Synopsis of the proposal for the research work towards Ph.D

entitled

Development of Some Mathematical Models in Inventory Control

by

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Contents

Published papers

- Khanra, S., Mandal, B., Sarkar, B.: An inventory model with time dependent demand and shortages under trade credit policy. *Economic Modellling* 35, 349-355 (2013).Impact factor 0.736
- Sarkar, B., Mandal, B., and Sarkar, S.: Quality Improvement and Backorder Price Discount under Controllable Lead Time in an Inventory Model, *Journal of Manufacturing Systems*, 35, 26-36 (2015). Impact factor 1.68
- Khanra, S., Mandal, B., Sarkar, B.: A Comparative Study Between Inventory Followed by Shortages and Shortages Followed by Inventory Under Trade-Credit Policy. *International Journal of Applied and Computational Mathematics*, DOI 10.1007/s40819-015-0024-z, 3, 399-426 (2015).
- Sarkar, B., Mandal, B., and Sarkar, S.: Preservation of Deteriorating Seasonal Products With Stock-Dependent Consumption Rate and Shortages, *Journal of Industrial and Management Optimization*, doi:10.3934/jimo.2016011 (2016). Impact factor 0.84

List of communicated papers

- Sarkar, B., Mandal, B., and Sarkar, S.: Improved quality and setup cost reduction in an imperfect production process with warranty policy and shortages. Submitted in Journal of Intelligent Manufacturing.
- Mandal, B., Dey, B.K., Khanra, S., Sarkar, B.: An inventory model with advertisement cost as demand under trade credit policy. Submitted in RAIRO journal.

1 Chapter I

1.1 Abstract:

In this research work, our target is to develop some inventory problems taking into account these realistic features as far as possible which is entitled as **'Development of Some Mathematical Models in Inventory Control'**. Inventory is broadly defined as stock of goods, commodities or other economic resources that are stored as reserved at any given period for meeting future demand or future production, i.e. to ensure running of business affair. Mathematical model is based on various real factors like demand, costs deterioration, production, inflation, constraint, etc. The demand rate may be time-dependent, price-dependent or stock dependent and coupling of the phenomena is quite likely in reality. The various cost components associated with the system are set up (ordering) costs, holding (or caring) costs, purchase (or production) costs. If the shortages in inventory are allowed, the shortages costs have to be taken into account. Usually every physical commodity decays or deteriorates over time. If this aspect is considered, one has to accommodate deterioration cost also. In practice, it is observed that supplier offer the retailer a certain credit period in paying purchasing cost. During this delay period, retailer can earn revenue by selling items and by earning interest which is trade credit policy among whole-seller, retailer and customers. Now a day's these are important factors in inventory system. Now this research develop some various problems. We first consider the research model an inventory model with time dependent demand and shortages under trade credit policy. This type of demand rate is more realistic because it can represent both accelerated growth and retarded growth in demand as it has the general form . Here indicates linear time depend demand and as well as simultaneously indicate constant demand. The model has been developed under three circumstances, Case 1: The credit period is less than the time of shortage period, Case 2: The credit period is greater than the time of commencement of shortage period but less than cycle length and Case 3: Credit period is greater than the cycle for settling the account. In the second model we present a comparative study between inventory followed by shortages and shortages followed by inventory under trade credit policy. The models are develop for linear trend in demand. The stock deteriorates over time which follows a two parameter Weibull distribution. Both the models are assumed fixed trade credit period to the retailer from the supplier. The third research model contains Preservation of Deteriorating Seasonal Products with Stock-dependent Consumption Rate and Shortages. In real life situation, deterioration of goods can be reduced by adding some extra effective capital investment in preservation technology. In this paper we formulate a deteriorating inventory model with ramp-type demand under stock-dependent consumption rate by assuming preservation technology cost as a decision variable. Shortages are allowed and the unsatisfied demand is partially backlogged at a negative exponential rate with the waiting time. The objective of this study is to find the optimal replenishment and preservation technology investment strategies so that the total profit per unit time is maximum. The fourth research model is setup cost reduction and warranty policy in an imperfect production process with shortages. This study investigates the effect of setup cost reduction in an imperfect production system with allowable shortages in which the products are sold with free minimal repair warranty. Our intention is to minimize the total relevant cost of the system. The necessary and sufficient conditions are derived for the existence and uniqueness of the optimal solution. A logarithmic investment function is considered to reduce the total setup cost. The fifth research model consists quality improvement and backorder price discount under controllable lead time in an inventory model. The proposed study investigates a continuous review inventory model with order quantity, reorder point, backorder price discount, process quality, and lead time as decision variables. An investment function is introduced to improve the process quality. Two models are developed based on the probability

