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A Variant of the Complementary Pivot Algorithm for Solving the Linear Complementarity Problem under Fuzzy Environment

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ABSTRACT

In this paper, a variant of the complementary pivot algorithm is proposed to solve the fuzzy linear complementarity problems (FLCP). Here, the cost coefficients, constraint coefficients and the right hand side coefficients are represented by triangular fuzzy numbers. The effectiveness of the proposed methods is illustrated by means of a numerical example.

Keywords: fuzzy linear complementarity problem, triangular fuzzy numbers, variant of the complementary pivot algorithm.

1. Introduction

In traditional mathematical programming, the coefficients of the problems are always treated as deterministic values. However uncertainity always exists in practical engineering problem. In order to deal with the uncertain optimization problems, fuzzy and stochastic approaches are commonly used to describe the imprecise characteristics. In stochastic programming (e.g. Charnes and Cooper, 1959; Kall,1982; Liu et al., 2003; Cho 2005) the uncertain coefficients are regarded as random variables and their probability distributions are assumed to be known.In fuzzy programming (e.g.Slowinski,1986; Delgado et al., 1989; Luhandjula,1989; Liu and Iwamura, 2001) the constraints and objective function are viewed as fuzzy sets and their membership functions also need to be known. In these two kinds of approaches, the membership functions and probability distributions play important roles. However, it is sometimes difficult to specify an appropriate membership function or accurate probability distribution in an uncertain environment.

Many practical problems cannot be represented by Linear Programming Model. Therefore, attempts were made to develop more general Mathematical Programming Methods and many significant advances have been made in the area of non-linear programming.The Linear Complementarity Problem (LCP) is a general problem which unifies linear, quadratic programs and bimatrix games. The study of LCP has led to many far reaching benefits. An algorithm known as the complementarity pivot algorithm first developed for solving the LCPs, has been generalized in a direct manner to yield efficient algorithms for computing Brouwner and Kakutani fixed points, for computing economic equilibria, and for solving systems of nonlinear equations and nonlinear programming problems.Also, iterative methods developed for solving LCPs hold great promise for handling very large scale linear programs which cannot tackled with the well known simplex method because of their large size and the consequent numerical difficulties.In 1968, Lemke proposed a complementarity pivoting algorithm for solving complementarity problems.In 2012 [10], Nagoor Gani, Kumar, proposed an index method for solving the linear complementarity problem under fuzzy environment.In this paper, we proposed a variant of the complementary pivot algorithm for solving the linear complementarity problem under fuzzy environment.

This paper is organized as follows: Section 2 provides some basic idea about the triangular fuzzy number and its arithmetic operation.Linear Complementarity Problem is described in section 3.Fuzzy Linear Complementarity Problem is described in section 4.Section 5 deals with a variant of the complementary pivot algorithm for solving the linear complementarity problem under fuzzy environment.In section 6, the effectiveness of the proposed method is illustrated by an example. The results are tabulated and it is inferred that when $\tilde{d}$ is small,we get an exact solution.

2. Preliminaries

Fuzzy set 2.1. A fuzzy set $\tilde{A}$ is defined by

$$\tilde{A} = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}.$$

In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval [0, 1], called Membership function.

Interval number 2.2. If $\bar{A}$ is a triangular fuzzy number, we will let $\bar{A}\alpha = [A\alpha\text{-}\ A\alpha\text{+}]$ be the closed interval which is a α – cut for $\bar{A}$ where $A\alpha$-and $A\alpha$+are its left and right end points respectively. Let I and J be two interval numbers defined by ordered pairs of real number with lower and upper bounds. I = [a, b], where $a \le b$, J = [c, d] where $c \le d$, when a = b, and c = d, these interval numbers degenerate to a scalar real number.

Arithmetic operations on Interval numbers 2.3. The Arithmetic operations on I and J are given below

1. Addition : I + J = [a, b] + [c, d] = [a+b, c+d] where a, b, c and d are any real numbers.

2. Subtraction : I - J = [a, b] - [c, d] = [a - d, b - c] where a, b, c and d are any real numbers.

3. Multiplication : I . J = [a, b] . [c, d] = [min(ac, ad, bc, bd), max(ac, ad, bc, bd)] where ac, ad, bc, bd are all arithmetic products.

