A STUDY OF SOME MAGNETOHYDRODYNAMICS PROBLEMS WITH OR WITHOUT HALL CURRENTS

A SYNOPSIS

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Introduction

The magnetohydrodynamics (MHD) is the study of the interaction between magnetic fields and moving conducting fluids. The word MHD covers the phenomena, where, in an electrically conducting fluid, the velocity field and the magnetic field are coupled. The field of MHD was initiated by Hannes Alfvén for which he received the Nobel Prize in physics in *1970*. An induced electric current is generated due to the flow of an electrically conducting fluid in the presence of a magnetic field which modifies the electromagnetic field. The flow field in turn is modified by the force due to the electromagnetic field called Lorentz force. This force tends to modify the initial motion of the conductor.

Hartmann[1] initiated the study of the subject in the name Hg-dynamics in his efforts to pump mercury by exploitation of hydrodynamical pressure and electromagnetic fields. The systematic study under the present name began with discovery of transverse waves by Alfvén[2] while he was engaged in the theoretical investigations of sunspots. The study of magnetohydrodynamics in astrophysical and geophysical problem have been made by many authors, viz. Cowling[3], Bullard[4] and many others. Kakutani[5] has investigate the effect of tranverse magnetic field on the flow due to an oscillating plate. The Hall effects in the viscous flow of ionized gas between parallel plates under transverse magnetic field have been analyzed by Sato[6], Yamanishi[7], Sherman and Sutton[8]. With regard to external hydromagnetic flows, Katagiri[9] has discussed the effects of Hall current on the boundary layer flow past a semi-infinite flat plate. Soundalgekar et al.[10] have studied the combined effects of Hall current and ion-slip on MHD Couette flow with heat transfer. The effects of Hall current on hydromagnetic flow near an accelerated plate has been analyzed by Pop[11]. Pop and Soundalgekar[12] have discussed the effects of Hall current on hydromagnetic flow near a porous plate. The effects of Hall current and coriolis force on Hartmann flow under general wall conditions have been analyzed by Nagy and Demendy[13].

Earlier works relevant to the present investigations

A brief review of the earlier investigations of the problems which has bearing on the contents of the thesis.

Present Investigations

In this thesis, we studied some problems on magnetohydrodynamics with or without Hall current. Some problems are also studied in rotating environment.
The thesis consists of ten Chapters, the *Chapter 1* presents the introductory part. In this Chapter, basic equations on certain relevant topics governing the flow of an electrically conducting incompressible fluid are discussed briefly. Moreover, earlier works related to the present investigations have also been discussed. A study of some magnetohydrodynamics problems on fluid dynamics with or without Hall current have been presented in next *eight Chapters*. *Chapters 2 - 9* are developed by mathematical formulation based on the geometry of the problem and carried on graphical presentation. Furthermore, conclusion about the present investigation and suggestion for future works have been given in *Chapter 10*. The thesis is devoted to study the following problems: *(i)* Unteady MHD flow and heat transfer past a porous flat plate in a rotating system, *(ii)* Radiation effects on unsteady MHD free convective Couette flow of heat generation/Absorbing fluid, *(iii)* Hall effects on unsteady hydromagnetic flow past an accelerated porous plate in a rotating system, *(iv)* Effects of Hall current and radiation on unsteady MHD flow past a heated moving vertical plate, *(v)* Combined effects of Hall current and rotation on unsteady Couette flow in a porous channel, *(vi)* Combined effects of Hall current and radiation on MHD free convective flow in a vertical channel with an oscillatory wall temperature, *(vii)* Hall effects on hydromagnetic free convection in a heated vertical channel in the presence of an inclined magnetic field and thermal radiation and *(viii)* Hall effects on an unsteady magneto-convection and radiative heat transfer past a porous plate.

Chapter1: Introduction

Throughout, the subject is based on basic equations built up in the *first Chapter*. Some of more important topics like Maxwells equations, Ohm's law, Energy equation *etc.* have been summarised in this Chapter.