distribution of lead time demand. The lead time demand follows a normal distribution in the first model and in the second model it does not follow any specific distribution but mean and standard deviation are known.The sixth research model is an extended version of the first one with advertisement cost as demand.

1.2 Introduction:

Inventory control or inventory management is one of the most important field of research in the area of operation research. 'Inventory' is defined as stock of materials in hand to meet the anticipated demand. Inventory thus, is generally used for indicating different categories of resources such as raw materials, goods in process or semi-finished goods, finished goods, packing spars and other stock goods in order to meet an expected demand, or distribution in the market for future. Without inventory, the customers have to wait until their demand are met for delivery to them. According to customers psychology, they may not have such patient to wait and some of them may go to another resources to fulfil their urgent requirement. On the other hand, keeping high volume inventory in hand is not a good decision because there are carrying or holding cost in the system. Furthermore to store much inventory, manufacturer has to produce significant volume of goods in a short time and that requires high percentage of efficient labour, long-run capacity of machines, sufficient space to stock etc. Machine may produce defective items for long run-process and then manufacture has again to invest for conversion of these defective items to perfect items. When the items are placed in idle-situation, some items are deteriorated or damaged. So, in a competitive market, a manager has to make a balance between the situation of over stocking or under-stocking. The main objective of inventory problem is to minimise the total cost or to maximize total profit. In inventory system there are many cost parameters like holding cost, shortage cost, setup cost, purchase cost and deterioration cost etc. There are some other factors in the system like, inflation of money, deterioration, price discount, delay in payment others. Keeping all these situation, a manager is to decide the optimum size of the stock to be replenished, that can fulfil custer service and the time when the goods are to be replenish. Now-a-days, inventory problem has flourished in-any sector of economy besides the industry. The manager of a Government agency, Bank, College, Pollution agency use their own technique to maintain their inventories.

1. It acts as a buffer stock when raw materials are received late and shop rejections are too many.
2. Inventory can solve the problem of customers in such a way that, when customers have to wait for some basic items from the retailers i.e when a shortage occur of the items. This may cause real inconvenience to many customers. Thus without inventories, customers have to wait until the required items are procured and delivered to the customers.
3. It minimize the losses due to inadequate inspection of incoming materials and losses due to deterioration, obsolescence, damages or pilferage, inflation etc.
4. With the help of displaying inventories of goods retailers can attract customers. It is a matter of fact that large piles of goods displayed in a super market will motivate the consumers to buy more.
5. It ensures the supply of the items which are actually demanded by the customer at any time with minimum cost.
6. It gets the financial advantages of transporting or shipping cost to the retailers and the suppliers both.
7. During long run process, when machine produces defective items, a certain amount of production cost is necessary to reduced the defective items original, which itself increases the production costs. Inventory helps the system to control production in such a way that the defective items are not produced by the system. Thus it reduced the production cost and maximized the profit.
8. It ensures proper execution of policies covering procurement and use of materials. It also facilitates

timely adjustment with changing conditions in the market.
9. It help to plan over all operating strategy through decoupling of successive stages in the chain of acquiring goods, preparing products, shipping to branch ware-houses, and finally serving customers.
10. It provides adequate service to customers. If inventories are not maintained, goods have to be procured in a given system precisely when demands for them occur. This practice is neither physically possible nor economically sound. Frequent procurement actions are bound to invite many operational difficulties and higher costs.