4. Division : $I/J = \frac{[a,\ b]}{[c,\ d]}$ = [a, b] . $[\frac{1}{d}, \frac{1}{c}]$, provided 0 ∉ [c, d] where $\frac{1}{d}$ and $\frac{1}{c}$ are quotients.

Fuzzy number 2.4. The notion of fuzzy numbers was introduced by Dubois and Prade fuzzy subset $\tilde{A}$ of the real lineR with membership function $\mu_{\tilde{A}}$:R→[0,1] is called a fuzzy number if

i) A fuzzy set $\tilde{A}$ is normal.

ii) $\tilde{A}$ is fuzzy convex,
(i.e.)$\mu_{\tilde{A}}[\lambda x_1+(1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2), x_1, x_2 \in R, \forall \lambda \in [0,1]$.

iii) $\mu_{\tilde{A}}$ is upper continuous,and

iv) Supp$\tilde{A}$ is bounded,where supp $\tilde{A}$ ={x∈R:$\mu_{\tilde{A}}$(x)>0}.

Triangular fuzzy number 2.5. It is a fuzzy number represented with three points as follows : Ã = (a_1, a_2, a_3). This representation in interpreted as membership functions

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & for\ x > a_3 \end{cases}$$

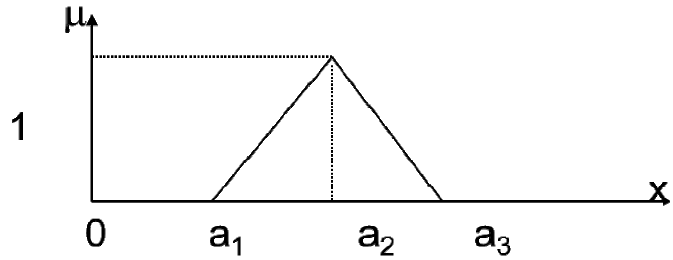

Figure 1: Triangular fuzzy number

Definition 2.5. A fuzzy number Ã= (a, b, c, d :ω) is said to be a ω −Trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}} = \begin{cases} \omega\left(\frac{x-a}{b-a}\right), & \text{if } a \le x \le b \\ \omega, & \text{if } b \le x \le c \\ \omega\left(\frac{d-x}{d-c}\right), & \text{if } c \le x \le d \\ 0, & \text{if } x \ge d \end{cases}$$

where $\omega \epsilon$ (0,1).

Operation of triangular fuzzy number using function principle 2.6.

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then

1. The addition of $\tilde{A}$ and $\tilde{B}$ is

   $\tilde{A} + \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

2. The product of $\tilde{A}$ and $\tilde{B}$ is

   $\tilde{A}$ x $\tilde{B} = (c_1, c_2, c_3)$, where $T = \{a_1b_1, a_2b_2, a_3b_3\}$ where $c_1 = \min\{T\}$, $c_2 = a_2b_2$, $c_3 = \max\{T\}$. If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non – zero positive real numbers, then

   $\tilde{A}$ x$\tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$

3. - $\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is

   $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are real numbers.

4. The division of $\tilde{A}$ and $\tilde{B}$ is

   $\frac{\tilde{A}}{\tilde{B}} = (c_1, c_2, c_3)$, where $T = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$, where $c_1 = \min\{T\}$, $c_2 = \frac{a_2}{b_2}$, $c_3 = \max\{T\}$.

   If $a_1, a_2, a_3, b_1, b_2, b_3$ are all non-zero positive real numbers then

   $\frac{\tilde{A}}{\tilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$.

L1 – Matrix 2.7. If for every $\tilde{y} \ge 0$, $\tilde{y} \in$ Rn, there is an i such that $\tilde{y}$ i > 0 and Mi.$\tilde{y} \ge 0$. If M is an L1- matrix, an i like it is called a defining index for M and $\tilde{y}$. These matrices are also called semimonotone matrices.

3. Linear complementarity problem

Given the nxn matrix M and the n – dimensional vector q, the Linear Complementarity Problem (LCP) consists in finding non – negative vectors w and z which satisfy,

$$w - Mz = q \qquad (1)$$

$$w_i, z_i \ge 0, \ \text{for } i = 1, 2, \ldots, n \qquad (2)$$

$$\text{and } w_iz_i = 0, \ \text{for } i = 1, 2, \ldots, n \qquad (3)$$

Given the non – negativity of the vectors w and z, (3) requires that $w_iz_i = 0$ for i = 1, 2, . .,. n.