Chapter2: Unsteady MHD flow and Heat Transfer past a Porous Flat Plate in a Rotating System*

Consider the unsteady flow of viscous incompressible electrically conducting fluid past an infinite heated porous flat plate with uniform suction or blowing at the plate. The plate oscillates in its own plane with the velocity $u_0\, e^{\beta^* t} \cos\omega t$ in a given direction, where ω is the frequency of temperature oscillation, β^* the accelerating index and u_0 a constant. The amplitude of oscillations decreases for acceleration ($\beta^* > 0$) and increases for deceleration ($\beta^* < 0$). We choose the x-axis along the plate, y- axis perpendicular to the plate and z-axis normal to the xy-plane. Both the plate and the fluid are in a state of rigid body rotation with uniform angular velocity Ω about the y-axis. An external uniform magnetic field of strength B_0 is imposed perpendicular to the plate [See Figure.1] and the plate is taken electrically non-conducting. The plate is kept at the constant temperature T_w. Since the plate is infinitely long along the x and z-directions, the velocity field and temperature distribution are functions of y and t only. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $\frac{\partial v}{\partial y} = 0$ which on integration yields $v = -v_0$(constant), where $\vec{q} \equiv (u, v, w)$. The constant v_0 denotes the normal velocity at the plate, is positive for suction and negative for blowing/injection. We shall assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. This assumption is justified since the magnetic Reynolds number is generally very small for partially ionized gases. The solenoidal relation $\nabla \cdot \vec{B} = 0$ for the magnetic field gives $B_y = B_0 =$ constant everywhere in the fluid, where $\vec{B} \equiv (0, B_y, 0)$.

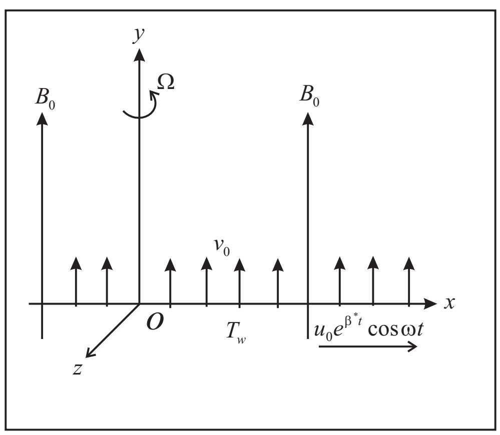

Figure 1: Geometry of the problem

*Published in Int. J. of Computer Appl.(IJCA): ISSN:0975-8887, 33(2)(2011), pp.17-26.

Conclusions: It is found that with an increase in either magnetic parameter M^2 or suction parameter S the primary velocity u_1 and the magnitude of secondary velocity w_1 of fluid decrease at a particular point in flow filed. The primary velocity u_1 and the magnitude of secondary velocity w_1 increase with an increase in either accelerated parameter β or frequency parameter n or time τ. It is found that with an increase in magnetic field intensity the mean temperature $\theta_0(\eta, \tau)$ increases at a particular point in flow filed. Further, it is found that the magnitude of tangent of the phase angle of the rate of heat transfer oscillations $\tan\psi$ decreases with an increase in either rotation parameter K^2.

Chapter3: Radiation Effects on unsteady MHD free convective Couette Flow of Heat Generation/Absorbing Fluid†

Consider the unsteady free convective MHD Couette flow of a viscous incompressible radiative heat generating fluid between two infinite vertical parallel walls separated by a distance h. The flow is set up by the buoyancy force arising from the temperature gradient occurring as a result of asymmetric heating of the parallel plates as well as constant motion of one of the plates. Choose a cartesian co-ordinates system with the x-axis along the plate in the vertically upward direction and the y- axis normal to the plates [See Figure.2]. Initially, at time $t = 0$, both the plates and the fluid are assumed to be at the same temperature T_h and stationary. At time $t > 0$, the plate at $y = 0$ starts to move in its own plane with a velocity $U(t)$ and is heated with temperature T_0 whereas the plate at $y = h$ is stationary and maintained at a constant temperature T_h. A uniform magnetic field of strength B_0 is imposed perpendicular to the plates. It is also assumed that the radiative heat flux in the x-direction is negligible as compared to that in the y-direction. As the plates are infinitely long along the x-direction, the velocity field and temperature distribution are functions of y and t only.

