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Nonclassical States of Light in Parametric Down-Conversion Process

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ABSTRACT

We study nonclassical states of light like squeezing and sub-poissonian in spontaneous and stimulated parametric down-conversion process. The amplitude-squared and amplitude-cubed squeezing effects of the radiation field in the fundamental mode are investigated and found to be dependent on the number of pump photons of the system. The photon statistics of the fundamental mode in this process have also been investigated and found to be sub-poissonian in nature. It is shown that the amplitude squeezing and sub-poissonian photon statistics of light occur simultaneously. It is observed that degree of squeezing and occurrence of sub-poissonian is directly associated with photon number in the fundamental mode of the optical field. The dependence of the amplitude squeezing in terms of signal to noise ratio on the number of photons is studied. It is found that maximum signal to noise ratio is possible in lower order i.e. in normal squeezing.

Keywords: Squeezing of radiation; Parametric down-conversion process; Photon number operator; Sub-poissonian photon statistics; Nonclassical light.

1. Introduction

Over the past decades, the squeezing [2,17,18,23,33,34] in quantized electro-magnetic fields has received a great deal of attention because of its wide applications in many branches of science and technology, especially for low noise property [14,30,35] with an application in high quality telecommunication [36], quantum cryptography [1,11], and so forth. The basic concept of squeezed light is concerned with the reduction of quantum fluctuations in one of the quadrature at the expense of increased fluctuations in the other quadrature. Squeezing has been focused on theoretical investigations and experimental observations in a variety of nonlinear optical processes, such as harmonic generation [12,19], multiwave mixing processes [3,4,24,29], Raman [5,15,25], hyper-Raman [16] Hong and Mandel [9,10], Hillery [6-8], and Zhan [38] for improving the performance of many optical devices and optical communication networks. Squeezing and photon statistical effect of the field amplitude has also been reported by Perina [26]. Higher-order sub-poissonian photon statistics of light have also been studied by Kim and Yoon [13]. Recently, Prakash and Mishra [20,27] have reported the higher-order sub-poissonian photon statistics and their use in detection higher-order squeezing. Squeezing and photon statistical effect of the field amplitude in harmonic generation has also been reported by Pratap et.al [28]. More recently, Mukherjee et al. [21] has been reported squeezing and entanglement in quadratically-coupled optomechanical system and squeezing and antibunching in three-mode atom-molecule Bose-Einstein condensates has been studied by Mukhopadhyay et al [22].

The aim of this paper is to extend our study the properties of amplitude squeezing and sub-poissonian states of the electromagnetic field of the fundamental mode in spontaneous and stimulated parametric down-conversion process under short-time approximation. The paper is organized as follows: Section 2 gives the definitions of squeezing and sub-poissonian states of light. We establish the analytic expression of amplitude squeezing including higher-order squeezing and sub-poissonian light in the fundamental mode in section 3. The dependence of the amplitude squeezing in terms of signal to noise ratio on the number of photons is investigated. The photon statistics of the pump mode in this process have also been incorporated in this section and found to be sub-poissonian in nature. Finally, we conclude the paper in section 4.

2. Definitions of squeezing and sub-poissonian states of light

Squeezed states of light are characterized by reduced quantum fluctuations in one quadrature of the field at the expense of the increased fluctuations in the other quadrature. It is possible to characterize the amplitude by its real and imaginary parts as

$$X_1 = \frac{1}{2}(A + A^{\dagger}) \quad \text{and} \quad X_2 = \frac{1}{2i}(A - A^{\dagger}) \tag{1}$$

where A and $A^{\dagger}$ are the slowly varying operators useful in discussing squeezing effects. For a single mode of the electromagnetic field with frequency ω and creation (annihilation) operators $a^{\dagger}$ (a), they are given by

$$A = a\exp(i\omega t), \quad A^{\dagger} = a^{\dagger}\exp(-i\omega t) \tag{2}$$

The operators defined by equation (1) do not commute and obey the commutation relation

$$[X_1, X_2] = \frac{i}{2} \tag{3}$$

and, as a result, satisfy the uncertainty relation ($\hbar = 1$)

$$\Delta X_1 \, \Delta X_2 \geq \frac{1}{4} \tag{4}$$

where ΔX_1 and ΔX_2 are the uncertainties in the quadrature operators X_1 and X_2 respectively. A quantum state is squeezed in the X_1 direction if $\Delta X_1 < \frac{1}{2}$ and is squeezed in the X_2 direction if $\Delta X_2 < \frac{1}{2}$.

Amplitude squared squeezing [6-8] is defined in terms of operators Y_1 and Y_2 as

$$Y_1 = \frac{1}{2}(A^2 + A^{\dagger 2}) \quad \text{and} \quad Y_2 = \frac{1}{2i}(A^2 - A^{\dagger 2}) \tag{5}$$

These operators obey the commutation relation

$$[Y_1, Y_2] = i\,(2N_A + 1) \tag{6}$$

where $A^{\dagger}A = N_A$ is the photon number operator in mode A.

