Synopsis of proposed thesis entitled

Studies on Dynamics of Some Model Based Ecological and Epidemiological Problems

To be submitted to

VIDYASAGAR UNIVERSITY

For the Award of Degree of

DOCTOR OF PHILOSOPHY
(SCIENCE)

In
APPLIED MATHEMATICS

BY

Prabir Panja

Department of Applied Mathematics with
Oceanology and Computer Programming
Vidyasagar University
Midnapore-721102
West Bengal, INDIA

December, 2016

PUBLICATION

List of Published Papers

1. Stability Analysis of Coexistence of Three Species Prey-Predator Model. ***Nonlinear Dynamics: An International Journal of Nonlinear Dynamics and Chaos in Engineering Systems, 81 (2015) 373-382***, **SPRINGER**.

2. Dynamics of Cholera Outbreak with Bacteriophage and Periodic Rate of Contact. ***International Journal of Dynamics and Control, 4(3) (2016) 284-292***, **SPRINGER**.

3. Stability and Bifurcation Analysis of Japanese Encephalitis Model with/without Effects of Some Control Parameters. ***Computational and Applied Mathematics***, **(2016), DOI 10.1007/s40314-016-0400-2**, **SPRINGER**.

List of Communicated Papers

4. Stability and Bifurcation Analysis of a Stage Structured Prey-Predator Model with Ratio-dependent Functional Response and Anti-predator Behavior of Adult Prey. *Communicated.*

5. Effects of Supplying Additional Food in a Tritrophic Food Chain Model with Harvesting Only. *Communicated.*

6. Effects of Additional Food in a Predator-Prey System Incorporating Refuge and Harvesting. *Communicated.*

7. Dynamical Study in Fuzzy Threshold Dynamics of a Cholera Epidemic Model. *Communicated.*

8. Threshold Dynamical Behaviors of a Malaria Disease in Control Parameters Based Periodic Environment. *Communicated.*

Studies on Dynamics of Some Model Based Ecological and Epidemiological Problems

1 Overview of Thesis

By nature, ecology and epidemiology are two distinct fields of research. A lot of works have already been done in these two subjects.

The main objective of the present investigation is to observe the dynamics of some ecological and epidemiological issues with some ecological and epidemiological factors. The present thesis is divided into two parts based on ecological and epidemiological issues. The first four chapters deal with the problem of ecological issues such as Phytoplankton-Zooplankton-Fish dynamics; prey-predator dynamics where prey population is subdivided into juvenile prey and adult prey; effect of harvesting and additional food in a food chain model (devoted two chapters). The last four chapters deal with the problems of some epidemiological issues such as Cholera (devoted two chapters), Malaria and Japanese Encephalitis.

The above issues are modelled with the help of ordinary differential equations. Basically all the mathematical models represent dynamical systems. In most of the models, the dynamics have been represented through autonomous dynamical system and some are represented through non-autonomous dynamical systems (Chapters 6, 8, 9). The dynamics of such system have been analyzed through local and global stability analysis. To study the global dynamics a suitable Liapunov functions and geometric approach are used. Moreover, the bifurcation analysis and exclusive numerical analysis have been performed to substantiate the analytical findings.

Now, I like to take opportunity to describe the main motivation and objective of the thesis.

2 Motivation and Objective of the Thesis

Ecological system is concerned about the interaction among same or different organisms with abiotic components of their living environments. Practically, it is very necessary

to study such systems for keeping the existence of human beings. In this regard, there is going on different works on this field mathematically. But till now, there exists also some unexplored directions in this field. So, in this research works, one of the objectives is to integrate mathematically different factors influencing the dynamics of predator-prey ecological system.

With the development of human life, different infectious disease are attacking the human population around the globe. Epidemiology is the study on analysis of different patterns, causes and effects of such disease identifying the various risk factors and suggest some measures for control. These diseases makes several thousands of death due to their unknown behavior and due to lack of appropriate control strategies. For this reason, the study is necessary to explore the actual dynamics of these diseases. Since 1760, many research works have been performed mathematically. But, still there exists some directions which are yet to be unveiled in epidemiology. Therefore, our other objective is to investigate mathematically different epidemiological problems considering the influence of different factors on the system.

