Synopsis of Thesis Entitled

A Comprehensive Study on Lattice Under Uncertain Environments

submitted to the

Vidyasagar University, Midnapore

For award of the degree

of

Doctor of Philosophy

in Science

by

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INDIA

2017

Synopsis of Ph. D. Work

1 Introduction

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. Recently it became also a crucial issue for computer scientists, particularly in the area of Artificial Intelligence. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. There are theories, as for example, theory of probability, theory of fuzzy sets [37], theory of intuitionistic fuzzy sets [2], theory of vague sets theory of interval mathematics [2] which can be considered as mathematical tools for dealing with uncertainties. Rough set theory was proposed by Pawlak in [21] presents still another attempt to this problem. Pawlak [22] introduced the theory of rough set as an extension of set theory for the study of incomplete information. The rough set theory is often a useful and powerful approach to deal with uncertainty but have some inherent difficulties mentioned by Molodtsov [17]. He further pointed out that the reason for these limitations is, possibly, the inadequacy of the parameterization tool of the theory. Soft set theory was introduced by Molodstov [17] which is quite different from these theories in this context. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable.
The study of the algebraic structure of the mathematical theory proves itself effective in making the applications more efficient. Studying on rough set, soft set and rough soft set combined with lattice is an interesting topic to the researchers.

1.1 Rough Set

Rough set was first introduced by Pawlak [21] which is a framework for systematic study of incomplete knowledge i.e., to vagueness. In this approach, vagueness is expressed by a boundary region of a set. Rough set theory consists of two key notions: rough set approximations and information systems. Rough set approximations are defined by mean of indiscernibility relations which are equivalently interpreted as two objects are equivalent

if we cannot distinguish them by using our information. That is our ability to discern object is limited. So we cannot observe individual object. For this reason we form equivalence classes of the equivalence indiscernibility relation called granules. Let U be the set of universe and ρ be an equivalence relation on U. An equivalence class of $x(\in U)$ is denoted by $[x]_\rho$ and is defined as follows: $[x]_\rho = \{y \in U : x\rho y\}$.
The lower and upper approximations of $X \subseteq U$ are denoted by $A_\star(X)$ and $A^\star(X)$ respectively and defined as follows:

$$A_\star(X) = \{x \in U : [x]_\rho \subseteq X\} \text{ and } A^\star(X) = \{x \in U : [x]_\rho \cap X \neq \phi\}.$$

The difference $B(X) = A^\star(X) - A_\star(X)$ is called boundary region of X. The pair $S = (U, \rho)$ is called an approximation space. Then $A(X) = (A_\star(X), A^\star(X))$ is called the rough set of X in S. The rough set approach seems to be of fundamental importance in artificial intelligence and cognitive sciences, especially in research areas such as machine learning, intelligent systems, inductive reasoning, pattern recognition, mereology, knowledge discovery, decision analysis, and expert systems. The main advantage of rough set theory in data analysis is that it does not need any preliminary or additional information about data as needed for probability and statistics.

1.2 Soft Set

Molodtsov [17] introduced a new approach called soft set theory for dealing with uncertainty to the real world problems to the economics, engineering and environmental areas, which approximates the universe of the fundamental problems. In soft set theory, the initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. We prefer any parameterization which can be used with the help of words and sentences, real numbers, functions, mappings and so on.
Let U be an initial universe, $P(U)$ is the power set of U and E is a set of the parameters. Let $A \subseteq E$. Then a pair (F, A) is called a soft set over U, where F is a set valued function defined by $F : A \to P(U)$. Clearly, a soft set over U is a parameterized family of subsets

of a given universe U. Also, for any $e \in E$, $F(e)$ is considered as the set of e- approximate element of the soft set (F, E).