Maintenance of inventories also costs money by way of expenses on stores, equipments, personnel, insurance. Thus excess inventories are undesirable. Thus inventories is to be maintain properly so that the profit is maximum. Taking into account these factors, the three fundamental questions in controlling the inventories of any physical good are: i) how much quantity should be ordered or purchased or produced at the beginning of each time interval? ii) when should the order of physical goods be replenished? Or, what is the optimal time of replenishing inventory? iii) how much safety stock should be kept? What quantity of an item in excess of the expected requirements should be held as buffer stock in anticipation of the variations in its demand and or the time involved in acquiring fresh supplier? However, these problems are easy to state in words. In practice, it is not an easy task to determine a suitable inventory policy. An inventory problem is a problem that deals with the decision to minimize the total average system cost or to maximize the overall profit while meeting the consumer's demand. Our aim is to develop operating rules for controlling inventory systems by using mathematical analysis. For this purpose, the first task is to express the inventory system in a mathematical framework. In other words, we have to construct a mathematical model of the inventory system. However, such a mathematical model is based on various assumptions and approximations that fit in the overall framework of the model. The optimal solution of an inventory problem is the set of specific values of the variables that minimizes the system cost or maximizes the total profit of the system under some restrictions, called constraints. We may take the differential calculus or different algorithms or various softwares to optimize the objective function under various constraints of an inventory system according to the nature of the problem.

Literature Review

There exists a vast literature over the historical review of various inventory models. Since the first formulation of economic order quantity (EOQ) model in 1915. Harris's (1915) was the first person who derived the famous square-root formula for the economic order quantity (EOQ) model, which was use in inventory literature for a pretty long time. To the author's knowledge a full length book on inventory problems was first written by F. E. Raymond (1931). It contains no theory or derivation but only have an attempt to explain how various extensions in a simple lot size model can be used in practice. During second World War, a groups of scientists in United Kingdom applied scientific techniques to research military operations to win the war and the techniques thus developed was named as operation research. Thus, the Operation Research started during World War II in Great Britain with the establishment of teams of scientists to study the strategic and tactical problems involved in military operations. The objective was to find the most effective utilization of limited military resources by the use of quantitative techniques. During second World War, operations research originated from the efforts of army advisors. After World War II, O.R. started to spread and expand different ways in the United States. The effectiveness of operations research in military spread interest in it to other governmental departments and industry. Also this Success of Operations Research in army attracted the attention of the industrial mangers who were seeking solutions to their complex business problems. In 1950 industry started to take in a few of the Operational Research scientists who left the army. The main techniques of Operation Research that is limited resources and more effectiveness were involved for decision making in industry, society, health and different business sectors. Also O.R. moving to a focus on the development of mathematical models that can be used to analyse and optimize complex systems, and has become an area of active academic and industrial research. The growth of operations research has not limited to the U.S.A. and U.K., it has reached many countries of the world. India was one the few first countries who started using operations research. In India, Regional Research Laboratory located at Hyderabad was the first Operations Research unit established during 1949. At the same time another unit was set up in Defence Science Laboratory (DSL) under the guidance of Prof. D. S. Kothari to solve the Stores, Purchase and Planning Problems. In 1953, Operations Research unit was established under the guidance of Prof. P. C. Mahalanobis in Indian Statistical Institute, Calcutta, with the objective of using Operations Research methods in National Planning and Survey. Some of the Indian organisations using operations research techniques to solve their varied complex problems are: Railways, Defence, Banking and Finance, production system, Indian Airlines, Fertilizer Corporation of India. A purpose of O.R. is to provide a rational basis for making decisions in the absence of complete information. O.R. can also be treated as science devoted to describing, understanding and predicting the behaviour of systems, particularly man-machine systems. Today Operations Research is a popular subject in management institutes and schools of mathematics.

