion of the bullet and is embedded in it. Show that the loss of K.E. is

$$\frac{1}{2}\frac{mM}{M+m}v^2.$$ 4