1.3 Lattice

Lattice is a simple algebraic structure since the basic concepts of the theory which include only orders, least upper bound and greatest lower bound. It is widely discussed and studied in classical algebraic theory. Partial order relation and lattice theory now play an important role in many disciplines of computer science and engineering. For example, they have various applications in distributed computing (vector clocks, global predicate detection), concurrency theory, programming language semantics (fixed-point semantics), and data mining (concept analysis). They are also useful in other disciplines of mathematics such as combinatorics, number theory and group theory. A partially ordered set (or a poset for short), is a non-empty set B equipped with a transitive, reflexive and antisymmetric relation $\leq$. B is totally ordered, or a chain, if all elements of B are comparable under $\leq$ (that is, $x \leq y$ or $y \leq x$ for all $x, y \in B$). The lattice is one of the most widely discussed and studied structure in the classical algebraic theory, both as a specific algebra with a carrier and two binary operations, and as a relational structure a specific ordered set [13]. In mathematics, a lattice is a partially ordered set in which any two elements have a unique supremum and an infimum. Lattices can also be characterized as algebraic structures satisfying certain axiomatic identities. Since the two definitions are equivalent, lattice theory draws on both order theory and universal algebra.

2 Literature Survey

Rough set theory has been attracted attention of many researchers and practitioners all over the world, who have contributed essentially to its development and applications. Pawlak [22] introduced the theory of rough set as an extension of set theory for the study of incomplete information. Pomykala and Pomykala [20] showed that the set of rough sets forms a stone algebra. Gehrke and Walker [12] introduced a precise structure theorem for the stone algebra of rough sets and a characterization of them in the category of all

stone algebras. Yao [36] described the notion of the formal concept analysis to rough set theory. Iwinski [14] defined rough lattice and rough order and he noticed that rough set can be viewed as a pair of approximations. In this definition, Iwinski defined rough lattice without using any indiscernibility concept of rough set. Rana and Roy [25] presented an unique approach to form lattice by choice function in rough set. Järvinen [15] proposed the lattice structure on rough sets which played an important role in rough set and Pawlak's information system. Biswas and Nanda [6] discussed the notion of rough substructure in groups. Rough set theory overlaps with many other theories. Despite this overlap, rough set theory may be considered as an independent discipline in its own right. A wide range of applications of methods based on rough set theory alone or in combination with other approaches have been discovered in the following areas as computer engineering, decision analysis and systems, economics, electrical engineering (e.g., control, signal analysis, power systems), environmental studies, digital image processing, informatics, medicine, molecular biology, robotics, social science, software engineering etc.

Molodtsov [17] incorporated the concept of soft set as a completely new mathematical tool with adequate parameterization for dealing with uncertainties. Maji et al. [18] studied on the theory of soft sets initiated by Molodtsov and developed several basic notions of soft set theory. Ali et al. [1] introduced the notion of restricted union, restricted intersection, restricted difference, and extended intersection between two soft sets. They established the notion of complement in soft set and also proved that certain De Morgan's laws hold in soft set theory. Furthermore, Babitha and Sunil [3] gave definitions for the soft set relation as a subset of cartesian product of two soft sets. After that, Babitha and Sunil [4] defined the partially ordered soft set by introducing ordering on soft sets. Park et al. [23] focused the discussion on equivalence relation and they established that complete lattice is defined on the poset of equivalence soft set relations under a soft set. Soft set theory has potential applications in many different fields due to its no necessity to describe the membership function. As a result, this makes that the soft set theory is so simple and popular in applications of various areas. Soft set theory based classification algorithm can be applied to texture classification. Application of soft set theory in various fields have been found in ([8]- [16]).

3 Objective and Scope of the Thesis

The main objective of the thesis has been defined after an extension literature survey based on the status in problems of mathematical extension and associated with some specific framework such as lattice structure of rough set in case of equivalence indiscernibility relation, homomorphism, rough lattice and rough ideal, hybridization of soft set, rough set and fuzzy set, lattice structure of soft set, soft set relation and approximation of soft set in modified soft rough approximation space. The novel and significant contribution in the present research work under report are summarized as follows:

1. In order to study the properties of lattice in an approximation space based on Pawlak's notion of indiscernibility relation among the objects in a set. Rough modular lattice and rough distributive lattice are defined.