The commutation relation (6) leads to the uncertainty relation

$$\Delta Y_1 \Delta Y_2 \geq \left\langle \left(N_A + \frac{1}{2} \right) \right\rangle \tag{7}$$

where ΔY_1 and ΔY_2 are the uncertainties in the quadrature operators Y_1 and Y_2 respectively. A quantum state is squeezed in the Y_1 direction if $(\Delta Y_1)^2 < \left\langle \left(N_A + \frac{1}{2} \right) \right\rangle$ and is squeezed in the Y_2 direction if $(\Delta Y_2)^2 < \left\langle \left(N_A + \frac{1}{2} \right) \right\rangle$.

Amplitude-cubed squeezing [38] is defined by the operators

$$Z_1 = \frac{1}{2}(A^3 + A^{\dagger 3}) \text{ and } Z_2 = \frac{1}{2i}(A^3 - A^{\dagger 3}) \tag{8}$$

The operators obey the commutation relation

$$[Z_1, Z_2] = \frac{i}{2}(9N_A^2 + 9N_A + 6) \tag{9}$$

which leads to the uncertainty relation

$$\Delta Z_1 \, \Delta Z_2 \geq \frac{1}{4} \langle\, (9N_A^2 + 9N_A + 6)\, \rangle \tag{10}$$

Amplitude-cubed squeezing is said to exist if

$$(\Delta Z_1)^2 \text{ or } (\Delta Z_2)^2 < \frac{1}{4} \langle\, (9N_A^2 + 9N_A + 6)\, \rangle \tag{11}$$

The quantum effect of sub-Poissonian photon statistics are the reduction of quantum fluctuations in photon number is reflected by an increase of fluctuations of phase of the field. Hence the photon number uncertainty [19] is

$$\left\langle (\Delta N_A)^2 \right\rangle < \left\langle N_A \right\rangle \tag{12}$$

3. Squeezing and sub-poissonian states of light in the fundamental mode

Parametric-down conversion (PDC) process, shown in fig.1, is a three-wave interaction process where a pump of photon of frequency ω_1 splits into two, signal and idler, photons with lower frequencies ω_2, ω_3 respectively and the corresponding Hamiltonian can be written as

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + \omega_3 c^\dagger c + g\,(a b^\dagger c^\dagger + a^\dagger bc) \tag{13}$$

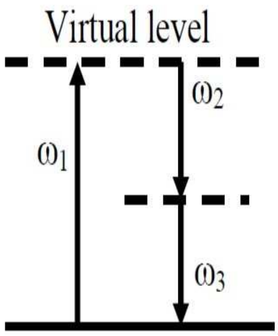

Figure 1: Schematic energy level diagram for PDC process

where $a^\dagger(a)$, $b^\dagger(b)$ and $c^\dagger(c)$ are the creation (annihilation) operators of the A, B and C modes respectively and g is the coupling constant in the interaction Hamiltonian, which is assumed to be real, describes the coupling between the two modes of the order of 10^2–10^4 per second and is proportional to the nonlinear susceptibility of the medium as well as the complex amplitude of the pump field [31,32]. However, to take care of complex g, we have used $|g|^2$ in the place of g^2 as we are not considering the phase terms. In the case of phase matching, g can also be treated as real [23].

Using the interaction Hamiltonian of the equation (13) in the coupled Heisenberg equation of motion

$$\dot{A}(t) = \frac{\partial A(t)}{\partial t} + i\,[\,H, A(t)\,] \qquad (\hbar = 1) \tag{14}$$

where the dot denotes time derivative.

Equation (14) leads to coupled Heisenberg equations of motion

$$\dot{A} = -igBC, \dot{B} = -igAC^\dagger \text{and}\, \dot{C} = -igAB^\dagger \tag{15}$$

where A, B and C are slowly varying operators because the interaction between modes, the operators A(t) and $A^\dagger(t)$ induces a slower dependence on time as compared to fast variation, which are defined by $A = a\exp(i\omega_1 t)$, $B = b\exp(i\omega_2 t)$ and $C = c\exp(i\omega_3 t)$, with the relation $\omega_1 = \omega_2 + \omega_3$.