Many researchers [2, 3, 4, 7] have worked on the dynamics of Phytoplankton and Zooplankton interaction model. Recently, Yunfei et al.[12] have discussed two species food chain model taking Phytoplankton as prey and Zooplankton as a predator with harvesting of both. It is known that Phytoplankton is the primary producer in a food chain. These are consumed by Zooplankton. Again, Phytoplankton, Zooplankton both are consumed by different Fish species. So, the growth rates of Fish species mainly depend on the availability of Phytoplankton and Zooplankton. Fish and other aquatic organisms are very essential to be processed into various food and non-food products such as shark skin leather, pigments made from the inky secretions of cuttlefish, isinglass used for the clarification of wine and beer, fish emulsion used as a fertilizer, fish glue, fish oil and fish meal. So, Fish is very important food source in human society. Though there are different predator-prey models, but no one consider Fish population separately as a predator. Hence, the study of dynamics of Fish species with Phytoplankton and Zooplankton is necessary. This point directs us to focus on the study of interactions among three species such as Phytoplankton, Zooplankton and Fish in a predator-prey model which has biologically importance on the real world.

The life of an organism has been divided into different stages in a stage structured mathematical model. Every species always experiences immature stage and mature stage of their life in the real world natural ecosystems and it performs different kinds of feature at each stage of growth. So, the ecological models with stage structure is more rational than without stage structure. In anti-predator behaviour, prey groups actively defend themselves by attacking or mobbing a predator and save their offsprings from predation. Tang et al.[9] investigated the bifurcation analysis of a predator-prey model with anti-predator behavior of prey species in which anti predator behavior had been considered for all prey members. But, in reality it is seen that only adult prey can protect their infants (Juvenile prey) from predation of predator by showing anti-predator behavior. There is no research papers including stage structure on prey species and anti-predator behavior of Adult prey population. This vacuum has motivated us to work on predator-prey mathematical model with stage structure and anti-predator behavior of adult prey population.

There are several motivations such as conservation of a species, control of a pest population, chaos control, disease control etc. behind the supply of additional food in a predator-prey system. Supplying additional food in a predator-prey system is the great topics to many researchers due to its eco-friendly nature. Recently, Sahoo and Poria [8] have published a research paper on the impacts of additional food in a predator-prey system with harvesting. They have considered the additional food on top predator population and top predator can consume only the middle predator. But, in reality it is seen that the top predator can consume both prey and middle predators. So, this idea has inspired us to work on the predator-prey mathematical model in the presence of additional food to the top predator.

Already, many research works have been done on the predator-prey system by taking refuge by the prey species. The using of refuge parameter decreases the predation rate of predator. Therefore, the concept of refuges by prey should have important effects on the predator-prey interaction. Many studies have suggested that refuges for prey are crucial in explaining prey persistence. Chakraborty and Das [1] have reported the impact of incorporating constant prey refuse parameter on a predator-prey system with alternative food to predators. In this model, it has been shown that the supply of alternative food to the predator has a significant effect to stabilize the predator-prey dynamics considering refuge on the prey species. But the effect of additional food on super predator and refuge

on predator may provide some interesting dynamics. We like to observe such dynamics by considering a food chain model.

It is well known that, Cholera is an infectious disease caused by the bacterium Vibrio Cholerae to be contaminated with food and drinking water. The poor sanitation, contaminated drinking water, poverty etc. are the main risk factors for the Cholera disease transmission. There are many mathematical models on Cholera disease dynamics and its control strategy. Experimentally, it has been proved in the paper of Jensen et al.[5] that bacteriophages can reduce the density of bacterium Vibrio Cholerae. So, the rapid decay of bacterial culturability and the predation of Vibrio Cholerae by bacteriophages have been observed in the dynamics of Cholera model. Therefore, bacteriophage has a great importance to control the Cholera disease transmission. This fact inspires us to incorporate bacteriophage in the Cholera disease transmission model including the time periodic nature of the disease transmission rates.