2. In order to study an algebraic connection between soft set and algebraic system like lattice theory in a soft approximation space. Notions of soft rough lattice are introduced.

3. To study the lattice theory in the framework of rough set. Rough ideal and rough homomorphism are studied in rough set environment.

4. To study rough set and soft set in different types of approximation spaces. Modified soft rough approximation space is defined using soft set. The concept of rough soft set is also introduced in modified soft rough approximation space.

5. To study soft set relation in a new way; and based on this relation, lattice theory on soft sets is discussed.

6. To form a hybridization structure between rough-soft set and fuzzy set. Fuzzy rough soft set is also introduced in modified soft rough approximation space.

7. To study congruence relation on soft set and to enrich the theoretical development of lattice theory under soft set environment.

4 Organization of the Thesis

The research work under report and evaluation are organized ten chapters. First Chapter presents a brief introduction related to my research work, the brief history and some preliminaries of rough set theory and soft set theory. Finally some definitions on order and lattices are also discussed.

In **Chapter 2**, rough modular lattice and rough distributive lattice are introduced based on Pawlak's indiscernibility relation. At first, the rough lattice is constructed and interpreted based on the equivalence relation. Then the different types of lattice under the rough set environment are established by incorporating a pair of sets in an approximation space. It is seen that the distributive property of lattice is extended to the area of uncertainty according to our defined Rough Distributive Lattice (RDL). We also show that modularity property of ordinary lattice in crisp set is extended to area of uncertainty for rough set which is the generalization of lattice theory. We make a connection between the rough set and the lattice theory both of which have wide fields of application in the areas of computer science. [One part of this chapter has been published in ***Journal of Uncertain Systems (World Academic Union)*, SCOPUS, Vol. 7, No. 4 (2013), pp. 289-293** and another part in ***Malaya Journal of Matematik (University Press, Singapore)*, Vol. 2, No. 3 (2014), pp. 273-276**].

In **Chapter 3**, we introduce the notion of rough ideals which is a generalized notion of ideals of a lattice. Important properties of rough ideals are also developed in this chapter. We consider the approximation space by means of an equivalence relation and also describe the rough set as pair of sets (lower and upper approximation sets). The objective of this chapter is to study the properties of lattice under an approximation space based on Pawlak's notion of indiscernibility relation between the objects in a set. Several important results are established. Also this chapter is devoted to study of homomorphism of rough lattice. [**A part of this chapter is Communicated to International Journal**].

In **Chapter 4**, an algebraic connection between the soft rough set and the algebraic structure named as lattice is established. As a result, lattice structure is developed on soft rough set; and the concept of soft rough lattice based on soft approximation space is defined. After that we investigate the several properties and theorems on soft rough

lattice. Finally we justify our proposed soft rough lattice with supporting examples by Hasse diagram. [A part of this chapter has been published in ***Kragujevac Journal of Mathematics (University of Kragujevac, Serbia)*, SCOPUS, Vol. 39, No. 1 (2015), pp. 15-20**].

Rough and soft sets are two different mathematical tools for dealing with uncertainty. Soft rough set, proposed by Feng et al. [**?**] is a study on roughness through soft set. In **Chapter 5**, we formulate a general mathematical concept defined on lattices in the framework of soft rough set. This chapter is devoted to study the lattice theory in the framework of soft rough set. The set from lattice structure is treated here as universal set and defined soft rough set on it. We construct the soft rough ideal and study their properties in soft approximation space. [A part of this chapter has been published in ***The Journal of Fuzzy Mathematics (International Fuzzy Mathematics Institute, USA)*, Vol. 24, No. 1 (2016), pp. 49-56**].

In **Chapter 6**, we introduce the concept of approximation on an information system with respect to another information one based on an MSR-approximation space. We construct the rough soft set and study their properties in MSR-approximation space. Besides, we establish the connection between a rough set and a lattice theory by measuring the roughness of a soft set. We endeavor to establish a link between soft set and rough set in connection with an application in lattice. [A part of this chapter has been published in ***Fuzzy Information and Engineering (Elsevier)*, Vol. 7, No. 3 (2015), pp. 379-387**].