Note that the system evolution during a short period of time is practically relevant because the actual interaction is in fact very short. Hence the interaction time is taken to be short, of the order of 10^{-10} sec and a nanosecond or picosecond pulse laser can be used as the pump field. For real physical situation in the short-time scale $gt << 1$ ($gt \sim 10^{-6}$) and the number of photons are very large ($|\alpha|^2 >> 1$), it is possible to obtain much simpler approximate analytical formulas describing the variances. Expanding A(t) in Taylor's expansion and keep terms up to second order in gt, we get

$$A(t) = A - igtBC - \frac{1}{2}|g|^2t^2 (N_BA + N_CA + A) \quad (16)$$

and $$A^\dagger(t) = A^\dagger + igtB^\dagger C^\dagger - \frac{1}{2}|g|^2t^2 (N_B A^\dagger + N_C A^\dagger + A^\dagger) \quad (17)$$

Similarly, $$B(t) = B - igtAC^\dagger - \frac{1}{2}|g|^2t^2 (N_C - N_A)B \quad (18)$$

and $$B^\dagger(t) = B^\dagger + igtA^\dagger C - \frac{1}{2}|g|^2t^2 (N_C - N_A)B^\dagger \quad (19)$$

also $$C(t) = C - igtAB^\dagger - \frac{1}{2}|g|^2t^2 (N_B - N_A)C \quad (20)$$

and $$C^\dagger(t) = C^\dagger + igtA^\dagger B - \frac{1}{2}|g|^2t^2 (N_B - N_A)C^\dagger \quad (21)$$

In order to examine the squeezing of the field amplitude of the fundamental mode A as a function of time, we define two general quadrature components,

$$X_{1A}(t) = \frac{1}{2}[A(t) + A^\dagger(t)] \quad (22)$$

and $$X_{2A}(t) = \frac{1}{2i}[A(t) - A^\dagger(t)] \quad (23)$$

Using equations (16) and (17) in equations (22) and (23), we obtain

$$X_{1A}(t) = \frac{1}{2}\left[(A + A^\dagger) - \frac{(gt)^2}{2}(A + A^\dagger)\right] \quad (24)$$

and $$X_{2A}(t) = \frac{1}{2i}\left[(A - A^\dagger) - \frac{(gt)^2}{2}(A - A^\dagger)\right] \quad (25)$$

Keeping terms up to second order in (*gt*) and assuming an initial quantum state as a product of coherent states $|\alpha>$ for the fundamental mode A, $|0>$ for the signal mode B and $|0>$ for the idler mode C, i.e.

$$|\psi> = |\alpha>_A|0>_B|0>_C \quad (26)$$

Using equations (24) and (26), we obtain

$$\left\langle\psi\middle|X_{1A}^{2}(t)\middle|\psi\right\rangle=\frac{1}{4}[\alpha^{2}+\alpha^{*2}+2|\alpha|^{2}+1-|g|^{2}t^{2}(\alpha^{2}+\alpha^{*2}+2|\alpha|^{2}+1)] \quad (27)$$

and $$\left\langle\psi\middle|X_{1A}(t)\middle|\psi\right\rangle^{2}==\frac{1}{4}[\alpha^{2}+\alpha^{*2}+2|\alpha|^{2}-|g|^{2}t^{2}(\alpha^{2}+\alpha^{*2}+2|\alpha|^{2})] \quad (28)$$

Hence the field variance is

$$[\Delta X_{1A}(t)]^{2}=\langle X_{1A}^{2}(t)\rangle-\langle X_{1A}(t)\rangle^{2}=\frac{1}{4}[1-|g|^{2}t^{2}] \quad (29)$$

From equations (4) and (29) yields

$$[\Delta X_{1A}(t)]^{2}-\frac{1}{4}=-\frac{1}{4}\left(|g|^{2}t^{2}\right) \quad (30)$$

To study squeezing in stimulated interaction in PDC process we assume an initial quantum state as a product of coherent states $|\alpha>$ for the fundamental mode A, $|\beta>$ for the mode B and vacuum state $|0>$ for the mode C

i.e. $|\psi>=|\alpha>_{A}|\beta>_{B}|0>_{C}$ (31)

We obtain

$$S_{X}=[\Delta X_{1A}(t)]^{2}-\frac{1}{4}=-\frac{1}{4}\left(|g|^{2}t^{2}\right)\left(|\beta|^{2}+1\right) \quad (32)$$

where $|\alpha|^{2}=<N_{A}>$ and $|\beta|^{2}=<N_{B}>$ are measure of average photon number of the coherent part of the single mode squeezed state field.

The right hand sides of equations (30) and (32) are always negative, showing the existence of squeezing in the amplitude of the fundamental mode in spontaneous and stimulated interaction respectively. The factor $(|\beta|^{2}+1)$ in equation (32) is the effect of stimulated interaction.

Similarly, for X_{2A} quadrature as

$$S'_{X}=[\Delta X_{2A}(t)]^{2}-\frac{1}{4}=+\frac{1}{4}\left(|g|^{2}t^{2}\right)\left(|\beta|^{2}+1\right) \quad (33)$$

From equations (32) and (33) we infer that only one quadrature can be squeezed at a time hence it follows the Heisenberg's uncertainty principle.

