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Systems of Equations with Fuzzy Coefficients

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ABSTRACT

In this work, we consider and solve systems of equations having triangular or trapezoidal fuzzy numbers as coefficients. The central idea applied here for solving a system of such type is the ranking of the fuzzy numbers involved and the solution of the obtained in this way ordinary system with a traditional method. Finally, one may convert the real solution(s) of the ordinary system to fuzzy numbers with the desired degree of fuzziness, which is a new notion introduced here. This is not compulsory, but it is suggested to be applied in typical scenarios where ambiguity or vagueness prevails and therefore a fuzzy expression of the corresponding solution could be preferable than the crisp one. Examples and real life applications are also presented illustrating our method.

Keywords: System of Equations (SoE), Fuzzy Set (FS), Fuzzy Number (FN), Triangular FN (TFN), Trapezoidal FN (TpFN), Centre of Gravity (COG) Defuzzification Technique, Ranking Functions, Degree of Fuzziness (DoF), Fuzzy SoE (FSoE).

1. Introduction

The long list of the available methods for the solution of a *System of Equations (SoE)* shows the rich history of the problem and its continuing importance for applications in the every day life, as well as in science and technology. It is recalled that such methods include substitution, elimination, the Cramer's rule for solving a n X n linear SoE, graphical and approximation procedures, etc.

However in large and complex management systems, like the socio-economic, the biological ones, etc., it is often very difficult to treat satisfactorily problems by solving a SoE with the traditional methods, because the necessary data cannot be easily determined precisely and therefore estimates of them are used in practice. The reason for this is that such kind of systems usually involve many different and constantly changing factors, the relationships among which are indeterminate resulting to non clear operation mechanisms. In order to obtain good results in such cases one could apply either principles of *Fuzzy Logic (FL)* (e.g. see [10 and its references, etc.), or of the *Grey Systems (GS)* theory (e.g. see [5, 11], etc.).

In this work a new method is developed for solving a SoE with fuzzy coefficients, referred here for brevity as a *Fuzzy SoE (FSoE)*. The rest of the paper is formulated as follows: In the second Section the background information about *Fuzzy Numbers (FNs)* is recalled which is necessary for the understanding of the paper. In the third Section our method for solving a FSoE is developed and examples are presented illustrating it. Finally, the fourth Section of the paper includes our conclusion and some suggestions for future research on the subject.

2. Fuzzy numbers

2.1. Introductory concepts

FL, due to its nature of characterizing the ambiguous real life situations with multiple values, offers rich resources for handling problems with approximate data. This multiple-valued logic, being an extension and complement of the classical bi-valued Logic of Aristotle, is based on the notion of *Fuzzy Set (FS)*, introduced by Zadeh in 1965 [14]. For general facts on FS we refer to the book [4].

FNs play an important role in fuzzy mathematics analogous to the role played by the ordinary numbers in crisp mathematics. The general definition of a FN is the following:

Definition 1. A FN is a FS A on the set $\boldsymbol{R}$ of real numbers with membership function m_A: $\boldsymbol{R} \rightarrow [0, 1]$, such that:

- A is *normal*, i.e. there exists x in $\boldsymbol{R}$ such that $m_A(x) = 1$.
- A is *convex*, i.e. all its *a-cuts* $A^a = \{x \in U\colon m_A(x) \geq a\}$, with a in [0, 1], are closed real intervals.
- Its membership function $y = m_A(x)$ is a *piecewise continuous* function.

Note that one can define the basic *arithmetic operations* on FNs with two different, but equivalent to each other methods [3]. However, since both methods involve laborious calculations in the general case, in practical applications it is usually preferred to utilize special forms of FNs, for which these operations can be performed easily.
For general facts on FNs we refer to the book [3].

