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Libration Points of Cable-Connected Satellites System Under the Influence of Solar Radiation Pressure, Earth's Magnetic Field, Shadow of the Earth and Air Resistance: Circular Orbit

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ABSTRACT

The present paper deals with the study of libration points of the motion of a system of two artificial satellites connected by a light, flexible, inextensible and non-conducting cable under the influence of solar radiation pressure, earth's magnetic field, shadow of the earth and air resistance. The case of circular orbit of centre of mass of the system is discussed. Differential equations of motion of the system are derived. To obtain the general solutions of the differential equations is beyond our reach. On the other hand, the general solutions do not serve our purpose. Jacobian integral of the system has also been obtained. Thereafter equilibrium positions of the motion of the system have been obtained. In non-linear oscillations of the system, one equilibrium position exists when all the perturbations mentioned above act on the system simultaneously.

Keywords: Libration points; Two cable-connected satellites; Circular orbit

1. Introduction

The present work is an attempt towards the generalization of work done by Novikova [6] and Novoorebelskii [7]. They studied the motion of a system of two satellites connected by a light, flexible and inextensible string in the central gravitational field of force relative to its centre of mass. This study assumed that the two satellites are moving in the plane of the centre of mass. Demin[3] and Singh[8] investigated the problem in two and three dimensional cases. Singh[9] studied the effect of magnetic force on the motion of a system of two cable-connected satellites in orbit. Bhattacharya[1] studied the stability of equilibrium positions of two cable-connected satellites under the influence of solar radiation pressure, earth's oblateness and earth's magnetic field. Bhattacharya[2] obtained the equations of motion of a system of two cable-connected artificial satellites under the influence of solar radiation pressure, earth's oblateness and shadow of the earth. The works of Uddin[10] and Yang[11] are reviewed for the present problem.

Libration points of the motion of a system of two cable-connected artificial satellites under the influence of solar radiation pressure, earth's magnetic field, shadow of the earth and air resistance is studied. The case of circular orbit of the centre of mass of the system is discussed. Shadow of the earth is taken to be cylindrical and the system is allowed to pass through the shadow beam. The satellites are connected by a light, flexible, inextensible and non-conducting cable. The satellites are taken as charged material particles. Since masses of the satellites are small and distances between the satellites and other celestial bodies are very large, the gravitational forces of attraction between the satellites and other celestial bodies including the sun have been neglected.

2. Equations of motion and Jacobian integral of the system

Equations of motion of one of the satellites when the centre of mass moves along Keplarian elliptical orbit in Nechvile's co-ordinate system[5] is written as Kumar[4]

$$X''-2Y'-3X\rho=\lambda_a X-\frac{A}{\rho}\cos i-\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\cos\in\cos(\nu-\alpha)-f\rho\rho'$$

and

$$Y''+2X'=\lambda_a Y-\frac{A\rho'}{\rho^2}\cos i+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\cos\in\sin(\nu-\alpha)-f\rho^2 \qquad (1)$$

with the condition of constraint

$$X^2+Y^2\le\frac{1}{\rho^2} \qquad (2)$$

Also,

$$\rho=\frac{1}{(1+e\cos\nu)},\ f=\frac{a_1 p^3}{\sqrt{\mu p}},$$

$$a_1=\rho_a\dot{R}(c_2-c_1)\left(\frac{m_1}{m_1+m_2}\right),\ A=\left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1}-\frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu\rho}}$$

$$\lambda_a=\frac{p^3\rho^4}{\mu}\left(\frac{m_1+m_2}{m_1 m_2}\right)\lambda \qquad (3)$$

m_1 and m_2 are masses of the two satellites. B_1 and B_2 are the absolute values of the forces due to the direct solar pressure on m_1 and m_2 respectively and are small. Q_1 and Q_2 are the charges of the two satellites. μ_E is the magnitude of magnetic moment of the earth's dipole. p is the focal parameter. μ is the product of mass of the earth and gravitational constant. λ is undermined Lagrange's multiplier. g_e is the force of gravity. e is ecentricity of the orbit of the centre of mass. v is the true anomaly of the centre of mass of the system. ∈ is inclination of the oscillatory plane of the masses m_1 and m_2 with the orbital plane of the centre of mass of the system. α is the inclination of the ray. γ is a shadow function which depends on the illumination of the system of satellites by the sun rays. If γ is equal to zero, then the system is affected by the shadow of the earth. If γ is equal to one, then the system is not within the said shadow. $\dot{R}$ is the first order time derivative of R. R is the modulus of position vector of the centre of mass of the system. c_1 and c_2 are the Ballistic co-efficients. ρ_a is the average density of the atmosphere. Prime denotes differentiation with respect to v.

If motion of one of the satellites m_1 be determined with the help of equations (1), motion of the other satellite of mass m_2 can be determined by Kumar[4].

$$m_1\overline{\rho_1}+m_2\overline{\rho_2}=0 \tag{4}$$

where, $\overline{\rho_j}\,(j=1,2)$ is the radius vector in the centre of mass System.