The variation of S_{x} and $|gt|^{2}$ of equation (32) is shown in fig 2.

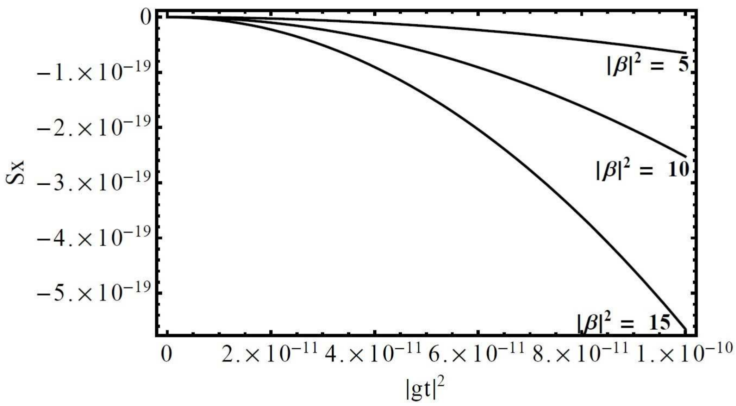

Figure 2: Variation of first-order squeezing S_x with $|gt|^2$

The results show the presence of squeezing in fundamental mode in spontaneous ($|\beta|^2 = 0$) and stimulated PDC process. It is clear from equation (32) that first-order (normal) squeezing increases nonlinearly and directly depends upon the coupling of the field amplitude and interaction time. The steady fall of the curve shows an increase in the degree of squeezing with $|\beta|^2$.

Further, in order to measure the degree of amplitude squeezing, we define normalized parameter [13] as

$$Q_X = \frac{[\Delta X_{1A}(t)]^2 - 1/4}{1/4} = -\left(|g|^2 t^2\right) \tag{34}$$

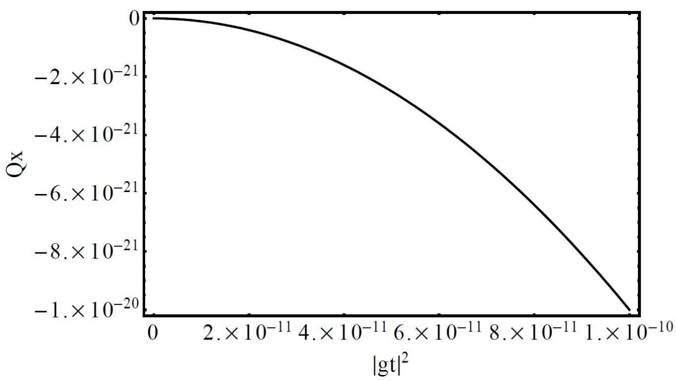

Figure 3: Degree of squeezing (first-order) Qx with $|gt|^2$

The plot of equation (34) is shown in fig. 3. It shows that the degree of squeezing increases with an increase of interaction time in fundamental mode. Hence the maximum reachable degree of squeezing is dependent upon the interaction time and will be limited by short interaction time.

Now, the mean field or coherent signal S carried by a light beam can be defined as the expectation value of the field operator in equation (1), hence

$$S = < X_{1A}(t) > = \left[\frac{1}{4}\left[\alpha^2 + \alpha^{*2} + 2|\alpha|^2 - |g|^2 t^2(\alpha^2 + \alpha^{*2} + 2|\alpha|^2)\right]\right]^{1/2} \quad (35)$$

and the field variance or quantum noise, is

$$N = \left[\Delta X_{1A}(t)\right]^2 = \frac{1}{4}[1 - |g|^2 t^2] \quad (36)$$

Hence the signal-to-noise ratio (SNR) is [31],

$$SNR_X = \frac{S^2}{N} = \frac{\langle X_{1A}(t)\rangle^2}{\left[\Delta X_{1A}(t)\right]^2} = 4|\alpha|^2 \quad (37)$$

The variation of SNR with $|\alpha|^2$ is shown in fig.8.

Similarly for studying the class of higher-order squeezing likes squeezing of amplitude-squared of the fundamental mode as a function of time, we define a real quadrature component for the pump mode as

$$Y_{1A}(t) = \frac{1}{2}[A^2(t) + A^{\dagger 2}(t)] \quad (38)$$

and $$Y_{2A}(t) = \frac{1}{2i}[A^2(t) - A^{\dagger 2}(t)] \quad (39)$$

Using equations (16) and (17) in equation (38), we get

$$Y_{1A}(t) = \frac{1}{2}\left[A^2 + A^{\dagger 2} - (gt)^2\left(A^2 + A^{\dagger 2}\right)\right] \quad (40)$$