2.2. Triangular and trapezoidal fuzzy numbers

In this work we are going to make use of the two simpler forms of FNs, the *Triangular FNs (TFNs)* and the *Trapezoidal FNs (TpFNs)*.
The definition of a TFN is given as follows:

Definition 2. Let a, b and c be real numbers with $a < b < c$. Then the TFN $A = (a, b, c)$ is a FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ \dfrac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \text{ and } x > c \end{cases} \tag{1}$$

It can be shown [3] that the two general methods for performing operations on FNs lead to the following simple rules for the addition and subtraction of TFNs:

Proposition 1. Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two TFNs. Then:

- The *sum* of them is the TFN $A + B = (a_1+b_1, a_2+b_2, a_3+b_3)$.
- The *difference* of them is the TFN $A - B = (a_1-b_3, a_2-b_2, a_3-b_1)$.

On the contrary, the *product* $A \,.\, B$ and the *quotient* $A : B$ are FNs which are not TFNs in

general, apart from some special cases. For example, if all the entries of A and B are positive numbers, then we approximately can write that
$A \,.\, B = (a_1b_1, a_2b_2, a_3b_3)$ and $A : B = (a_1/b_3, a_2/b_2, a_3/b_1)$.
A TpFN is defined as follows:

Definition 3. Let $a < b \leq c < d$ be real numbers. Then the TpFN (a, b, c, d) is a FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b] \\ x = 1, & x \in [b,c] \\ \dfrac{d-x}{d-c}, & x \in [c,d] \\ 0, & x < a \text{ and } x > d \end{cases} \tag{2}$$

It is easy to observe that the TFN (a, b, d) is a special case of the TpFN (a, b, c, d) with $c=b$, therefore the TpFNs are generalizations of the TFNs.
Similarly with the TFNs it can be shown [3] that, if $A = (a_1, a_2, a_3, a_4)$ and
$B = (b_1, b_2, b_3, b_4)$ are given TpFNs, then $A + B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$ and.
$A - B = (a_1-b_4, a_2-b_3, a_3-b_2, a_3-b_4)$, whereas $A \,.\, B$ and $A : B$ are not TpFNs in general.

Remark 1. The TFNs and the TpFNs are special cases of the *LR – FNs* of Dubois and Prade [2]. Generalizing the definitions of TFNs and TpFNs one can define *n-agonal FNs* of the form $(a_1, a_2, \dots, a_n)$ for any integer n, $n \geq 3$ (e.g. see Section 2 of [1] for the definition of hexagonal FNs).
For the needs of the present work we introduce the following concept:

Definition 4. The *Degree of Fuzziness (DoF)* of the n-agonal FN $A = (a_1, a_2, \dots, a_n)$ is defined to be the real number $D = a_n - a_1$. We write then DoF $(A) = D$.

2.3. Defuzzification and ranking of TFNs /TpFNs

The general approach for solving a problem using principles of FL involves the following steps:

- *Fuzzification* of the problem's data by representing them with properly defined FS.
- *Evaluation of the fuzzy data* by applying principles and methods of FL in order to find the problem's solution in the form of a unique FS.
- *Defuzzification* of the problem's solution in order to "translate" it in the natural language for applying it to the original real-life problem.

The most popular defuzzification method is perhaps the *Centre of Gravity (COG) technique*, according to which the problem's fuzzy outcomes are represented by the coordinates of the COG of the membership's function graph of the FS representing the solution [8]. The following two propositions enable the defuzzification of a given TFN or TpFN respectively with the COG technique:

Proposition 2. The coordinates (X, Y) of the COG of the graph of the TFN (a, b, c) are calculated by the formulas $X = \frac{a+b+c}{3}$,$Y = \frac{1}{3}$.

Proof: The graph of the TFN (*a*, *b*, *c)* is the triangle ABC of Figure 1, with A (*a*, 0), B (*b*, 1) and C (*c*, 0).