In 1951, the first book on the subject "Methods of Operations Research", by Morse and Kimball, was published. Few years later, a numerous publications have been devoted solely to this subject. Whitin (1957), Arrow et al. (1958), and Hirshleifer (1958) published a full length book considering mathematical properties of inventory systems. Wagner and Whitin (1958) were able to explain a dynamic version of the economic lot-size model. After this, Holt et al. (1960), Miller (1962), Hanssmann (1962), Hardly and Whitin (1963), Naddor (1966), Silver and Peterson (1985) wrote a book on Operations Research in production and inventory control. Recently Several books devoted in the direction of inventory control are written by Silver, Peterson, Pyke and Pyke (1998), Zipkin(2000), Porteus (2002), Axsater (2006), Golsttein and Levitt (2006), Fogarty (2007), Mcdonald (2009), Frey (2010), Babar et al. (2011), Lee (2011), Hua et al. (2012), Benjaafar and Elhafsi (2012), Dantan et al. (2013), Yu and Benjaafar (2013), Chen and Benjaafar (2013), Benjaafar et al. (2013), Ruths and Ruths (2014), Natarajan et al. (2014), Chakrabarti and Blessing (2015), Luo (2015), Che et al. (2015), Lee and Quek. (2015), Taylor et al. (2015), Cho and Kim (2016), Yan and Luo (2016), among others. The following important factors related to inventory system are discussed below:

Lead time

Most of the researchers consider in their inventory model either zero or fixed lead time. An uncertain gap of time is observed in between the placement of an order and its delivery which leads to consideration of the lead time as a random variable. For this reason, lead time has a topic of interest to many researches like Naddor(1966), Das(1975), Ben-daya and Raouf(1994).

Deterioration

Deterioration refers to decay, damage, spoilage, dryness, and vaporization of the products. The products like foodstuffs, green vegetables, human blood, photographic film etc. having a maximum usable lifetime are known as perishable products and products like alcohol, gasoline, radioactive substances etc. having no shelf life at cell are known as decaying products. To meet the future demands, most of the inventory models in the literature assume that items can be stored indefinitely. However, after a certain time, certain types of inventories either deteriorate or become obsolete. Its effect can be neglected if the rate of decay is extremely low. This type of decay plays a major role in some practical situations and its impact must be incorporated in such inventory models. Realization of this fact, researchers are motivated to take the deterioration factor into their models. Deterioration factor can be found in the review article of Nahmias (1982). More recently, Raafat (1991) provided a comprehensive survey on continuously deteriorating inventory models where deterioration is considered as a function of the on hand inventory.

Inventory cost

The main three types of inventory cost are (a) Inventory carrying cost or stock holding costs. (b) Procurement costs or setup costs. (c) Shortage costs or stock out cost. (a) Inventory carrying cost or stock holding costs They arise on account of maintaining the stocks and the interest paid on the capital tied up with the stocks. They vary directly with the size of the inventory as well as the time for which the item is held in stock. Various components of the stock holding cost are:

(b) Procurement costs or setup costs These include the fixed cost associated with placing of an order or setting up machinery before starting production. They include costs of purchase, requisition, follow up, receiving the goods, quality control, etc. Also called order costs or replenishment costs, they are assumed to be independent of the quantity ordered or produced but directly proportional to the number of orders placed.

(c) Shortage costs or stock-out cost These costs are associated with either a delay in meeting demands or the inability to meet it at all. Therefore, storage costs are usually interpreted in two ways. In case the unfilled demand can be filled at a later stage(backlog case), these costs are proportional to quantity that is short as well as the delay time. They represent loss of goodwill and cost of idle equipment. In case the unfilled demand is lost (no backlog case), these costs become proportional to only the quantity that is short. These result in cancellation of orders, lost sales, profit and even the business itself.

Backlogging

Most of the effort in determining economic lot sizes has been concentrated on two general situations regarding the demand process. When the system is out of stock, either all the demand during the stockout period is back ordered or all the demand during the stockout period is lost forever. These two cases result in models on back orders or lost sales respectively.