Using equation (26) in equation (40), we obtain the expectation value as

$$\left\langle\psi\left|Y_{1A}^2(t)\right|\psi\right\rangle = \frac{1}{4}[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 + 4|\alpha|^2 + 2 - 2|g|^2 t^2(\alpha^4 + \alpha^{*4} + 2|\alpha|^4 + 4|\alpha|^2 + 2)] \quad (41)$$

and

$$\left\langle\psi\left|Y_{1A}(t)\right|\psi\right\rangle^2 = \frac{1}{4}[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 - 2|g|^2 t^2(\alpha^4 + \alpha^{*4} + 2|\alpha|^4)] \quad (42)$$

and the expectation value of the time dependent mean photon number is

$$\langle N_A(t)\rangle = |\alpha|^2 - |g|^2 t^2|\alpha|^2 \quad (43)$$

Using equations (41) and (42) in equation (7), we get

$$[\Delta Y_{1A}(t)]^2 - \left\langle N_A(t) + \tfrac{1}{2} \right\rangle = -|g|^2 t^2 \left(|\alpha|^2 + 1 \right) \quad (44)$$

Similarly using equation (31) which gives for stimulated process

$$S_y = [\Delta Y_{1A}(t)]^2 - \left\langle N_A(t) + \tfrac{1}{2} \right\rangle = -|g|^2 t^2 \left(|\alpha|^2 + 1 \right)\left(|\beta|^2 + 1 \right) \quad (45)$$

The right hand sides of equations (44) and (45) are always negative, showing the existence of squeezing in amplitude-squared of the fundamental mode in spontaneous and stimulated interaction respectively. The multiplication factor $(|\alpha|^2 + 1)$ and $\left(|\beta|^2 + 1 \right)$ are the nonlinear effect due to strong pump field interaction and the effect of stimulated interaction respectively.

Similarly, we get for Y_{2A} quadrature as

$$S'_y = [\Delta Y_{2A}(t)]^2 - \left\langle N_A(t) + \tfrac{1}{2} \right\rangle = +|g|^2 t^2 \left(|\alpha|^2 + 1 \right)\left(|\beta|^2 + 1 \right) \quad (46)$$

From equations (45) and (46) we infer that only one quadrature can be squeezed at a time hence it follows the Heisenberg's uncertainty principle.

To study higher-order (amplitude-squared) squeezing, we denote the right hand side of equation (45) by S_y and plot with $|\alpha|^2$ as shown in fig.4. It shows that the squeezing increases non-linearly with $|\alpha|^2$ which is directly dependent upon the mean number of photons. Squeezing also increases with $|\beta|^2$ that is squeezing is more in stimulated interaction than spontaneous one. A comparison between figs. 2 and 4 show greater noise reduction in second-order than in first-order, having same number of photons.

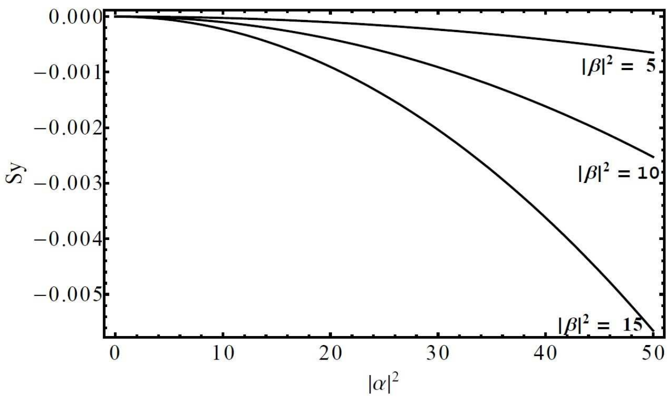

Figure 4: Variation of second-order squeezing Sy with $|\alpha|^2$ (when $|gt|^2 = 10^{-8}$)

Now, we define the normalized parameter [13] for amplitude-squared squeezing as,

$$Q_y = \frac{[\Delta Y_{1A}(t)]^2 - \left\langle \left(N_A(t) + \tfrac{1}{2} \right) \right\rangle}{\left\langle \left(N_A(t) + \tfrac{1}{2} \right) \right\rangle}$$

$$= \frac{-|g|^2 t^2 \left(|\alpha|^2 + 1 \right)}{\left(|\alpha|^2 + \frac{1}{2} \right) - |g|^2 t^2 |\alpha|^2} \quad (47)$$

The variation of Q_y with $|\alpha|^2$ is shown in fig.5.