There exists many research papers of the Cholera disease transmission and its control strategies. But, the parameters involved in these model are crisp in nature. In the real worlds, every parameters related to the disease transmission model are changing with respect to time due to human activities and natural disasters. Thus, these parameters should not be crisp in nature always i.e., these may be uncertain in nature. There are very few works on fuzzy parameters based disease transmission model. Recently, Pal et al.[6] reported the impact of taking fuzzy parameter on predator-prey harvesting model. This article intends us to consider the parameters involved in a Cholera disease transmission to be uncertain in nature.

The other important disease to study is Malaria. Malaria is a vector borne fatal disease caused by a parasite. It spreads in human population through the bites of infected mosquitoes. There are mainly four types of Malaria parasites such as Plasmodium falciparum, P. vivax, P. ovale and P. malariae, infect humans. It causes several thousands of death in every year. It still remains global threat for humans along the whole world though the medical science has been developed. Therefore, the actual dynamics of the Malaria disease transmission and new possible effective control strategies need further investigation. Wang et al. [11] investigated a mathematical model on Malaria disease transmission in a periodic environment. According to the transmission mechanism of

Malaria disease they divided mosquito population into two subpopulations such as susceptible mosquito and infected mosquito. Again, human population is also divided three subpopulations such as susceptible human, infected human and recovered human. Here, it has been considered that the disease transmission rates are time periodic. In their model, the control parameters were not considered to eradicate the Malaria disease from human population, which are most important parameters. This idea motivated us to develop the interaction between mosquito and human population with some control parameters which are time dependent.

Japanese Encephalitis (JE) virus is a single-stranded RNA virus which belongs to the genus Flavivirus. JE virus is transmitted to humans through the bite of an infected mosquito mainly Culex species. It grows in the body of the amplifying vertebrate hosts primarily pigs and wading birds. It makes several thousands of death in every year in India and other parts in the world due to lack of investigation of proper dynamical behavior using suitable control strategies. So, the study about the authentic dynamics of this disease and its control strategies are necessary. Tapaswi et al.[10] developed a three-populations such as mosquito, reservoir and human interaction Japanese Encephalitis disease transmission model. In their model, only stability analysis of the system around different equilibrium points was discussed and no control strategies are considered to control the disease. This influence us to develop a Japanese Encephalitis disease transmission mathematical model among mosquito, reservoir and human populations with some suitable control parameters.

3   Organization of the Thesis

In the proposed thesis, some real life Ecological and Epidemiological problems are considered and solved. The proposed thesis has been divided into following four parts and eleven chapters.

Part I: General Introduction

- **Chapter 1:** Introduction and Organization of the Thesis

Part II: Studies on Dynamics in Some Ecological Problems

- **Chapter 2:** Stability Analysis of Coexistence of Three Species Prey-Predator Model

- **Chapter 3:** Stability and Bifurcation Analysis of a Stage Structure Prey-Predator Model with Ratio-dependent Functional Response and Anti-predator Behavior of Adult Prey

- **Chapter 4:** Effects of Supplying Additional Food in a Tritrophic Food Chain Model with Harvesting Only

- **Chapter 5:** Effects of Additional Food in a Predator-Prey System Incorporating Refuge and Harvesting

Part III: Studies on Dynamics in Some Epidemiological Problems

- **Chapter 6:** Dynamics of Cholera Outbreak with Bacteriophage and Periodic Rate of Contact

- **Chapter 7:** Dynamical Study in Fuzzy Threshold Dynamics of a Cholera Epidemic Model

- **Chapter 8:** Threshold Dynamical Behaviors of a Malaria Disease in Control Parameters Based Periodic Environment

- **Chapter 9:** Stability and Bifurcation Analysis of Japanese Encephalitis Model with/without Effects of Some Control Parameters

Part-IV: Summary, Extension and Bibliography

- **Chapter-10:** Summary and Future Research Work

- **Chapter-11:** Bibliography

<div align="center">

Part I
(General Introduction)

</div>

The Part I contains only one chapter (Chapter 1).

Chapter 1: Introduction and Organization of the Thesis

This chapter contains an introduction giving an overview of the development along with the historical literature reviews on Ecological and Epidemiological problems.