Demand

First analytical solution of inventory replenishment policy for a linear trend in demand was done by Donaldson (1977). Later, some researches proposed simple heuristic procedures with less conceptual effort. Many supermarket managers have observed that, for some items, the demand rate is directly related to the amount of inventory displayed. Empirical observations indicate that a price reduction results in an increase in demand (except for some luxury goods). This discount in price motivates the retailers to increase their order quantity and the retailers in turn offer a price discount to their customers to increase demand and profit. Demand for the units stocked in an inventory can seldom be predicted with certainty. They must be described in probabilistic terms. Realistic inventory models must account for uncertainty in demand. One problem that arises is that of providing a buffer stock to protect against shortages when uncertain demand is taken into account. With fixed demand under conditions of certainty, the probability of the demand occurring is one because the number of units to be demanded and used in each period is certain and known in advance; there is no problem of the order size. The order size in such a situation is made equal to the number of units demanded or required. In other words, if the demand for one period is known and remains stable overtime, this same number of units would be acquired at the beginning of each period. The beginning inventory would be consumed during the period, and there would never be an excess or a shortage of inventory at the end of the period. But such a situation is seen very in real life.

Planning horizon

Planning horizon may be finite or infinite. In this case of a fixed planning horizon, the condition of equal replenishment cycles is imposed for the purpose of model-building. But this is not the practical solution. Some researchers like Chung and Ting (1993), suggested to relax the condition of equal duration of replenishment cycles and to adjust the scheduling periods over the fixed planning horizon.

Delay in payments

Before the end of the credit period, the retailer can sell the goods and accumulate revenue and earn interest. He also can get a price discount per unit purchase from the wholesaler.

Lead time reduction

To reduce the safety stock and to improve customer service level, lead time should be shortened. Therefore, controlling the lead time plays an important factor. In modern manufacturing systems like just in time (JIT) or agile, the main objective is to reduce the delivery lead time from one facility to another. Therefore, this research investigates the procedure to reduce the delivery lead time.

Quality improvement

In any production system with long run process, there is a possibility to produce defective items. These items are reworked or refused if sent to customers. Therefore, it is realistic that a cost to be incurred in order to improve the quality of products. This proposal tries to reduce the probability of the occurrence of defective items.

Distribution free approach

In global marketing environment, forecasting of demand, especially the lead time demand, is difficult to achieve. Managers have to pay a lot of money to collect the proper information to know the probability distribution of demand. If any procedure can be obtained to suggest the managers whether or not to invest money for the information, then it would be profitable to the managers. Thus, this research investigates the approach of obtaining the optimal result of a supply chain model without considering any specific probability distribution where the demand can follows any probability distribution.

2 Chapter II

An inventory model with time dependent demand and shortages under trade credit policy

This model investigates an economic order quantity (EOQ) model over a finite time horizon for an itemwith a quadratic time dependent demand by considering shortages in inventory under permissible delay in payments. Shortages are assumed after variable time $T_1(< T$ cycle period). The credit period (m) is known and fixed. The model is derived under three different circumstances depending on the time of occurrence of shortages, credit period, and cycle time. The results are illustrated with the help of numerical examples. The sensitivity analysis of key parameters of the optimal solution is studied with respect to changes in different parametric values. Some special features of the model are discussed.

2.1 Author(s) contribution

Author(s)	Time dependent quadratic demand	Shortages	Trade credit policy
Khanra and Chaudhuri (2003)	√		
Ghosh (2006)	√	√	
Khanra, Ghosh and Chaudhuri (2011)	√		√
This paper	√	√	√