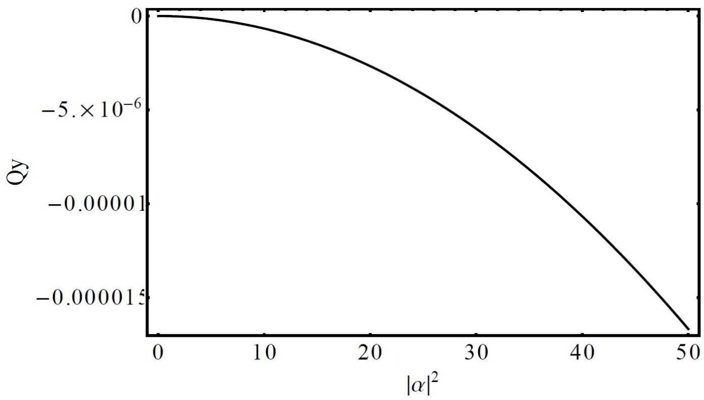

Figure 5: Degree of squeezing (second-order) Qy with $|\alpha|^2$ (when $|gt|^2 = 10^{-8}$)

The steady fall of the curve shows an increase in the degree of squeezing with number of photons. This confirms that the squeezed states are associated with large number of photons.

The signal-to noise ratio in second-order squeezing is obtained as

$$SNR_y = \frac{S^2}{N} = \frac{\langle Y_{1A}(t) \rangle^2}{[\Delta Y_{1A}(t)]^2} = \frac{2|\alpha|^2}{\left(|\alpha|^2 + 1 \right)} \quad (48)$$

The variation of SNR with $|\alpha|^2$ is shown in fig.8.

Further, using equations (8), (16) and (17), the real quadrature component for the fundamental mode A in third-order squeezing may be written as

$$Z_{1A}(t) = \frac{1}{2}[A^3(t) + A^{\dagger 3}(t)] = \frac{1}{2}[A^3 + A^{\dagger 3} - \frac{3}{2}|g|^2 t^2 (A^3 + A^{\dagger 3})] \quad (49)$$

Using equation (26) in equation (49), we obtain the expectation value as

$$\left\langle \psi \left| Z_{1A}^2(t) \right| \psi \right\rangle = \frac{1}{4}[\alpha^6 + \alpha^{*6} + 2|\alpha|^6 + 9|\alpha|^4 + 18|\alpha|^2 + 6$$

$$-3|g|^2 t^2(\alpha^6+\alpha^{*6}+2|\alpha|^6+9|\alpha|^4+18|\alpha|^2+6)] \quad (50)$$

and

$$\left\langle\psi\middle|Z_{1A}(t)\middle|\psi\right\rangle^2=\frac{1}{4}[\alpha^6+\alpha^{*6}+2|\alpha|^6-3|g|^2 t^2(\alpha^6+\alpha^{*6}+2|\alpha|^6)] \quad (51)$$

Using equations (50) and (51), we get

$$[\Delta Z_{1A}(t)]^2=\left\langle Z_{1A}^2(t)\right\rangle-\left\langle Z_{1A}(t)\right\rangle^2=\frac{1}{4}[9|\alpha|^4+18|\alpha|^2$$

$$+6-3|g|^2 t^2(9|\alpha|^4+18|\alpha|^2+6)] \quad (52)$$

$$\text{and } \left\langle N_A^2(t)\right\rangle=|\alpha|^4+|\alpha|^2-2|g|^2 t^2\left(|\alpha|^4+|\alpha|^2\right) \quad (53)$$

Using equations (43) and (53), we get expectation value as,

$$\frac{1}{4}\left\langle 9N_A^2(t)+9N_A(t)+6\right\rangle=\frac{1}{4}[9|\alpha|^4+18|\alpha|^2+6-9|g|^2 t^2(2|\alpha|^4+3|\alpha|^2)] \quad (54)$$

Hence from equations (52) and (54),

$$[\Delta Z_{1A}(t)]^2-\frac{1}{4}\left\langle 9N_A^2(t)+9N_A(t)+6\right\rangle=-\frac{9}{4}|g|^2 t^2[|\alpha|^4+3|\alpha|^2+2] \quad (55)$$

Similarly for stimulated process, using equation (31)

$$S_Z=[\Delta Z_{1A}(t)]^2-\frac{1}{4}\left\langle 9N_A^2(t)+9N_A(t)+6\right\rangle=-\frac{9}{4}|g|^2 t^2[|\alpha|^4+3|\alpha|^2+2]\left(|\beta|^2+1\right) \quad (56)$$

The right-hand side of equations (55) and (56) are always negative, showing the existence of squeezing in amplitude-cubed of the fundamental mode in spontaneous and stimulated interaction respectively. The multiplication factor ($|\beta|^2+1$) is the effect of stimulated interaction.

Similarly, for Z_{2A} quadrature,

$$S'_Z=[\Delta Z_{2A}(t)]^2-\frac{1}{4}\left\langle 9N_A^2(t)+9N_A(t)+6\right\rangle=+\frac{9}{4}|g|^2 t^2[|\alpha|^4+3|\alpha|^2+2]\left(|\beta|^2+1\right) \quad (57)$$

From equations (56) and (57) we infer that only one quadrature can be squeezed at a time hence it follows the Heisenberg's uncertainty principle.