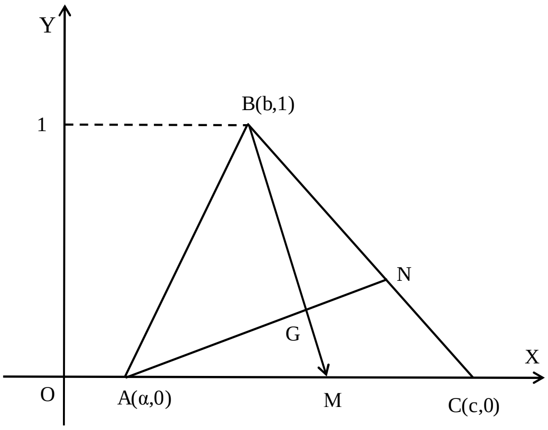

Figure 1: Graph and Centre of Gravity (COG) of the TFN (*a*, *b*, *c*)

Then, the COG, say G, of ABC is the intersection point of its medians AN and BM, where N ($\frac{b+c}{2}$, $\frac{1}{2}$) and M ($\frac{a+c}{2}$, 0). Therefore, it is a routine process of Analytic Geometry (see proof of Proposition 2 in [9]) to form the equations of the straight lines defined by the line segments AN and BM and then to determine the coordinates of G by solving the linear system of those equations.

Proposition 3. Consider the graph of the TpFN (*a*, *b*, *c*, *d*) (Figure 2). Let G_1 and G_2 be the COGs of the rectangular triangles AEB and CFD and let G_3 be the COG of the rectangle BEFC respectively. Then $G_1G_2G_3$ is always a triangle, whose COG G has coordinates $X = \frac{2(a+d)+7(b+c)}{18}$, $Y = \frac{7}{18}$.

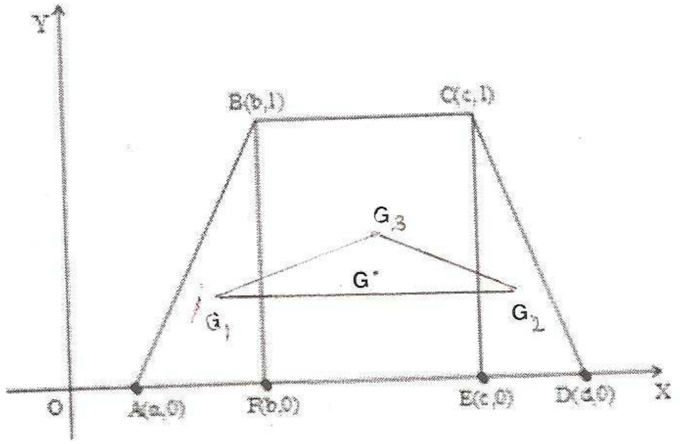

Figure 2: Graph of a TpFN *(a, b, c, d)* and COGs of its parts

Proof: By Proposition 2 one finds that G_1 $(\frac{a+2b}{3},\frac{1}{3})$ and G_2 $(\frac{d+2c}{3},\frac{1}{3})$. Further, it is easy to check that the GOG G_3 of the rectangle BCFD, being the intersection of its diagonals, has coordinates $(\frac{b+c}{2},\frac{1}{2})$. The y – coordinates of all points of the straight line defined by the line segment G_1G_2 are equal to $\frac{1}{3}$, therefore the point G_3, having y – coordinate equal to $\frac{1}{2}$, does not belong to this line. Hence, by Proposition 2, the COG G of the triangle $G_1G_2G_3$ has coordinates $X = (\frac{a+2b}{3}+\frac{d+2c}{3}+\frac{b+c}{2}): 3 = \frac{2(a+d)+7(b+c)}{18}$ and

$Y = (\frac{1}{3}+\frac{1}{3}+\frac{1}{2}): 3 = \frac{7}{18}$.

Remark 2. Since the COGs G_1, G_2 and G_3 are the balancing points of the triangles AEB and CFD and of the rectangle BEFC respectively, the COG G of the triangle $G_1G_2G_3$, being the balancing point of the triangle formed by those COGs, may be considered instead of the COG of the trapezoid ABCD as the tool for defuzzifying the TpFN (*a*, *b*, *c*, *d*). An advantage of the choice of G is that the formulas calculating the coordinates of it are simpler than those calculating the COG of the trapezoid ABCD (see Proposition 11, Chapter 7 of [10]).