Conclusions: It is found that the velocity u_1 decreases with an increase in magnetic parameter M^2. It is also observed that the velocity u_1 increases with an increase in either heat generation parameter or Grashof number Gr or time τ for both impulsive motion as well as for accelerated motion. An increase in either radiation parameter R or Prandtl number Pr leads to fall in the fluid temperature θ. Further, it is seen that the absolute value of the shear stress τ_x at the moving plate increases for the impulsive and accelerated motion with an increase in either magnetic parameter or radiation parameter while it decreases with an increase in either heat generation parameter or

† *Published in Advances in Int. J. of Computer Appl.(IJCA): ISSN:0975-8887, 39(3)(2012), pp.42-51.*

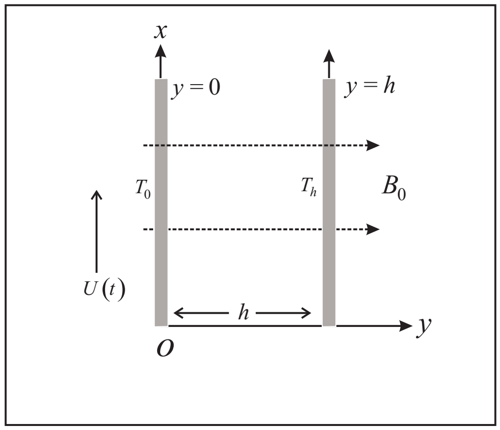

Figure 2: Geometry of the problem

Prandtl number.

Chapter4: Hall effects on unsteady hydromagnetic flow past an accelerated porous plate in a rotating system‡

Consider the unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid past an accelerated porous flat plate in the presence of a uniform transverse magnetic field in a rotating system. Choose a Cartesian co-ordinates system with x-axis along the plate in the direction of the flow, z-axis is normal to the plate and y-axis is perpendicular to the xz-plane [See Figure.3]. Initially, at time $t = 0$, both the plate and the fluid are assumed to be at rest. At time $t > 0$, the plate at $z = 0$ starts to move in its own plane with the velocity at, where t is the time and a being a constant. A uniform magnetic field of strength B_0 is imposed perpendicular to the plate. The effects of Hall current and rotation give rise to a force in y-direction, which induces a cross flow in that direction. Since the plate is infinitely long along x and y-directions and is electrically non-conducting, the velocity field and temperature distribution are functions of z and t only. Also, no applied or polarized voltages exist, so the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cowling 1957). It is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is justified because magnetic Reynolds number is very small for partially ionized fluids which are commonly used in industrial applica-

‡ *Published in J. of Appl. Fluid Mechanics (JAFM): ISSN:1735-3572, IF:0.89, 8(3)(2015), pp.409-417.*

tions. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $w = -w_0$, where u,v and w_0 being the velocity components along the coordinate axes. Here $w_0 > 0$ for suction and $w_0 < 0$ for blowing/injection at the plate.

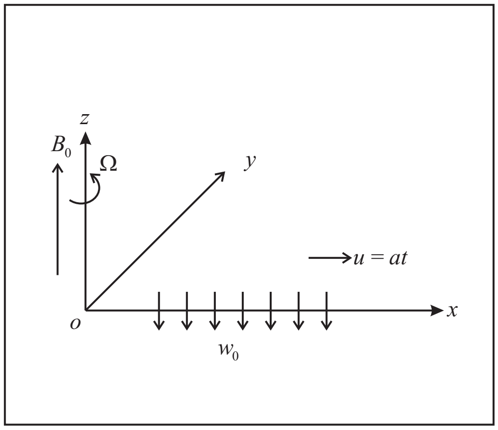

Figure 3: Geometry of the problem

Conclusions: It is seen that Hall current tends to accelerate the primary and secondary fluid velocities. Rotation has tendency to retard primary fluid velocity and tends to accelerate secondary fluid velocity. The primary and secondary fluid velocities are getting accelerated with the progress of time. Rotation tends to enhance both the shear stresses at the plate. On the other hand, the absolute value of the shear stress τ_x due to the primary flow decreases whereas the absolute value of the shear stress τ_y due to the secondary flow increases with an increase in Hall parameter m.

Chapter5: Effects of Hall current and radiation on unsteady MHD flow past a heated moving vertical plate[§]

Consider the unsteady hydromagnetic viscous incompressible electrically conducting fluid past a moving vertical plate with variable plate temperature. Choose a Cartesian co-ordinates system with x- axis along the plate in the direction of the flow, z-axis is normal to the plate and y-axis is perpendicular to the xz-plane [See Figure.4]. Initially, at time $t = 0$, both the plate and the fluid are assumed to be at the same temperature T_∞ and stationary. At time $t > 0$, the plate at $z = 0$ starts to move in its own plane with a uniform velocity U_0 and is heated with temperature $T_\infty + (T_w - T_\infty)\,\frac{t}{t_0}$, where T_w is the plate temperature. A uniform magnetic field of strength B_0 is imposed

[§] *Published in J. of Applied Fluid Mechanics (JAFM): ISSN:1735-3572, IF:0.89, 7(4)(2014), pp.683-692.*

perpendicular to the plate. It is also assumed that the radiative heat flux in the x-direction is negligible as compared to that in the z- direction. Since the plate is infinitely long along x and y-directions, the velocity field and temperature distribution are functions of z and t only. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $w = 0$ everywhere in the fluid, where $\vec{q} \equiv (u, v, w)$.