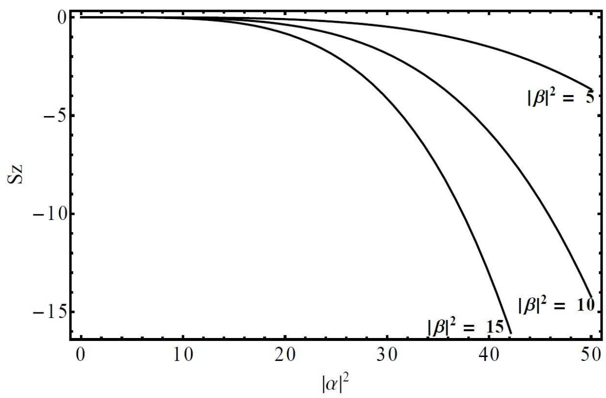

Figure 6: Variation of third-order squeezing Sz with $|\alpha|^2$ (when $|gt|^2 = 10^{-8}$)

The above plotted fig.6 between S_z and $|\alpha|^2$ of equation (56) shows that the higher order squeezing directly depends on number of photons. A comparison between second-order and third-order squeezing (figs. 4 and 6) shows greater squeezing in the latter in both spontaneous and stimulated processes for same value of $|\alpha|^2$ and $|\beta|^2$ respectively. It also infer that in comparison of first-, second- and third-order that squeezing is maximum in amplitude-cubed (third-order) followed by amplitude squared (second-order) and amplitude-squeezing (first-order) having the same number of photons.

The normalized parameter [13] for amplitude-cubed squeezing is as,

$$Q_Z = \frac{[\Delta Z_{1A}(t)]^2 - \frac{1}{4}\left\langle 9N_A^2(t) + 9N_A(t) + 6 \right\rangle}{\frac{1}{4}\left\langle 9N_A^2(t) + 9N_A(t) + 6 \right\rangle}$$

$$= \frac{-\frac{9}{4}|g|^2 t^2 [|\alpha|^4 + 3|\alpha|^2 + 2]}{\frac{1}{4}[9|\alpha|^4 + 18|\alpha|^2 + 6 - 9|g|^2 t^2 (2|\alpha|^4 + 3|\alpha|^2)]} \tag{58}$$

Fig. 7 shows degree of squeezing in third order is more pronounced than second-order, having same number of photons.

For third-order squeezing, signal-to noise ratio [31] is obtained as

$$SNR_Z = \frac{S^2}{N} = \frac{\langle Z_{1A}(t)\rangle^2}{[\Delta Z_{1A}(t)]^2} = \frac{4|\alpha|^6}{\left(9|\alpha|^4 + 18|\alpha|^2 + 6\right)} \tag{59}$$

The variation of SNR with $|\alpha|^2$ is shown in fig.8.

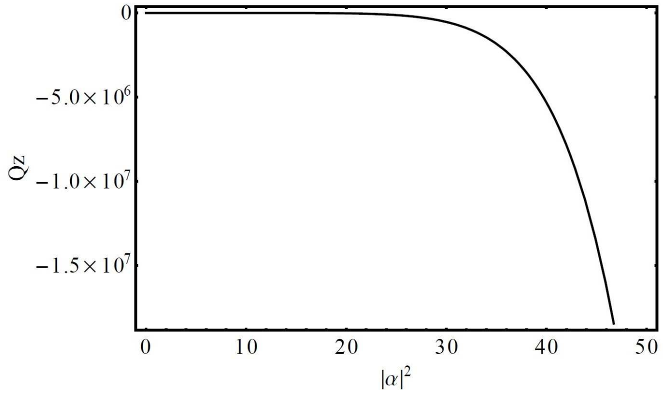

Figure 7: Degree of squeezing (third-order) Qz with $|\alpha|^2$ (when $|gt|^2 = 10^{-8}$)

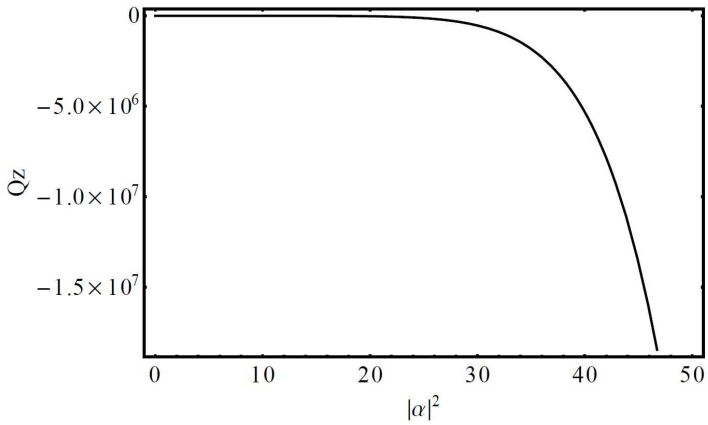

Figure 8: Variation of signal to noise ratio SNR with $|\alpha|^2$ (when $|gt|^2 = 10^{-8}$ and SNRx, SNRy and SNRz represent first, second and third-order signal to noise ratio respectively)