2.2 Conclusions

The proposed model is based on a quadratic time-varying demand rate. The rationale for considering a quadratic demand instead of a linear demand or an exponential demand has been explained at the beginning of this model. While dealing with time-varying demand patterns, the researchers usually take the demand rate to be a linear function of time. This type of demand of the form $R(t) = a + bt \;\; a \geq 0, \; b \neq 0$,implies steady increase $(b \geq 0)$ or decrease $(b \neq 0)$ in the demand rate, which is rarely seen to occur for any product. Some researchers adopted an exponential functional form like $R(t) = ae^{bt} \;\; a > 0, \; b \neq 0$, implying exponential increase $(b > 0)$ or decrease $(b < 0)$ in the demand rate. An exponential rate being very high, it is also rarely seen in the real market that the demand of any product can really rise or fall exponentially. A better alternative would be to think of accelerated rise or fall in demand. Accelerated growth in the demand rate takes place in case of the state of art of aircrafts, computers, machines, and their spare parts. Accelerated decline in the demand rate is found to occur in the case of obsolete aircrafts, computers, machines, and their spare parts. The demand of a seasonal product rises rapidly to a peak in the mid-season and then falls rapidly as the season wanes out. These different types of demand can be better represented by the functional form $R(t) = a + bt + ct^2, a \geq 0, b \neq 0, c \neq 0$.We have $\frac{dR(t)}{dt} = b + 2ct$,for $b > 0, c > 0$, the rate of increase of the demand rate $R(t)$ is itself an increasing function of time. we call it accelerated growth in demand. For $b > 0, c < 0$, there is retarded growth in demand for all times $t \in (0, -\frac{b}{2c})$. For $b > 0, c < 0$, the demand rate falls at an increasing rate for $t > -\frac{b}{2c}$. We call it accelerated decline in demand. For $b > 0, c < 0$, there is retarded decline in the demand rate for all times. Thus, we may have different types of realistic demand patterns from the functional form $R(t) = a + bt + ct^2$ depending on the signs of b and c. This advantage of the time-quadratic demand has motivated authors to adopt it in the present model. Shortages are allowed and are completely backlogged. In many practical situations, stock out is unavoidable due to various uncertainties. Also it is important from the managerial point of view to reduce average total cost. In the real market, we see that suppliers offer their customers a certain credit period without interest during the permissible delay time period. As an outcome, it motivates customer to order more quantities because paying later indirectly reduces the purchase cost. There are several extensions of this model are to consider multi-item EOQ model with variable lead time by considering shortages, inflation, reliability, and partial delay in payments.

3 Chapter III

A comparative study between inventory followed by shortages and shortages followed by inventory under trade credit policy

This paper deals with a comparison between inventory followed by shortages model and shortage followed by inventory model with variable demand rate. It is assumed that the stock deteriorates over time which follows a two parameter Weibull distribution. Both the models are assumed fixed trade credit period to the retailer from the supplier.

3.1 Author(s) contribution

Author(s) Name	Linear demand	IFS	SFI	Variable deterioration	Trade credit policy
Covert and Philip (1973)	√			√	
Goyal *et al.* (1992)	√	√			
Sing and Sing (2009)	√				√
This paper	√	√	√	√	√

3.2 Conclusion

Most of the physical goods under decay or deterioration over time. Electronics goods, radio-active substances, photographic film, grain etc. deteriorate through a gradual loss of potential with passage of time. Fruits, vegetables, foodstuff etc. suffer depletion by direct spoilage when kept in stock. Gasoline, petroleum, alcohol, etc. are highly perishable items. Thus decay or deterioration of physical goods held in stock is a very realistic features. Here we assume that the deterioration rate is a two-parameter Weibull distribution. This type of deterioration is justified for increasing rate or decreasing rate or constant rate of deterioration for different choice of parameters of the distribution function.

The demand rate is taken to be time dependent in contract to constant demand rate in other models. A linear trend in demand is of the form $D(t) = a+bt$ and it represents steady increase or decrease of demand rate over time. Thus it is certainly more realistic than constant demand in the real market. In practice, the supplier offers the retailer a delay period, known as trade credit period, in paying purchasing cost. During the trade credit period, the retailer can accumulate revenues by earning interest of selling items. In a competitive market, the supplier offers different delay-period under different situation to encourage the retailer to run his smooth business.

Shortages are very important for a managerial view. Some retailer allows shortages at the beginning of the planning period to avoid large holding cost or maintenance cost. Many retailers are compelled to allow shortage at the end of planning period.

This model studied a comparative study between IFS policy and SFI policy with the presence of stochastic deterioration of products. The deterioration rate followed a two-parameter Weibull deterioration. An attempt was made by assuming trade-credit financing in this comparative study. This study found that the total cost of the model starting with shortages is less than the total cost of the model starting with no shortages.