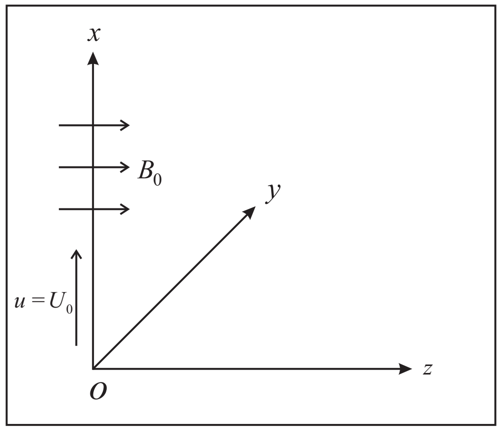

Figure 4: Geometry of the problem

Conclusion: It is found that the Hall parameter m accelerates the primary velocity u_1 as well as the magnitude of the secondary velocity v_1. The temperature θ reduces with an increase in radiation parameter R. This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid. Further, the absolute value of the shear stress τ_x due to the primary flow at the moving plate $(\eta = 0)$ decreases whereas the absolute value of the shear stress τ_y due to the secondary flow at the moving plate $(\eta = 0)$ increases with an increase in Hall parameter m.

Chapter6: Combined Effects of Hall Current and Rotation on Unsteady Couette Flow in a Porous Channel¶

Consider the unsteady MHD flow of a viscous incompressible electrically conducting fluid between two infinite parallel porous plates separated by a distance h when both the fluid and channel are rotate in unison about an axis normal to the plates with a uniform angular velocity Ω. Choose a Cartesian co-ordinate system with x-axis along the lower stationary plate in the direction of flow, z-axis is normal to the plates and y-axis is perpendicular to the zx-plane. A uniform magnetic field of strength B_0 is

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imposed perpendicular to the plates. The flow within the channel is induced due to the movement of the upper plate ($z = h$) parallel to itself in x-direction with a uniform velocity U_0. Initially, at time $t = 0$, both the fluid and the plates of the channel are assumed to be at rest. At time $t > 0$, the upper plate ($z = h$) suddenly starts to move with uniform velocity U_0 along x-direction in its own plane while the lower plate ($z = 0$) is held at rest [See Figure.5]. Since the plates of the channel are infinitely long parallel to the x and y-directions and are electrically non-conducting, all the physical quantities, except pressure, will be the functions of z and t only. Suction/injection of the fluid takes place through the porous plates of the channel with uniform velocity w_0 which is $w_0 > 0$ for suction and is $w_0 < 0$ for blowing/injection. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $w = -w_0$ everywhere in the fluid, where $\vec{q} \equiv (u, v, w)$.

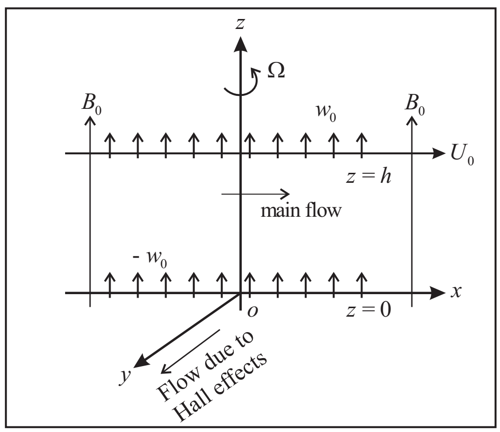

Figure 5: Geometry of the problem

Conclusions: It is found that the primary velocity u_1 and the magnitude of the secondary velocity v_1 decrease with an increase in Hall parameter m. It is also found that the primary velocity u_1 decreases while the magnitude of the secondary velocity v_1 increases with an increase in rotation parameter K^2. It is seen that the solution for small time converges more rapidly than the general solution. It is interesting to note that for large values of M^2, the boundary layer thickness is independent of the rotation parameter. It is found that both the shear stress τ_{x_0} due to the primary flow and magnitude of the shear stress τ_{y_0} due to secondary flow decrease with an increase in M^2 when m is fixed while τ_{x_0} first decreases, reaches a minimum and then increases and the magnitude of τ_{y_0} increases with an increase in m when M^2 is fixed.