In **Chapter 7**, the concept of cartesian product on soft sets is introduced in an another way. Besides this, based on this cartesian product, a soft set relation is defined. Soft set relation is also constructed based on the induced binary relation in the set of parameters of soft sets. A connection between the relations is also established. Moreover, lattice theory is studied on soft sets considering with soft set relation. Based on the ideas of the Cartesian product and soft set relation, we newly formulate the soft lattice, soft modular lattice, soft distributive lattice and soft equivalent relation which are the unique characteristic of this chapter. **[A part of this chapter is Communicated to International Journal].**

In **Chapter 8**, we establish the soft congruence relation over lattice. Several properties

of soft congruence relation are studied. Approximations of subset of a lattice are studied with respect to soft congruence relation. That is the roughness of a subset of lattice is discussed using the soft set relation. We also discuss the properties of lattice ideal with respect to the soft congruence relation. [A part of this chapter has been published in ***Hacettepe Journal of Mathematics and Statistics (Hacettepe University, Turkey), SCIE, IF: 0.277, 2017, DOI: 10.15672/HJMS.2017.436***].

Soft set theory, rough set theory and fuzzy set theory are all treated as mathematical tools to deal with uncertainty for variety of problems. A possible hybridization of these theories is an interesting topic to the researchers. In **Chapter 9**, we propose the concept of fuzzy rough soft set in MSR approximation space which can be viewed as a pair of soft set and its roughness. We define the union and the intersection of fuzzy rough soft set with several examples. Also we establish the important properties of fuzzy rough soft set with respect to fuzzy rough soft union and intersection. **[A part of this chapter is Communicated to International Journal].**

Chapter 10 contains conclusion of the whole study presented in this thesis and direction of the future work emerging from this thesis.

5 Conclusions and Scope of Future Works

Conclusions:

Lattice and order set have wide fields of applications in computer science, engineering, discrete mathematics, data mining, number theory, group theory etc. In addition to the above, many applications utilize lattices and ordered set in fundamental ways. These include such areas as knowledge representation, text categorization and data mining, where order plays an fundamental organizing principle. Also, for the application of lattice and ordered set to inductive logic programming, ordered set form basic models. On the other hand in our complex world, there are many situations occur, where we cannot use traditional methods to solve problems in economics, engineering, environment, social science, medical science etc. because of various types of uncertainties present in these problems. Probability theory, fuzzy set theory, rough set theory, soft set theory are novel

mathematical tools to solve real world uncertain problems approximately. That is why, the study on lattice theory under uncertain environments with the help of rough set, soft set and their hybridizations i.e., rough soft sets are initiated. We present a general frame work for the study of approximation in lattice. We have studied the properties of lattice in an approximation space based on Pawlak's notion of indiscernibility relation among the objects in a set. Rough modular lattice and rough distributive lattice are defined in Pawlak's approximation space. In order to study an algebraic connection between soft set and algebraic system like lattice theory in a soft approximation space. Notions of soft rough lattice are introduced. In this thesis, rough-ideal and rough-homomorphism are studied in rough set environments. We have initiated to study rough set and soft set in different types of approximation spaces. Rough soft set is defined in a modified soft rough approximation space. We have presented soft set relation in a new way; and based on this relation we have introduced lattice theory on soft sets. In this thesis we have presented a hybridization structure between fuzzy set and rough-soft set and, as a result fuzzy rough soft set is introduced. To enrich the theoretical development of lattice theory under soft set environment, the notion of soft congruence relation is introduced. In this study, we have tried to made a fusion between fuzzy set and rough soft set. Here we have measured the roughness of rough soft set and introduced the concept of fuzzy rough soft set in MSR-approximation space. Moreover, lattice theory is studied on the fuzzy rough soft set. In the whole thesis we have tried to incorporate lattice theory in uncertain environments.