An important problem of the fuzzy arithmetic is the *ordering of FNs*, i.e. the process of determining whether a given FN is larger or smaller than another one. This problem can be solved through the introduction of a *ranking function*, say R, which maps each FN on the real line, where a natural order exists. Several ranking methods have been

proposed until today, like the lexicographic screening [13], the use of an area between the Centroid and original points [12], the subinterval average method [1], etc.
Here, under the light of Propositions 2 and 3 respectively, we define the ranking functions of a TFN and of a TpFN as follows:

Definition 5. i) If *A* {*a, b, c*) is a TFN, then R(*A*) = $\frac{a+b+c}{3}$.

ii) If *A* {*a, b, c, d)* is a TpFN, then R(*A*) = $\frac{2(a+d)+7(b+c)}{18}$.

Proposition 4. Let *A* be a TFN with DoF (*A*) = D and R(*A*) = R. Then *A* can be written in the form *A* = (*a*, 3R-2*a*-D, *a* + D), where *a* is a real number such that

$$\mathrm{R} - \frac{2\mathrm{D}}{3} < a < \mathrm{R} - \frac{\mathrm{D}}{3}.$$

Proof: Let *A* (*a, b, c*) be the given TFN, where *a, b, c* are real numbers such that *a*<*b*<*c*. Then, since D*(A)* = *c* - *a* = D, it is *c* = *a* + D.

Therefore, R(*A*) = $\frac{a+b+c}{3} = \frac{2a+b+\mathrm{D}}{3}$ = R, which gives that *b* = 3R-2*a*-D.

Consequently we have that *a* < 3R-2*a*-D < *a* + D. The left side of the last inequality implies that 3*a* < 3R-D, or *a*< R-$\frac{\mathrm{D}}{3}$. Also its right side implies that -3*a* < 2D-3R, or

a > R-$\frac{2\mathrm{D}}{3}$ and this completes the proof.

The corresponding situation is more complicated if *A* is a TpFN. Then we have:

Proposition 5. Let *A* be a TpFN with DoF (*A*) = D and R(*A*) = R. Then *A* can be written in the form *A* = (*a*, b, c, *a* + D), where *a, b and c* are real numbers such that

$$a < b \leq c < a + \mathrm{D} \text{ and } b + c = \frac{18\mathrm{R} - 4a - 2\mathrm{D}}{7}.$$

Proof: Let *A* (*a*, *b*, *c*, *d*) be the given TFN, with *a*, *b*, *c*, *d* real numbers such that *a*<*b*<*c*<*d.* Since D*(A)* = *d* - *a* = D, it is *d* = *a* + D. Also, by Definition 5(ii) we get that R=$\frac{2(2a+D)+7(b+c)}{18}$, which provides the expression of *b* + *c* in the required form.

3. A method of solving fuzzy systems of equations

3.1. The method

The FSoEs considered in our examples here are of the form $F_i(x_1, x_2, \ldots, x_n) = 0$, i = 1, 2,…, m, where the coefficients of the equations $F_i = 0$ with respect to the unknown variables $x_1, x_2, \ldots, x_n$ are TFNs or TpFNs and n, m are integers such that n, m$\geq$ 2.

Our method proposed for solving such a FSoE involves the following steps:

1. Ranking the coefficients of the equations $F_i = 0$, i = 1, 2,…. , m..
2. Solving the obtained in step 1 ordinary SoE with one of the existing standard methods.
3. Converting the real solution(s) of the ordinary SoE found in step 2 to FN(s) with

the desired DoF.

The last step is not compulsory, but it is suggested to be applied to typical scenarios where ambiguity or vagueness prevails and therefore a fuzzy expression of the problem's solution is preferable than a crisp one.

3.2. Examples

The following examples illustrate our method:

Example 1. A market's research resulted to the following fuzzy data: The sum of the quantities *x* and *y* of two imported goods are expected to be between 2 and 7 units but most probably between 4 and 5 units. Also their product is expected to be between 0.2 and 0.8 units, but most probably between 0.25 and 0.75 units. Find all the possible values of x and y with DoF equal to 2.