Chapter7: Combined effects of Hall current and radiation on MHD free convective flow in a vertical channel with an oscillatory wall temperature‖

Consider the unsteady MHD flow of a viscous incompressible electrically conducting radiative fluid between two infinitely long vertical parallel walls separated by a distance h. The flow is set up by the buoyancy force arising from the temperature gradient. Choose a Cartesian co-ordinates system with x- axis along the channel wall in the vertically upward direction, y-axis perpendicular to the channel walls and z-axis is normal to the xy-plane [See Figure.6]. Initially, at time $t = 0$, both the walls and the fluid are assumed to be at the same temperature T_h and stationary. At time $t > 0$, the wall at $y = 0$ starts to move in its own plane with a velocity $U(t)$ and its temperature raised to $T_h + (T_0 - T_h)\cos\omega t$ whereas the wall at $y = h$ is stationary and maintained at a constant temperature T_h, where ω is the frequency of the temperature oscillations. A uniform transverse magnetic field of strength B_0 is applied perpendicular to the channel walls. We assume that the flow is laminar and the pressure gradient term in the momentum equation can be neglected. Here we also neglect the viscous and Joule dissipations. It is also assumed that the radiative heat flux in the x-direction is negligible as compared to that in the y-direction. As the channel walls are infinitely long along x and z-directions, the velocity field and temperature distribution are functions of y and t only.

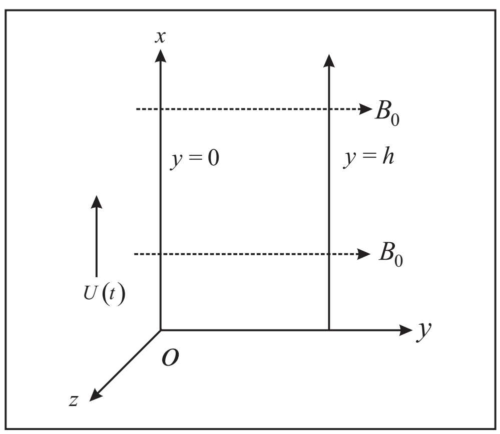

Figure 6: Geometry of the problem

Conclusions: It is observed that the primary velocity u_1 and the magnitude of the secondary velocity v_1 decrease with an increase in either radiation parameter R or

‖ *Published in Open J. of Fluid Dynamics (OJFD): ISSN:2165-3852, 3(1)(2013), pp.9-22.*

frequency parameter n or Prandtl number Pr for both the impulsive as well as accelerated motions of one of the channel walls. The fluid temperature θ increases with an increase in Prandtl number Pr whereas it decreases in either radiation parameter R or frequency parameter n. Further, the shear stress τ_x due to the primary flow and the absolute value of the shear stress τ_y due to the secondary flow at the wall $\eta = 0$ decrease with an increase in either radiation parameter R or frequency parameter n or Prandtl number Pr for both the impulsive as well as accelerated motions of one of the channel walls.

Chapter8: Hall effects on unsteady hydromagnetic free convective flow between two heated vertical plates in presence of thermal radiation**

Consider the unsteady hydromagnetic free convective flow of a viscous incompressible electrically conducting fluid between two infinite vertical parallel plates separated by a distance $2h$. The channel plates are electrically nonconducting. Choose a Cartesian co-ordinates system with the x-axis along vertically upward direction, z-axis normal to the plates and y-axis is perpendicular to the xz-plane [See Figure.7]. Initially, at time $t = 0$, both the plates and the fluid are assumed to be at the same temperature T_h. At time $t > 0$, the plate at $(z = -h)$ is heated with the variable temperature $T_h + (T_0 - T_h)(1 - e^{-n\tau})$, where T_0 is the temperature at the plate at $z = h$ and $n(> 0)$,a real number, denotes the decay factor. The plate at $z = h$ is thermally insulated. A uniform magnetic field of strength B_0 is imposed at an angle ϕ to the x-axis. It is also assumed that the radiative heat flux in the x-direction is negligible in comparison with that in the z- direction. As the plates are infinitely long along the x and y directions, the velocity field and temperature distribution are functions of z and t only. In accordance with the Boussinesq approximation, we assume that all fluid properties are constant, except the density, which varies with temperature only in the body force term.