Two such vectors are said to be complementarity. A solution (w, z) to the LCP is called a complementarity feasible solution, if it is a basic feasible solution to (1) and (2) with one of the pairs (w_i, z_i) is basic

4. Fuzzy linear complementarity problem

Assume that all parameters in (1.1) – (1.3) are fuzzy and are described by fuzzy numbers. Then the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with fuzzy number

$$\tilde{w} - \tilde{M}\tilde{z} = \tilde{q} \tag{4}$$

$$\tilde{w}_i, \tilde{z}_i \geq 0, \text{ for } i = 1,2, \ldots\ldots, n.$$

$$\text{and } \tilde{w}_i\tilde{z}_i = 0, \text{ for } i = 1,2, \ldots\ldots, n. \tag{6}$$

The pair ($\tilde{w}_i, \tilde{z}_i$) is said to be a pair of fuzzy complementarity variables.

5. A variant of the complementary pivot algorithm

Step 1 : Introduce the fuzzy artificial variable $\tilde{z}_0$ for the purpose of obtaining a feasible basis

Step 2 : Given a column vector$\tilde{d} \in R^n$ satisfying $\tilde{d}$> 0, clearly we can choose the original column vector associated with $\tilde{z}_0$ to be -$\tilde{d}$.The original tableau turns out to be

$\tilde{w}$	$\tilde{z}$	$\tilde{z}_0$	
I	-$\tilde{M}$	-$\tilde{d}$	$\tilde{q}$

Table 1:

Step 3 : If $\tilde{q} \geq 0$, ($\tilde{w} = \tilde{q}$, $\tilde{z} = 0$) is a solution of the FLCP ($\tilde{q}$, $\tilde{M}$) and we are done. So assume $\tilde{q} \ngeq 0$.

Step 4 : To find leaving variable : Determine$\tilde{t}$ to satisfy $(\frac{\tilde{q}_t}{\tilde{d}_t})$= Minimum {$(\frac{\tilde{q}_i}{\tilde{d}_i})$: i = 1 to n}. Ties for$\tilde{t}$can be broken arbitrarily.

Step 5: If a pivot step is performed, with the column vector of $\tilde{z}_0$ as the pivot column and the$\tilde{t}$th row as the pivot row; the RHS constants vector becomes non – negative after the pivot step. So ($\tilde{w}_1$, $\tilde{w}_2$, . . . ,$\tilde{w}_{t-1}$,$\tilde{z}_0$, $\tilde{w}_{t+1}$, . . . , $\tilde{w}_n$) is a feasible basic vector. It is an almost complementary feasible basic vector.

Step 6: Choose$\tilde{z}_t$ as the entering variable into this initial almost complementary feasible basic vector $(\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_{t-1}, \tilde{z}_0, \widetilde{w}_{t+1}, \ldots, \widetilde{w}_n)$ and continue by choosing entering variables using the complementary pivot rule.

Step 7: Suppose all the elements of column vector of $\tilde{z}_t$ are negative then the algorithm ends up in ray termination.

Theorem 5.1. If the variant of the complementary pivot algorithm starting with an arbitrary positive vector $\tilde{d}$ for the column of the artificial variable $\tilde{z}_0$ in the original tableau ends up in ray termination when applied on the FLCP $(\tilde{q}, \widetilde{M})$ in which $\widetilde{M}$ is copositive, there exists a$\tilde{z}$ satisfy

$$\begin{aligned} &\widetilde{M}\tilde{z} \geq 0 \\ &\widetilde{q}^T\tilde{z} < 0 \\ &\tilde{z}^T\widetilde{M}\tilde{z} = 0 \\ &\tilde{z} \geq 0 \end{aligned} \tag{7}$$

Proof :
Let the terminal extreme half – line obtained in the algorithm be
$\{(\widetilde{w}, \tilde{z}, \tilde{z}_0) = (\widetilde{w}^k + \lambda\widetilde{w}^h, \tilde{z}^k + \lambda\,\tilde{z}^h, {\tilde{z}_0}^k + \lambda\,{\tilde{z}_0}^h) : \lambda \geq 0\}$
where $(\widetilde{w}^k, \tilde{z}^k, {\tilde{z}_0}^k)$ is the BFS of (1.4) and $(\widetilde{w}^h, \tilde{z}^h, {\tilde{z}_0}^h)$ is a homogeneous solution corresponding to (1.4), that is,