4 Chapter IV

Preservation of Deteriorating Seasonal Products with Stock-dependent Consumption Rate and Shortages

Most of the researchers in the past have developed their inventory models by assuming deterioration of items as either a variable or constant. But in real life situation, deterioration of goods can be reduced by adding some extra effective capital investment in preservation technology. In this paper we formulate a deteriorating inventory model with ramp-type demand under stock-dependent consumption rate by assuming preservation technology cost as a decision variable. Shortages are allowed and the unsatisfied demand is partially backlogged at a negative exponential rate with the waiting time. The objective of this study is to find the optimal replenishment and preservation technology investment strategies so that the total profit per unit time is maximum. Further, the necessary and sufficient conditions are considered to prove the existence and uniqueness of the optimal solution. Some numerical examples along with graphical representations are provided to illustrate the proposed model. Sensitivity analysis of the optimal solution with respect to major parameters of the system has been carried out and the implications are discussed.

4.1 Author(s) contribution

Author(s) Name	Ram-type demand	Stock dependent consumption rate	Deterioration	Partial backlogges
Deng *et al.* (2007)	√		√	
Wu *et al.* (2008)	√	√	√	
Skouri *et al.* (2001)	√		√	√
This paper	√	√	√	√

4.2 Conclusions

The effect of deterioration cannot be ignored in inventory system because there are sufficient amounts of goods deteriorate during the normal storage period. Increasing deterioration rate decreases the total profit. Thus it is very essential to control the deterioration of goods. We can control the deterioration rate by adding some extra effective capital investment in preservation technology. In this direction the proposed study considered the concept of preservation technology investment to reduce the deterioration rate. It is a very common phenomenon that any customer would like to choose a single type of item from a large amount of stock. Therefore to keep sell higher, retailers have to store a huge amount of stock. In this regard we considered stock dependent consumption rate in our model. In this marketing environment, when a new brand of consumer goods are launched, the demand of goods increases quickly to a certain moment and after sometime it stabilizes. Finally, it becomes almost constant. Keeping in mind this type of demand pattern, we consider the demand as a ramp-type function of time. Therefore, this model is described with the combination of stock dependent consumption rate, ramp-type demand, and time varying backorder rate with preservation technology investment. We provide a useful solution procedure to find the optimal replenishment policy and preservation technology investment. The proposed model can be used in inventory control of certain deteriorating items such as food items, electronic components, fashionable commodities, and others. There are several extensions of this work that could constitute future research related in this field. This model can be extended in several ways, like multi-item inventory models, inflations, variable deterioration, etc. The research can also be extended to consider fuzzy demand case.

5 Chapter V

Improved quality and setup cost reduction in an imperfect production process with warranty policy and shortages

The main purpose of this paper is to extend [Chung KJ, Hou KL (2003) An optimal production run time with imperfect production processes and allowable shortages. Comp & Oper Res 30: 483-490] with setup cost reduction and warranty policy. We seek to minimize the total cost by simultaneously optimizing the production run time and setup cost. The necessary and sufficient conditions are derived for the existence and uniqueness of the optimal solution. A logarithmic investment function is considered to reduce the total setup cost. Finally, numerical example, sensitivity analysis along with graphical representations are shown to illustrate the practical application of the proposed model.

5.1 Author(s) contribution

Author(s) Name	Setup cost reduction	Warranty policy	Shortges
Chung and Hou (2003)			√
Sana (2011)		√	
Lin (2012)	√		
This paper	√	√	√

5.2 Conclusions

In most of the inventory model, it is considered so far that the produced products are perfect in nature. But in reality, due to the different types of machinery problems during production run-time, it is often observe that some of the items may be defective in nature which are reworked at a cost to make them perfect. In this paper, these defective items are immediately reworked at a cost in a parallel system. To attract customers, a warranty period is provided. We proved two effective lemmas which gave the optimal solutions that satisfied the existence and uniqueness property of the solutions. We obtained more savings than the exiting model (See for instance Chung and Hou (2003)) for setup cost reduction with warranty policy. In basic inventory models, setup cost was treated as a constant. Moreover, if the setup cost could be reduced effectively, the total relevant cost per unit time could be automatically improved. This strategy is supported by the numerical examples, so from the practical point of view, this strategy is valid and useful to the competitive business. The numerical results showed that the savings of expected total annual cost are realized when the setup cost reduction could be achieved through extra investments (by comparing the results with fixed setup cost model).