Conclusions: It is found that both the primary velocity u_1 and the secondary velocity v_1 decrease with an increase in magnetic parameter M^2. An increase in angle of inclination of the magnetic field leads to accelerate the primary velocity while decelerate the secondary velocity. The primary velocity u_1 as well as the secondary velocity v_1 accelerate with an increase in Grashof number Gr. The fall of temperature θ will be shown with an increase in Prandtl number Pr. Prandtl number Pr is the ratio of the viscosity to the thermal diffusivity. An increase in thermal diffusivity leads to a

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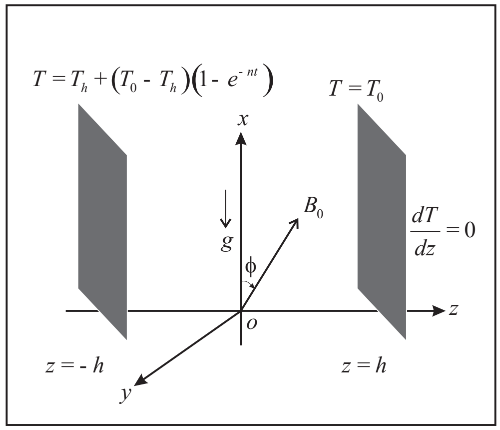

Figure 7: Geometry of the problem

decrease in Prandtl number. Therefore, thermal diffusion has a tendency to reduce the fluid temperature. The absolute values of the shear stress τ_{x_1} due to the primary flow as well as the shear stress τ_{y_1} due to the secondary flow reduce with an increase in either radiation parameter R or magnetic parameter M^2.

Chapter9: Hall effects on an unsteady magneto-convection and radiative heat transfer past a porous plate[††]

Consider the unsteady magnetohydrodynamic free convective flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in the presence of a uniform transverse magnetic field and thermal radiation. The motion of the fluid is induced by both pressure gradient and gravity. Choose a Cartesian co-ordinates system with x-axis along the plate in the direction of the flow vertically upward, y-axis is perpendicular to the plate and z-axis is normal to the xy-plane as shown in [See Figure.8]. The plate at $y = 0$ is at rest and heated with the temperature T_w. A uniform magnetic field of strength B_0 is imposed perpendicular to the plate. It is assumed that the radiative heat flux in the x-direction is negligible as compared to that in the y- direction. As the plate is infinitely long along x and z-direction, the velocity field and temperature distribution are functions of y and t only. The equation of continuity $\nabla \cdot \vec{q} = 0$ gives $\frac{\partial v}{\partial y} = 0$ which on integration yields $v = -v_0$(constant), where $\vec{q} \equiv (u, -v_0, w)$. The constant v_0 denotes the normal velocity at the plate, is positive for suction and negative for blowing/injection.

[††] *published in Alexandria Engineering Journal(AEJ)(Elsevier), ISSN:1110-0168, 55(2)(2016), pp.1321-1331.*

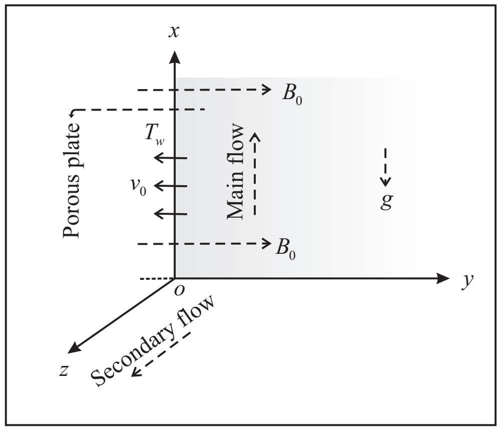

Figure 8: Geometry of the problem

Conclusions: It is found that the velocity components $f(\eta, \tau)$ and $g(\eta, \tau)$ accelerate with increase of the strength of magnetic field and buoyancy force. In the presence of thermal radiation, the temperature $\theta(\eta, \tau)$ of the fluid increases whereas it decreases when Prandtl number Pr enlarges. Also, the fluid temperature enhances for increasing values of temperature difference parameter. Suction (or injection) has a profound effect on the boundary layer thickness in which the suction reduces the thermal boundary layer thickness whereas injection thickens it. It is found that the shear stress $f'(0, \tau)$ increases for increasing values of magnetic parameter M^2 while it reduces with an increase in Hall parameter m. Thus, the magnetic field acts as the accelerating force that causes the shear stress $f'(0, \tau)$ to enhance significantly. Positive value of $f'(0, \tau)$ means the fluid exerts a drag force on the plate along x-direction, while a negative value of $f'(0, \tau)$ means the opposite.