Solution: A TpFN (a, b, c, d) expresses mathematically the fuzzy statement that the interval [b, c] is contained in the interval [a, d]. Therefore, utilizing TpFNs the given problem can be formulated as follows: Find all the TpFNs *A* and *B* with DoF equal to 2, such that *A* + *B* = (2, 4, 5, 7), and *A* **.** *B* = (0.2, 0.25, 0.75, 0.8).

The required TpFNs can be found by solving the FSoE $\left.\begin{array}{l} x+y=(2, 4, 5, 7) \\ xy=(0.2, 0.25, 0.75, 0.8) \end{array}\right]$.

We apply the steps of our method as follows:

1-2. Ranking the constants of the above FSoE (see Definition 6(ii)) one finds the traditional SoE $\left.\begin{array}{l} x+y=4.5 \\ xy=0.5 \end{array}\right]$, which has the solutions $x \approx 4.38$, $y \approx 0.12$ and $x \approx 0.12$, $y \approx 4.38$.

3. Converting with the help of Proposition 5 the values of *x* and *y* to TpFNs with DoF equal to 2 one finds that either *A* = (*a*, *b*, *c*, *a* + 2) with $b + c = \frac{74.84-4a}{7}$ and

B = (a, b, c, a + 2) with b + c = $\frac{74.84-4\mathrm{a}}{7}$ or vice versa, where *a, b, c,* a, b, c are positive numbers such that *a<b<c<a+2* and a<b<c<a+2.

Therefore the quantities *x* and *y* of the two imported goods will lie between *a* and *a*+2, but most probably between *b* and *c,* and between a and a+2, but most probably between b and c respectively, or vice versa.

Example 2. A cheese-making factory produces three different types of cheese, say T_1, T_2 and T_3, by mixing cow-milk (C), sheep-milk (S) and milk powder (P). The required quantities in kilos for producing a receptacle of each type of cheese are depicted in the form of TFNs in the following Table:

Table 1: Required quantities of milk[1]

	T_1	T_2	T_3
C	(2,3,7)	(1,3,5)	(1,2,3)
S	(0,2,4)	(0.5, 1,1.5)	(2,3,4}
P	(1,1.5,3.5)	(3,6, 9)	(0.2,1,1.8)

At the end of a certain day it has been estimated that the total stock of the cow-milk will be between 90 and 110 kilos, of the sheep-milk between 85 and 115 kilos and of the milk powder between 45 and 55 kilos. Find all the possible TFNs with DoF equal to 1 representing the production of the three types of cheese for the next day.
Solution: The total stocks of milk can be represented by the TFNs P = (90, 100, 110), Q = (85, 100, 115) and T = (45, 50, 55). Next we follow the steps of our method:

1-2. The ranking of the TFNs P, Q, T and of the fuzzy data of Table 1 leads to the ordinary linear SoE $\left.\begin{array}{l} 4x+3y+2z=100 \\ 2x+y+3z=100 \\ 2x+6y+z=50 \end{array}\right]$., which has the unique solution x = 12.5, y = 0, z = 25.

3. Applying Proposition 4, one can convert x to a TFN, say *X*, with DoF equal to 1 of the form *X* = (*a*, 36.5-2*a*, *a*+1), with $\frac{35}{3}<a<\frac{36}{3}$. Similarly y can be converted to a TFN of the form *Y* = (*a*, 74-2*a*, *a*+1), with $\frac{73}{3}<a<\frac{74}{3}$

