OLD

2017

Part-II 3-Tier

MATHEMATICS

(General)

PAPER—III

Full Marks : 90

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group—A

(Linear Programming)

[Marks : 36]

1. Answer any *one* question : 1×15

(a) (i) Old hens can be bought for Rs. 5 each but young one costs Rs. 12 each. The old hens lay 3 eggs per week and young ones 5 eggs per week, each egg being worth Re. 1. A hen costs Rs. 2 per week to feed.

A person has Rs. 200 to spend for purchasing the hens.

Obtain the linear programming model of the problem to purchase the number of hens of each category so that the profit is maximized. 5

(ii) Show that although (2, 3, 2) is a feasible solution to the system of equations

$$x_1 + x_2 + 2x_3 = 9,$$
$$3x_1 + 2x_2 + 5x_3 = 22,$$
$$x_1, x_2, x_3 \geq 0,$$

it is not a basic solution. How many basic solution of this system may have ? Find all the basic feasible solutions of the given system. 5

(iii) Solve the L.P.P by simplex method

Maximize $Z = 2x_1 - 3x_2$

Subject to $-x_1 + x_2 \geq -2$

$5x_1 + 4x_2 \leq 46$

$7x_1 + 2x_2 \geq 32$

$x_1, x_2 \geq 0.$ 5

(b) (i) Solve the following L.P.P graphically

Maximize $Z = 10x_1 + 15x_2$

Subject to $x_1 + x_2 = 2$

$3x_1 + x_2 \leq 6$

$x_1 \geq 0, x_2 \geq 0.$ 5

(ii) $x_1 = 1$, $x_2 = 2$, $x_3 = 1$ is a feasible solution of the following set of linearly independent equations

$$2x_1 + 3x_2 + 5x_3 = 13$$
$$3x_1 - x_2 + 3x_3 = 4$$

Reduce the feasible solution to a basic feasible solution. 5

(iii) Find the extreme points, *f* any, of the set $S = \{(x, y) : x^2 + y^2 \leq 25\}$. Prove that in E^2, the set $X = \{(x_1, x_2) : x_2^2 \geq 4x_1\}$ is not a convex set. 2+3

2. Answer any *two* questions : 8×2

(a) Find the optimal solution of the following L.P.P, by solving its dual :

$$\text{Maximize } Z = 10x_1 + 40x_2$$
$$\text{Subject to } x_1 + 2x_2 \geq 2$$
$$x_1 + x_2 \leq -1$$
$$x_1, x_2 \geq 0$$

8

(b) Find the optimal solution of the following transportation problem : 8

	D_1	D_2	D_3	D_4	Capacities
O_1	5	4	6	14	15
O_2	2	9	9	6	4
O_3	6	11	7	13	8
Demands	9	7	5	6	

(c) Consider the following problem of assigning four operators to four machines the assignment costs in rupees are given below. Find the optimal assignment and cost of assignment. 8

Operators	MACHINES 1	2	3	4
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

3. Answer any *one* question : 1×3

(a) Find the dual of the following L.P.P,

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to

$$x_1 - 5x_2 + 3x_3 = 7$$
$$2x_1 - 5x_2 \leq 3$$
$$3x_1 - x_3 \geq 5$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

(b) If the objective function of a L.P.P assumes its optimal value at more than one extreme point, then prove that every convex combination of these extreme points also gives the optimal value of the objective function.

4. Answer any *one* question ; 1×2

(a) What is the difference between convex null and convex polyhedron.

(b) Show that the transportation problem always has a feasible solution.

Group—B

(Numerical Analysis)

[Marks : 18]

5. Answer any *two* questions : 2×8

(a) Compute the value of x for $y = 0.6742$ from the table

x :	3.5	4.0	4.8	5.6
y :	0.5441	0.6020	0.6812	0.7482

8

(b) Derive the formula of Newton-Raphson method for computing a simple real root of an equation $f(x) = 0$. Also, derive the condition of convergence and give a geometrical interpretation of this method. 8

(c) Compute the integral $\int_0^1 \frac{dx}{1+x^2}$, by Simpson's $\frac{1}{3}$ rule and then use it to compute the value of π correct to four decimal places. 8

6. Answer any *one* question : 1×2

(a) Taking $h = 2$ what will be the value of

$$\Delta^{10}[(1 - ax)(1 - bx^2)(1 - cx^3)(1 - dx^4)] ?$$ 2

(b) Round off the following number's to three decimal places :

0.20093, 32.00995, 5.0565, 0.99356 2

Group—C

(*Analytical Dynamics*)

[Marks : 36]

7. Answer any *one* question : 1×15

(a) (i) A particle moves from rest in a straight line unde· an attractive force μx (distance)$^{-2}$ per unit mass to fixed point on the line. Shat that if the initial distance from the centre of force be 2a, then the distance will be a after a time

$$\left(\frac{\pi}{2}+1\right)\left(\frac{a^3}{\mu}\right)^{\frac{1}{2}}.$$ 7

(ii) A smooth tube of length $2a$ is capable of turning in a horizontal plane about an extremity and has a particle of mass m inside it. If initially the particle is at rest at the middle point of the tube, and the tube rotates with a uniform angular velocity w, show that the pressure of the tube on the particle when it is at a distance r from the fixed end is $2mw^2\sqrt{r^2-a^2}$. Find also the time when it emerges out of the tube. 8

(b) (i) A particle is projected from an apse at a distance 'a' under the law of force $\frac{\mu}{r^5}$. If the velocity of projection be $\sqrt{\frac{\mu}{2}}\cdot\frac{1}{a^2}$. Show that the orbit described is $r = a\cos\theta$. 7

(ii) A particle moves under a repulsive force $m\mu \times (\text{distance})^{-3}$ and is projected from an apse at a distance a with velocity V; show that the equation of the path is $r \cos p\theta = a$ and that the angle θ described in time t is $\frac{1}{p}\tan^{-1}\left(\frac{pV}{a}t\right)$ where

$$p^2 = \frac{\mu + a^2V^2}{a^2V^2}.$$ 8

8. Answer any *two* questions : 2×8

(a) A particle is projected vertically upwards with a velocity v, and the resistance of the air produces a retardation kv^2, where v is the velocity. Show that the velocity U with which the particle will return to the point of projection is given by $\frac{1}{U^2} = \frac{1}{V^2} + \frac{k}{g}$. Find also the greatest height attained by the particle. 8

(b) An engine working at a constant rate H draws a load of mass M against a resistance R. Show that the maximum speed attained is H/R and the time taken to attain half this speed is $\frac{MH}{R^2}\left[\log 2 - \frac{1}{2}\right]$. 8

(c) A ball falls from a height h upon a fixed horizontal plane. If e be the co-efficient of restitution, show that the whole distance described before the ball has finished rebounding is $h(1 + e^2) / (1 - e^2)$, and that the whole time taken is $\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$. 8

9. Answer any *one* question : 1×3

(a) State Kepler's laws of planetary motion.

(b) A bullet of mass m moving with a velocity u, strikes a block of mass M, which is free to more in the direction of the motion of the bullet and is embedded in it. Show that the loss of K.E. is

$$\frac{1}{2}\cdot\frac{mM}{m+n}\cdot u^2$$ 3

10. Answer any *one* question : 1×2

(a) The law of motion in a straight line being $x = \frac{1}{2}vt$; show that the acceleration is constant.

(b) Define escape velocity. Calculate the escape velocity at an attitude of 900 km above the surface of the earth. [Radius of earth = 6400 km, mass of earth = 6×10^{27} gm, $G = 6.66 \times 10^{-8}$ C.G.S. unit]

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