Chapter10: Conclusion and suggestion for future work

Chapter 10 covers with the conclusion and suggestion for the future work. In this last Chapter to conclude this dissertation, we would first like to remind the geometry of problems performed during this research work and in particular, the flow characteristics which have lead to novel ideas or particularly interesting results. Then we will sketch the ideas that have been arisen and that we have considered, are important in the frame of a future continuation of this work.

Bibliography

[1] Hartmann J., Kgl. Danske Vidensk. Selsk. Mat. Fys. Medd: **15(6)**(1937).

[2] Alfvén H., Nature: **150**(1942), pp. 405.

[3] Cowling T.G., Magnetohydrodynamics, Interscience Publisher, Inc, New York: (1957).

[4] Bullard E.C., Mon. Nat. Roy. Astro. Soc., Geophys. Suppl: **5**(1948), pp. 245.

[5] Kakutani, T., J. Phys. Soc. Jpn.: **13**(1958), pp. 1504.

[6] Sato, H., J. Phys. Soc. of Japan: **16**(1961), pp. 1427.

[7] Yamanishi, T., Preprint, 17 th annual meeting, Phys. Soc. of Japan, Osaka: **5**(1962), pp. 27.

[8] Sherman, A. and Sutton, G. W., Engeneering magnetohydrodynamics, McGral-Hill book company, Inc., New York: (1965), pp. 363.

[9] Katagiri, M., J.Phys. Soc. Japan: **27**(1969), pp. 1051.

[10] Soundalgekar, V. M., Vighnesam, N. V. and Takhar, H. S., IEEE, Transactions on Plasma Sci.: **7**(1978), pp. 178.

[11] Pop, I., J. Math. Phys. Sci.: **5**(1971), pp. 375.

[12] Pop, I. and V. M. Soundalgekar, Acta Mechanica: **20**(1974), pp. 316.

[13] Nagy, T. and Demendy, Z., Acta Mechanica: **113**(1995), pp. 77.

Published Papers

The thesis includes eight papers published in peer reviewed journals. List of publications during the thesis work as follows:

(i) Unsteady MHD flow and heat transfer past a porous flat plate in a rotating system, *published in International Journal of Computer Applications (IJCA), ISSN:0975-8887, 33(2)(2011), pp.17-26.*

(ii) Radiation effects on unsteady MHD free convective Couette flow of heat generation / absorbing fluid, *published in International Journal of Computer Applications (IJCA), ISSN:0975-8887, 39(3)(2012), pp.42-51.*

(iii) Hall effects on unsteady hydromagnetic flow past an accelerated porous plate in a rotating system, *published in Journal of Applied Fluid Mechanics (JAFM), ISSN:1735-3572, IF:0.89, 8(3)(2015), pp.409-417.*

(iv) Effects of Hall current and radiation on unsteady MHD flow past a heated moving vertical plate, *published in Journal of Applied Fluid Mechanics (JAFM), ISSN:1735-3572, IF:0.89, 7(4)(2014), pp.683-692.*

(v) Combined effects of Hall Currents and rotation on unsteady MHD Couette flow in a porous channel, *published in World Journal of Mechanics (WJM), ISSN:2160-049X, 1(3)(2011), pp.87-99.*

(vi) Combined effects of Hall current and radiation on MHD free convective flow in a vertical channel with an oscillatory wall temperature, *published in Open Journal of Fluid Dynamics (OJFD), ISSN:2165-3852, 3(1)(2013), pp.9-22.*

(vii) Hall effects on unsteady hydromagnetic free convective flow between two heated vertical plates in the presence of thermal radiation, *published in Turkish Journal of Engineering and Environmental Sciences (TJEES), ISSN:1300-0160, 38(3)(2014), pp.434-454.*

(viii) Hall effects on an unsteady magneto-convection and radiative heat transfer past a vertical porous plate, *published in Alexandria Engineering Journal (AEJ)(Elsevier), ISSN:1110-0168, 55(2)(2016), pp.1321-1331.*