In the case of circular orbit of centre of mass of the system, we put $e=0$, $\rho=1$ and $\rho'=0$ in equations (1) and write

$$X''-2Y'-3X=\lambda_a X-A\cos i-\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\cos\in\cos(\nu-\alpha)$$

and

$$Y''+2X'=\lambda_a Y+\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\cos\in\sin(\nu-\alpha)-f \tag{5}$$

with the condition of constraint

$$X^2+Y^2\le 1 \tag{6}$$

For circular orbit,

$$X^2+Y^2=1\text{ ; whence } XX'+YY'=0 \tag{7}$$

The system of two satellites is allowed to pass through the shadow beam during its motion. Let us assume that θ_2 is the angle between the axis of the cylindrical shadow beam and the line joining the centre of the earth and the end point of the orbit of the centre of mass within the earth's shadow, considering the positive direction towards the motion of the system. The system starts to be influenced by the solar pressure when it makes an angle θ_2 with the axis of the shadow beam and remains under the influence of solar pressure till it makes an angle $(2\pi-\theta_2)$ with the axis of the cylindrical shadow beam. Thereafter, the system will enter the shadow beam and the effect of solar pressure will come to an end.

Next, the small secular and long periodic effects of solar pressure together with the effects of earth's shadow on the system may be analyzed by averaging the periodic terms in (5) with respect to ν from θ_2 to $(2\pi-\theta_2)$ for a period when the system is under the influence of the sun rays directly i.e. $\gamma = 1$ and from $-\theta_2$ to $+\theta_2$ for a period when the system passes through the shadow beam i.e. $\gamma = 0$.

Thus after averaging the periodic terms, (5) may be written as

$$X'' - 2Y' - 3X = \beta X - A\cos i + \left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right) \cdot \frac{\cos\in \cos\alpha \sin\theta_2}{\pi}$$

and

$$Y'' + 2X' = \beta Y + \left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right) \cdot \frac{\cos\in \sin\alpha \sin\theta_2}{\pi} - f \tag{8}$$

where $\beta = \dfrac{p^3\lambda}{\mu}\left(\dfrac{m_1 + m_2}{m_1 m_2}\right)$ (9)

These equations do not contain the time explicity. Therefore, Jacobian integral of the motion exists.

Multiplying the first and second equations of (8) by X' and Y' respectively, adding them and then integrating the final equation, we get the Jacobian integral in the form

$$X'^2 + Y'^2 - 3X^2 = \beta - 2AX\cos i + \frac{2}{\pi}\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)\cos\in \sin\theta_2 \left(X\cos\alpha + Y\sin\alpha\right) - 2fY + h \tag{10}$$

The surface of zero velocity can be obtained in the form

$$3X^2 - 2AX\cos i + \frac{2}{\pi}\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right) \cdot \cos\in \sin\theta_2 \left(X\cos\alpha + Y\sin\alpha\right) - 2fY + \beta + h = 0 \tag{11}$$

$h =$ constant of Integration called Jacobian constant.

It is, therefore, concluded that satellite m_1 moves inside the boundary of different curves of zero velocity, represented by (11) for different values of Jacobian constant *h*.

3. Equilibrium solution of the problem

A set of equations (8) for motion of the system in the rotating frame of reference has been obtained. It is assumed that the system is moving with the effective constraint and the connecting cable of the two satellites always remains tight.

The equilibrium positions of motion of the system are given by the constant values of the co-ordinates in the rotating frame of reference. Let us take

$$X = X_0 \text{ and } Y = Y_0 \tag{12}$$

X_0 and Y_0 are constant, give the equilibrium positions. Therefore, we get

$$X' = X'_0 = 0 \quad ; \quad X'' = X''_0 = 0$$
$$Y' = Y'_0 = 0 \quad ; \quad Y'' = Y''_0 = 0 \tag{13}$$

Putting (12) and (13) in the set of equations (8), we get

$$\left(3+\beta\right)X_0 = A\cos i-\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\cos\in\cos\alpha\sin\theta_2}{\pi}$$

and

$$\beta Y_0 = f-\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\cos\in\sin\alpha\sin\theta_2}{\pi} \tag{14}$$

Actually it is very difficult to obtain the solution of (14). Hence, we are compelled to make our approaches with certain limitations. In addition to this, we are interested only in the case of the maximum effect of the earth's shadow on motion of the system.

In the further investigation, we put $\in = 0$ and $\alpha = 0$ as because $\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)$ or θ_2 cannot be zero. Clearly equations (14) become

$$\left(3+\beta\right)X_0 = A\cos i-\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\sin\theta_2}{\pi}$$

and

$$\beta Y_0 = f \tag{15}$$

All the two equations of (15) are independent of each other.
With the help of the two equations of (15), the equilibrium position is obtained as

$$\left(X_0,Y_0\right)=\left[\frac{1}{\left(3+\beta\right)}\left\{A\cos i-\frac{1}{\pi}\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\sin\theta_2\right\},\frac{f}{\beta}\right] \tag{16}$$

4. Conclusion

Only one equilibrium position of motion of the system under the combined influence of solar radiation pressure, earth's magnetic field, shadow of the earth and air resistance is obtained, when the orbit of centre of mass of the system is circular in nature.

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