Example 3. Find all the solutions of the linear FSoE

$$\left.\begin{array}{l} (l-3,l,l+3)x+(m-2,m-1,m+3]y+(0.4,1,1.6)z=(-2,1,4) \\ (0.4,1,1.6]x+(lm-3,lm+1,lm+2)y+(-1,0,4)z=(-2,-1,6) \\ (-2,-1,6]x+[m-4,m+1,m+3]y+[l-2,l,l+2]z=(-3,2,4) \end{array}\right]$$

with DoF equal to 4 for the several real values of the parameters *l* and *m*.
Solution: We follow the steps of our method:
1. The ranking of the coefficients and the constants of the given FSoE leads to the ordinary linear SoE

$$\left.\begin{array}{l} lx+my+z=1 \\ x+lmy+z=1 \\ x+my+lz=1 \end{array}\right] \qquad (3)$$

[1] The fuzzy data of Table 1 show that the production of a receptacle of T_1 requires quantities of cow-milk between 2 and 7 kilos of sheep-milk between 0 and 4 kilos and of milk powder between 1 and 3.5 kilos, etc.

2. The determinant of the above SoE is $D = \begin{vmatrix} l & m & 1 \\ 1 & lm & 1 \\ 1 & m & l \end{vmatrix} = m(l\text{-}1)^2(l\text{+}2)$.

Therefore we distinguish the following cases:

I) If $l \neq 1, -2$ and $m \neq 0$, then $D \neq 0$ and applying the Cramer's rule one finds the unique solution x = z = $\frac{1}{l+2}$, y = $\frac{1}{m(l+2)}$.

3. Thus, converting x, y, z to TFNs with DoF equal to 4 one finds, by Proposition 4, that the given FSoE has the solutions $X =$ $Z = (a,\ \frac{3}{l+2}$ - $2a$-4, a+ 4] with

$\frac{1}{l+2}$ - $\frac{8}{3} < a < \frac{1}{l+2} - \frac{4}{3}$, Y = (a, $\frac{3}{m(l+2)}$ -2a-4, a+4)

with $\frac{1}{m(l+2)}$ - $\frac{8}{3} <$ a $< \frac{1}{m(l+2)} - \frac{4}{3}$.

The steps of our method are repeated in the other cases as it is presented below:

II) If m = 0, then the SoE (3) takes the form $\left.\begin{matrix} lx + z = 1 \\ x + z = 1 \\ x + lz = 1 \end{matrix}\right]$.

Denote by L_1, L_2 and L_3 the first, second and third row of the system's augmented matrix $\begin{bmatrix} l & 1 & | & 1 \\ 1 & 1 & | & 1 \\ 1 & l & | & 1 \end{bmatrix}$. Applying successively the linear transformations $L_1 \leftrightarrow L_2$, $L_2 \rightarrow L_2$ - lL_1, $L_3 \rightarrow L_3$ – L_1 and $L_3 \rightarrow L_3$ + L_2 one finds the equivalent matrix $\begin{bmatrix} l & 1 & | & 1 \\ 0 & 1-l & | & 1=l \\ 0 & 0 & | & 1-l \end{bmatrix}$ from the third row of which it turns out that, if $l \neq 1$ then the SoE (1) has no solution. Therefore the same happens with the corresponding FSoE.

On the other hand, if l =1, then the SoE (1) becomes equivalent to the equation x + z = 1, which has infinite solutions of the form (1-z, z) for the several real values of z.

Therefore, the corresponding FSoE has also infinite solutions of the form X = (a, -1-3z-2a, a+4), -z-$\frac{5}{3} < a <$ -z-$\frac{1}{3}$, Z = (a, 3z-2a-4, a+40, z-$\frac{8}{3}$ < a < z-$\frac{4}{3}$, for the several real values of z.

III) If l = 1, then the SoE (3) becomes equivalent to the equation x + my + z = 1, which has infinite solutions of the form (1 – my – z, y, z) for the several real values of y and z. Therefore the corresponding FSoE has also infinite solutions of the form

X = (a, -1-3 my–3z, a+4), $-\frac{5}{3}$-my-z < a < $-\frac{1}{3}$-my-z, Y = (a, 3y-2a-4, a+4), y-$\frac{8}{3}$ < a < y-$\frac{4}{3}$, Z = (b, 3y-2b-4, b+4), z-$\frac{8}{3}$ < b < y-$\frac{4}{3}$, for the several real values of y and z.