From fig. 8, it is evident that maximum signal to noise ratio is possible in lower-order i.e. in normal squeezing. The result agrees with the result of Yuen [37]. The steady increase of the curve shows the reduction of quantum noise with an increase of the number of photons $|\alpha|^2$. Thus, noise reduces and the degree of squeezing increases with the photon number in fundamental mode. We also observe that the greater noise reduction in third-order followed by second-order and first-order, having same number of photons.

Now, using equation (43), we obtain as

$$\langle N_A(t)\rangle^2 = |\alpha|^4 - 2|g|^2 t^2 \left(|\alpha|^4\right) \tag{60}$$

Hence the fluctuation of time dependent mean photon number is

$$[\Delta N_A(t)]^2 = \left\langle N_A^2(t) \right\rangle - \left\langle N_A(t) \right\rangle^2 = |\alpha|^2 - 2|g|^2 t^2 |\alpha|^2 \quad (61)$$

and the photon statistics of pump mode is found to be sub-Poissonian, as

$$N = [\Delta N_A(t)]^2 - \left\langle N_A(t) \right\rangle = -|g|^2 t^2 |\alpha|^2 \quad (62)$$

The right-hand side of equation (62) is always negative, showing the existence of sub-poissonian light in the fundamental mode under short-time approximation.

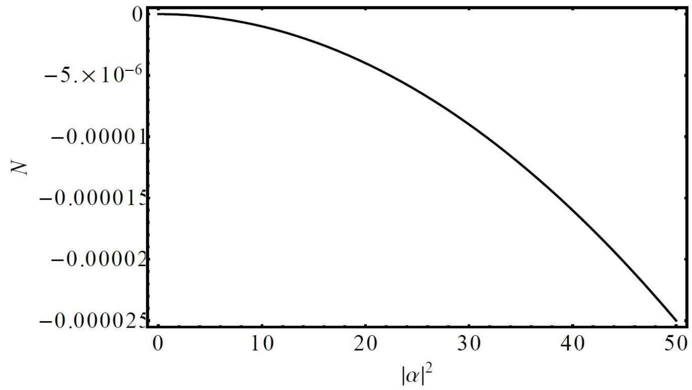

Figure 9: Variation of sub-poissonian states *N* with $|\alpha|^2$ (when $|gt|^2 = 10^{-8}$)

For studying sub-Poissonian photon statistics nature in nonclassical state, let us take the variations of N with $|\alpha|^2$ as shown in fig 9. In fig 9, sub-poissonian behaviour takes place and shows the depth of nonclassicality. It also shows that the sub-poissonian statistics properties of light is directly proportional to the number of photons of fundamental mode i.e. sub-poissonian statistics of light increases with increasing $|\alpha|^2$. Thus we infer that sub-poissonian effects of light appear simultaneously with the amplitude squeezing.

4. Conclusion

In the present paper, the squeezing and sub-poissonian states of light in spontaneous and stimulated parametric down-conversion process are investigated.

It is shown that amplitude, amplitude-squared and amplitude-cubed squeezing as well as sub-poissonian photon statistics of light of the initial pump field is directly dependent upon coupling of the field amplitude and interaction time as well as the number of photons. A comparison between first-, second- and third-order squeezing shows greater squeezing in third-order followed by second and first-order squeezing, having the same number of photons. It is inferred that higher-order squeezing (amplitude-squared & amplitude-cubed) makes it possible to achieve significantly larger noise reduction than ordinary (normal) squeezing. The occurrence of multiplication factor

$\left(|\alpha|^4 + 3|\alpha|^2 + 2\right)$ and $\left(|\alpha|^2 + 1\right)$ in third-order and second-order squeezing respectively is the nonlinear effect due to strong pump field interaction, which shows that squeezing is found to be maximum in amplitude cubed or amplitude-squared followed by amplitude-squeezing. The multiplication factor ($|\beta|^2+1$) is due to the effect of stimulated interaction and it shows that squeezing is more in stimulated process than spontaneous one.

It is found that the degree of squeezing and occurrence of sub-poissonian light is directly depends upon the photon number of the fundamental field. It is observed that degree of squeezing increases and lowers the depth of classicality of field amplitude with photon number. Hence the amplitude squeezing and sub-poissonian photon statistics of light appear simultaneously.

The above results arrived at may help in selecting suitable process to generate optimum squeezing in the radiation field and can be useful in optical telecommunication.

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