<div align="center">

Part II

(Studies on Dynamics in Some Ecological Problems)

</div>

This part is divided into four chapters (Chapter 2, 3, 4 and 5) and in those chapters different ecological models are derived and solved.

Chapter 2: Stability Analysis of Coexistence of Three Species Prey-Predator Model

In this chapter, we have proposed a prey-predator model for the study of dynamical behaviors of three species such as toxin producing Phytoplankton, Zooplankton and Fish in a Fishery system. The stability condition, existence condition of equilibrium and bifurcation have been also established. In this chapter, Holling type II functional response has been considered to analysis the proposed model. All equilibriums of the proposed system are determined and the behavior of the system is also investigated near the positive equilibrium point. Hopf bifurcation analysis has been done with respect to the consumption rate of Zooplankton (β) and releasing rate of toxin substances (ρ) produced by unit biomass of Phytoplankton. Finally, some numerical simulations has been performed to verify our theoretical results.

Chapter 3: Stability and Bifurcation Analysis of a Stage Structured Prey-Predator Model with Ratio - dependent Functional Response and Anti-predator Behavior of Adult Prey

In this chapter, a three species predator-prey model such as (i) juvenile prey and (ii) adult prey and (iii) predator population has been developed. It is considered that the growth rate of juvenile prey depends on the adult prey populations and then the juvenile prey population becomes adult. The functional responses for predator to consume both the juvenile prey and adult prey population have been considered as ratio dependent. Also, the anti-predator behavior has been considered on adult prey population. Then, the boundedness of all solutions of our proposed mathematical model has been discussed. Also, we determine different equilibria and there existence conditions. Then, the stability

conditions of the system around these equilibria have been analyzed. Global stability analysis of the interior equilibrium has been done. After that, the Hopf bifurcation analysis with respect to the anti-predator behavior of adult prey has been discussed. Finally, some numerical simulations have been performed to test our theoretical results.

Chapter 4: Effects of Supplying Additional Food in a Tritrophic Food Chain Model with Harvesting Only

In this chapter, we propose and analyze a three species predator-prey system in presence of additional food for predators. It is assumed that the middle predator is acting as a prey as well as predator and the top predator consumes both prey as well as middle predator. It is also considered that a constant amount of additional food for the top predators exists in the ecosystem. The effects of top predator harvesting are investigated. Then the existence and stability conditions of the equilibria have been discussed analytically. The Hopf bifurcation analysis of the system with respect to predation rate of prey to the top predator and the harvesting effort have been analyzed both analytically and numerically. Pontryagins maximum principle is used to determine the optimal harvesting of top predator population to maximize the discounted net revenue. From our analysis, it is seen that the additional food has significant effects to prevent the extinction risk of top predator population and also to increase revenue collection. Finally, some numerical results have been given in support of our analytical findings.

Chapter 5: Effects of Additional Food in a Predator-Prey System Incorporating Refuge and Harvesting

In this chapter, a food chain model has been developed among three species such as prey population, predator population, super predator population. Here, it is assumed that prey population grows logistically and predator population consumes prey only. But, the predator population is consumed by a super predator population. In this model, it is assumed that the predator population shows refuge behavior to the super predator population. Due to having the refuge characteristic of the predator population, the super

predator feels the lack of food. To maintain the growth of the super predator properly, a constant amount of additional food is supplied to the system. Henceforth, partially the predator population is also benefited with this additional food. On the basis of this notions, a food chain model has been derived in which the extinction conditions of super predator population has been explored. Also, stability analysis of the model has been shown along with Hopf bifurcation analysis to examine some parametric values for which the system losses its stability. To get optimal harvesting of super predator, the Pontryagins maximum principle has been used. Finally to study the feasibility of the model, some numerical simulations have been presented.

Part III
(Studies on Dynamics in Some Epidemiological Problems)

This part is divided into four chapters (Chapter 6, 7, 8 and 9) and in those chapters different epidemiological models are derived and solved.

Chapter 6: Dynamics of Cholera Outbreak with Bacteriophage and Periodic Rate of Contact

In this chapter, a Cholera epidemic model with periodic transmission rate has been considered and discussed. It is shown that the disease free equilibrium point is globally asymptotically stable and also seen that the Cholera disease is disappeared if the basic reproduction number is less than one. When the basic reproduction number is grater than one, then the endemic equilibrium is globally asymptotically stable. Finally, numerical simulations have been given for the existence of the analytical results.