JV) If l = =2, then the SoE (3) takes the form $\left.\begin{array}{l} -2x+my+z=1 \\ x-2my+z=1 \\ x+my-2z=1 \end{array}\right]$.

Then, working similarly with case II it is easy to check that its augmented matrix is equivalent to the matrix $\begin{bmatrix} 1 & -2m & 1 & | & 1 \\ 0 & -3m & 3 & | & 3 \\ 0 & 0 & 0 & | & 3 \end{bmatrix}$. From the third row of this matrix it turns out that the system has no solution, therefore the same happens with the corresponding FSoE.

Example 4. Find with DoF equal to 1 a solution (X, Y) of the FSoE

$$\left.\begin{array}{l} (2,4,6)x^3+(-150,2,67)xy^2+(15,29,31]=(-2,-1,3) \\ (1,2,9)x^2y+(-15,1,5)y^3-(0.6,1,1.4]=(-3,-2,5) \end{array}\right]$$

near to the TFN (0, 1, 2).

Solution: Following the steps of our method we have:

1. The ranking of the coefficients and the constants of the given FSoE, as well as of the given TFN (0, 1, 2 leads to the following problem:

Find a solution of the SoE $\left.\begin{array}{l} 4x^3-27xy^2+25=0 \\ 4x^2y-3y^3-1=0 \end{array}\right]$ (2), near to the point (1, 1).

2. The required solution of the SoE (2) can be found by applying the *Newton's iterative method* (e.g. see [7], Chapter 25, p. 312). The above method solves a SoE of the form $\left.\begin{array}{l} f(x,y)=0 \\ g(x,y)=0 \end{array}\right]$ through the convergence of the sequences defined by $x_{n+1} = x_n + k_n$, $y_{n+1} = y_n + l_n$, where k_n and l_n are determined by the solution of the linear with respect to them SoE $\left.\begin{array}{l} \frac{\partial f}{\partial x}(x_n,y_n)k_n+\frac{\partial f}{\partial y}(x_n,y_n)l_n=-f(x_n,y_n) \\ \frac{\partial g}{\partial x}(x_n,y_n)k_n+\frac{\partial g}{\partial y}(x_n,y_n)l_n=-g(x_n,y_n) \end{array}\right]$, n = 0, 1, 2,.... .One continues the process until reaching the required approximation.

In our case, setting $x_0 = y_0 = 1$ in the above SoE and replacing $f(x, y)$, $g(x, y)$ from SoE (2), one finds that $\left.\begin{matrix} -15k_0 - 54l_0 = -2 \\ 8k_0 - 5l_0 = 0 \end{matrix}\right]$, which has the solution

$k_0 \approx 0.02$, $l_0 \approx 0.03$. This gives that $x_1 \approx 1.02$ and $y_1 \approx 1.03$.

3. Therefore the corresponding solutions of the given FSoE are $X_1 = (a, 2.06\text{-}2a, a+1)$, with $0.35 < a < 0.69$), $Y_1 = (a, 2.09\text{-}2a, a+1)$ with $0.36 < a < 0.7$. Since $R(X_1) = 1.02$ and $R(Y_1) = 1.03$, all these solutions are near to the TFN (0, 1, 2).

4. Conclusion

A new method was developed in this work for solving a FSoE by ranking its coefficients and constants and by solving the obtained in this way ordinary SoE with a traditional method. When it is desirable, the real solutions found may be turned to FNs with the desired DoF. The examples presented here involved systems of the form $F_i(x_1, x_2, \ldots, x_n) = 0$, $i = 1, 2, \ldots, m$, where the coefficients of the equations $F_i = 0$ are TFNs or TpFNs and n, m are integers with $n, m \geq 2$.

One of the targets of our future research is the application of a similar procedure for solving *Fuzzy Linear Programming* problems, whereas other types of FNs or GNs could be also used in the examples instead of TFNs and TpFNs (e.g. see [6], etc.).

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