$$\begin{aligned} &\widetilde{w}^h - \widetilde{M}\tilde{z}^h - \tilde{d}{\tilde{z}_0}^h = 0 \\ &\widetilde{w}^h, \tilde{z}^h, {\tilde{z}_0}^h \geq 0 \\ &(\widetilde{w}^h, \tilde{z}^h, {\tilde{z}_0}^h) \neq 0 \end{aligned} \tag{8}$$

and every point on the terminal extreme half – line satisfies the complementarity constraint, that is,

$$(\widetilde{w}^k + \lambda\,\widetilde{w}^h)^{\mathrm{T}}(\tilde{z}^k + \lambda\tilde{z}^h) = 0 \text{ for all } \lambda \geq 0 \tag{9}$$

Clearly$\tilde{z}^h \neq 0$ (otherwise the terminal extreme half line is the initial one, a contradiction) so $\tilde{z}^h \geq 0$.
By complementarity, we have

$$(\widetilde{w}^h)^{\mathrm{T}}\tilde{z}^h = 0,$$

from (2) this implies that

$$(\tilde{z}^h)^T\widetilde{M}\tilde{z}^h = -\tilde{d}^T\tilde{z}^h{\tilde{z}_0}^h \leq 0,$$

(since $\tilde{d} > 0, \tilde{z}^h \geq 0$

implies that $\tilde{d}^T\tilde{z}^h > 0$) which implies by the copositivityof $\widetilde{M}$, that
$(\tilde{z}^h)^T\widetilde{M}\tilde{z}^h = 0$ and ${\tilde{z}_0}^h = 0$.
Using this in (5) we conclude that

$$\widetilde{M}\tilde{z}^h = \widetilde{w}^h \geq 0 \tag{10}$$

Since $(\widetilde{w}^k, \tilde{z}^k, {\tilde{z}_0}^k)$ is a BFS of (3) we have

$$\widetilde{w}^k = \widetilde{M}\tilde{z}^k + \tilde{d}{\tilde{z}_0}^k + \tilde{q}.$$

Using this and (10) in (9) we get, for all $\lambda \geq 0$,

$$(\tilde{z}^k + \lambda\tilde{z}^h)^T\tilde{d}{\tilde{z}_0}^k + (\tilde{z}^k + \lambda\,\tilde{z}^h)^T\tilde{q} = -(\tilde{z}^k + \lambda\tilde{z}^h)^T\widetilde{M}\,(\tilde{z}^k + \lambda\,\tilde{z}^h) \leq 0$$

(Since $\widetilde{M}$ is copositive and $\tilde{z}^k + \lambda\tilde{z}^h \geq 0$). Make $\lambda > 0$, divide this inequality by λ and take the limit as λ tends to $+\infty$. This leads to

$$(\tilde{z}^h)^T\tilde{d}{\tilde{z}_0}^k + (\tilde{z}^h)^T\tilde{q} \leq 0 \tag{11}$$

But ${\tilde{z}_0}^k > 0$ (otherwise $(\widetilde{w}^k, \tilde{z}^k)$ will be a solution to the FLCP $(\tilde{q}, \widetilde{M})$, contradicting the hypothesis that the algorithm terminated with ray termination without leading to a solution of the FLCP), $\tilde{d} > 0, \tilde{z}^h \geq 0$. Using these facts in (11)we conclude that $\tilde{q}^T < 0$. All these facts imply that $\tilde{z}^h = \tilde{z}$ satisfies (7).

6. Numerical example

Example 6.1. Consider the FLCP $(\tilde{q}, \widetilde{M})$ where

$$\widetilde{M} = \begin{pmatrix} (-1.75, -1.5, -1.25) & (1.75, 2, 2.25) \\ (-4.25, -4, -3.75) & (3.75, 4, 4.25) \end{pmatrix}$$

$$\tilde{q} = \begin{pmatrix} (-5.25, -5, -4.75) \\ (16.75, 17, 17.25) \end{pmatrix}$$

Solution :

Let $\tilde{d} = \begin{pmatrix} (0.75, 1, 1.25) \\ (0.75, 1, 1.25) \end{pmatrix}$

From the Table 2 given below, we get
The solution of the FLCP is

$\widetilde{w}_1 = (0, 0, 0)$,$\widetilde{w}_2 = (0, 0, 0)$, $\tilde{z}_1 = (-227.01, 27, 31.75)$ and
$\tilde{z}_2 = (-259.7, 22.75, 22.75)$