For future research, the model can be extended by considering preventive maintenance. Another interesting research may be conducted by considering an inspection policy.

6 CHAPTER VI

Quality improvement and backorder price discount under controllable lead time in an inventory model

The proposed study investigates a continuous review inventory model with order quantity, reorder point, backorder price discount, process quality, and lead time as decision variables. An investment function is introduced to improve the process quality. Two models are developed based on the probability distribution of lead time demand. The lead time demand follows a normal distribution in the first model and in the second model it does not follow any specific distribution but mean and standard deviation are known. We prove two lemmas to obtain optimal solutions for the normal distribution model and distribution free model. Finally, some numerical examples are given to illustrate the model.

6.1 Author(s) contribution

Author(s) Name	Quality improvement	Controllable lead time	Back-order price discount	Reorder point
Porteus (1986)	√			
Pan and Hsiao (2005)		√	√	
Ouyang *et al.* (2002)	√	√		√
This paper	√	√	√	√

6.2 Conclusions

This study discussed about two models as first model with normally distributed lead time demand and second model without any specific distribution with known mean and standard deviation. The purpose of this study was to minimize the total expected cost with order quantity, reorder point, backorder price discount, process quality, and lead time as decision variables. A logarithmic investment function was considered to improve the quality of products. For each model, a lemma was derived to show the global optimality of decision variables. Two separate computationally efficient algorithms were provided to obtain optimal solutions of the model. Furthermore, some numerical examples were given to compare with the existing models. For future research, the model can be extended by considering fuzzy demand and delay-in-payments concepts in this model. This study can be extended further if we consider the above-mentioned extensions with real data from an industry. Then we can allow more managerial insights based on that real data.

7 CHAPTER VII

An inventory model with advertisement cost as demand under trade credit policy

This paper deals with an inventory model for items with stock level and advertising dependent demand pattern by considering shortages under trade-credit policy. Based on the time of occurrence of shortages, credit period, and cycle time the model is derived under three different circumstances. We establish the necessary and sufficient conditions for each case to show the existence and uniqueness of the optimal solution. The main purpose is to minimize the retailer's annual total cost with finite replenishment rate. Some numerical examples for different cases are provided to illustrate the model. The sensitivity analysis of key parameters of the optimal solution is studied with respect to changes in different parametric values.

7.1 Author(s) contribution

Author(s) Name	Stock/time dependent demand	Advertisement cost dependent demand	Time,Stock, advertisement demand	Deterioration	Trade credit policy
Chang *et al.* (2010)	√			√	
Singh (2013)	√			√	√
Giri and Sharma (2014)		√			
This paper			√	√	√

7.2 Conclusions

The business sectors invest more finance in advertising their products to sell their products more and to gain more profits at optimum level. Thus advertising by the sales team is one of the most important factor to increase the companies profit in modern marketing system. The purpose of the advertising is to enhance potential buyers responses to a business organization. In general, this strategy is only for to sell more items at short time. More demand implies more selling of the items. The sells team helps to identify their products with different facilities and more reliable among all the products in the market. The advertising intensity increases not only the probability of successful marketing targets but also the demand of the customers. Therefore, the more investment in advertising implies more profits for the company. In the competitive marketing environment, every supplier wants to sell more items to earn more revenues, as a result, most of the suppliers offer a delay period to stimulate the retailer to buy more items. Before the end of the delay period, the retailer can sell his products, accumulates revenue and earns interest. Thus, the delay in payment by the supplier is a kind of price discount which encourages the retailers to increase their order quantity. Several papers discussing the topic have been mentioned in the literature that investigates inventory problems under varying conditions. The proposed model extends the model of existing literature with finite replenishment rate, stock-dependent and advertising demand and delay in payments with two progressive periods. The model may be generalized by considering shortages, time-varying deterioration for multi-item perishable products.

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