In this thesis we have studied uncertainty in algebra with the help of rough set, soft set and soft rough set. We have constructed different approximation spaces based on Pawlak's approximation space and then have approximated a subset of the universal set. In this study, hybrid model combining rough sets with soft sets, rough soft sets are exploited to extend many practical applications based on rough sets or soft sets. As a consequence, the output results in each chapter certainly have been arrested the attention of researchers who are highly hopeful that this thesis will widely help in and contribute to growth and development of interest among interested researchers who are involving in such areas.

Scope of Future Works

There are many directions of future works emerging from this thesis. A few of them are appended below:

• The concept of rough modular and rough distributive lattice can be extended in knowledge representation problems.

• Soft rough ideal and soft rough homomorphism in connection with other algebra may be designed.

• Researchers may put their attention on soft rough approaches to ring ideal.

• Researchers can apply the hybrid structure (Rough-Soft) in decision making problem.

• Researchers can formulate soft rough ideal to some applied fields such as knowledge representation theorems, information system etc.

• The concept of soft rough ideal may be extended to more results on lattices under soft rough environment.

• Researchers can implement rough congruence relation and soft rough congruence relation over lattice.

• Soft congruence relation of soft lattice and fuzzy soft congruence relation can be designed based on our defining soft congruence relation.

• The researchers can define soft rough lattice newly based on our proposed soft set relation.

• Researchers may defined fuzzy rough soft relation, congruence relation and lattice ideal under fuzzy rough soft set, which may be the extended work of this thesis.

• In this thesis, we have studied lattice theory under uncertain environment and the obtained results may be apply in that situation where the information about the data are imprecise.

• Finally, one can applied lattice theory for practical problems on engineering, computer science, data mining, decision making problem, knowledge representation system and other real-life problems under uncertain environment.

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List of Publications

1. Bera, S. and Roy, S. K. (2013) Rough modular lattice, *Journal of Uncertain Systems,* SCOPUS, Vol. **7**, No. 4, pp. 289-293.

2. Roy, S. K. and Bera, S. (2014) Distributive lattice: a rough set approach, *Malaya Journal of Matematik (University Press, Singapore)*, Vol. **2**, No. 3, pp. 273-276.

3. Roy, S. K. and Bera, S. (2015) Soft rough lattice, *Kragujevac Journal of Mathematics (University of Kragujevac, Serbia)*, SCOPUS, Vol. **39**, No. 1, pp. 15-20.

4. Roy, S. K. and Bera, S. (2015) Approximation of rough soft set and its application to lattice, *Fuzzy Information and Engineering (Elsevier)*, Vol. **7**, No. 3, pp. 379-387.

5. Bera, S. and Roy, S. K. (2016) Soft rough approach to lattice-ideal, *The Journal of Fuzzy Mathematics (International Fuzzy Mathematics Institute, USA,)* Vol. **24**, No. 1, pp. 49-56.

6. Bera, S., Roy, S. K. and Karaaslan, F. (2017) Soft congruence relation and its application to lattice, *Hacettepe Journal of Mathematics and Statistics (Hacettepe University, Turkey),* SCIE, IF: 0.277, DOI: 10.15672/HJMS.2017.436.

List of Communicated Papers

1. Bera, S. and Roy, S. K. Fuzzy rough soft set and its application to lattice.

2. Bera, S., Roy, S. K. and Çağman, N. An another approach for cartesian product on soft set relation and its application to lattice.

3. Bera, S. and Roy, S. K. Rough ideal and homomorphism and their applications to lattice.

List of Conferences/Seminar Attended

1. International Conference on **Frontier of Mathematical Sciences with Applications (ICFMSA),** Calcutta Mathematical Society (CMS), December 07-09, 2012.

2. National Seminar on **Emerging Trends in Mathematics,** Vidyasagar University, W.B. In collaboration with Calcutta Mathematical Society (CMS), December 19-20, 2012.

3. 1^{st} International Conference on **Recent Trends in Mathematics and Its Applications(ICRTMA),** Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, W.B., March 20-21, 2013.

4. International Conference on **Facets of Uncertainties and Applications (ICFUA),** ORSI, Calcutta Chapter and Department of Applied Mathematics, University of Calcutta, December 5-7, 2013.