$\tilde{z}_2$ $\tilde{z}_1$	$\tilde{z}_0$	$\tilde{z}_0$	$\tilde{w}_1$	**Basic vector**
(-2, -2, 15.5) (-2.25,-2, 1.7)	(-1.75,-,1.55) (-0.45,-,0.39)	(-1, -1, -1) (-1.25,-1,-.75)	(0.75,1, .25) (0, 0, 0)	$\tilde{w}_1$
(-10.5, 0.75, 0.75) (-9.22, 1, 1.35)	(0.38,0.6,1.05) (0.23,0.4,0.75)	(0, 0, 0) (0.75,1,1.25)	(0, 0, 0) (0.75,1,1.25)	$\tilde{w}_2$
(-13,0, 0) (-11.18, 1, 1.93)	(-0.8, 0, 1.3) (0.52, 1, 1.93)	(-1.67,-1.5, -1.4) **(1.66, 2.5, 3.2)**	(1.25,1.5,1.75) (3.75, 4, 4.25)	$\tilde{z}_1$
(-19.1, 1, 1) (-18.09,0,0.02)	**(-0.1,0.8, .91)** (-0.9,-.8,0.78)	(1.4, 2, 3) (-.2.5,-2, -1.5)	(-2.25,-2,-1.75) (-4.25,-4,-3.75)	$\tilde{z}_2$
(-0.9,1.25,1.25) (-18.97, 1, 1.3)	(0.33,1,2.09) (-0.16, 0, 0.3)	(0.6,1,1.67) (-0.5,0,0.5)	**(-1.25,-1,-.75)** (-1.25,-1,-.75)	$\tilde{z}_0$
(-59.7,22.75,22.75) (-227.01,27, 31.75)	(15.02,18.2,25.97) (6.72, 8.8,13.55)	(3.8, 5, 7) (21.5, 22, 22.5)	(-5.25, -5,-4.75) (16.75,17,17.25)	$\tilde{q}$

Table 2: Here the bold values are represented as pivot elements.

From the Table 3 given below, we get

The solution of the FLCP is

$\tilde{w}_1$ = (0, 0, 0) ,$\tilde{w}_2$ = (0, 0, 0), $\tilde{z}_1$ = (-227.01, 27, 31.75) and
$\tilde{z}_2$ = (-259.7, 22.75, 22.75)

The results obtained by different values of$\tilde{d}$:

$\tilde{d}$	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{z}_1$	$\tilde{z}_2$
(i) $\begin{bmatrix}(0.75,1,1.25)\\(0.75,1,1.25)\end{bmatrix}$	(0,0,0)	(0,0,0)	(-227.01,27,31.75)	(-259.7,22.75,22.75)
(ii) $\begin{bmatrix}(1.75,2,2.25)\\(1.75,2,2.25)\end{bmatrix}$	(0,0,0)	(0,0,0)	(15.76,27,75.8)	(10.38,22.75,84.85)
(iii) $\begin{bmatrix}(2.75,3,3.25)\\(2.75,3,3.25)\end{bmatrix}$	(0,0,0)	(0,0,0)	(16.34,26.78,69.65)	(11.08,22.48,77.78)
(iv) $\begin{bmatrix}(3.75,4,4.25)\\(3.75,4,4.25)\end{bmatrix}$	(0,0,0)	(0,0,0)	(16.75,28.57,65.72)	(11.57,24.32,72.71)
(v) $\begin{bmatrix}(4.75,5,5.25)\\(4.75,5,5.25)\end{bmatrix}$	(0,0,0)	(0,0,0)	(17.24,27,70.97)	(12.04,22.75,79.8)
(vi) $\begin{bmatrix}(4.75,5,5.25\\(15.75,16,16.2\end{bmatrix}$				Ray termination

Table 3: If the value of $\hat{d}$ is largest, then the above FLCP ends with in ray termination

7. Conclusion

In this paper,a new approach for solving a fuzzy linear complementarity problem is suggested.The minimum value of $\tilde{d}$ gives an exact solution of the given fuzzy linear complementarity problem.Here,A variant of the complementary pivot algorithm is proposed to solve the given fuzzy linear complementarity problem with triangular fuzzy number.The effectiveness of the proposd method is verified with the different values of $\tilde{d}$.

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