Chapter 7: Dynamical Study in Fuzzy Threshold Dynamics of a Cholera Epidemic Model

In this chapter, a fuzzy mathematical model on Cholera disease has been developed in which all parameters related to the Cholera disease have been considered as fuzzy numbers. Here, total human population is divided into three subpopulations such as susceptible human, infected human and recovered human. Also, the bacterial population is the vibrio

Cholerae in the environment. Then the existence condition and boundedness of solution of our proposed mathematical model have been discussed. Also, the different equilibrium points and the stability condition of the system around these equilibrium points have been analyzed. The global stability condition of the proposed system around the endemic equilibrium point has been also discussed. Finally, some numerical simulations have been shown to test the theoretical results of the system.

Chapter 8: Threshold Dynamical Behaviors of a Malaria Disease in Control Parameters Based Periodic Environment

In this chapter, a Malaria disease transmission model has been developed in which the transmission rates from mosquito to human as well as human to mosquito and death rate of infected mosquito have been constituted by two variabilities: one is periodicity with respect to time and another is based on some control parameters. Here, total vector population is divided into two subpopulations such as susceptible mosquito and infected mosquito as well as the total human population is divided into three subpopulations such as susceptible human, infected human and recovered human. The dynamical behaviors of the system associated with reproduction number with respect to control parameters have been investigated theoretically and numerically both. The biologically feasible equilibria and their stability properties have been discussed and the existence condition of the disease has been illustrated numerically. At last, Hopf bifurcation analysis has been done analytically and numerically for autonomous case of our proposed model.

Chapter 9: Stability and Bifurcation Analysis of Japanese Encephalitis Model with/without Effects of Some Control Parameters

In this chapter, a mathematical model on transmission of Japanese Encephalitis disease has been developed considering some control parameters and time dependent environmental carrying capacity. Here, total vector population is divided into two subpopulations

such as susceptible mosquito and infected mosquito. Here also, total reservoir population (i.e., the population in which the encephalitis virus grows) such as pig, horse etc has been considered which is divided into three subpopulations such as susceptible reservoir, infected reservoir and recovered reservoir. Total human population is also divided into three subpopulations such as susceptible human, infected human and recovered human. The dynamical behaviors of the system have been investigated. Here, the reproduction number associated with the system has been analyzed with respect to control parameters theoretically and numerically both. The biological feasible equilibria and their stability properties have been discussed and the existence condition of the disease has been illustrated numerically. For a certain set of parametric values, the effectiveness of control parameters in our proposed model has been checked numerically. At last, Hopf bifurcations have been made numerically without considering control parameters for the case of constant environmental carrying capacity of mosquito.

Part IV
(Summary, Extension and Bibliography)

This part is divided into two chapters (Chapter 10 and 11) and in those chapters summary, future research works and bibliography have been presented.

Chapter 10: Summary and Future Research Work

In this chapter, a summary of the thesis and the scope of future research work have been discussed.

Chapter 11: Bibliography

In this chapter, the references have been presented.

References

[1] Chakraborty, K. and Das, S.S. (2014). Biological conservation of a prey-predator system incorporating constant prey refuse through provision of alternative food to predators: a theoretical study, Acta Biotheoretica, 62, $183 - 205$.

[2] Chattopadhyay, J. and Pal, S. (2002). Viral infection on phytoplankton zooplankton system-A mathematical model, Ecological Modelling, 151(1), $15 - 28$.

[3] Chattopadhyay, J., Sarkar, R.R. and Pal, S. (2003). Dynamics of nutrient-phytoplankton interaction in the presence of viral infection, Biosystems, 68(1), $5 - 17$.

[4] Gakkhar, S. and Negi, K. (2006). A mathematical model for viral infection in toxin producing phytoplankton and zooplankton system, Applied Mathematics and Computation, 179(1), $301 - 313$.

[5] Jensen, Mark A., Faruque, Shah M., Mekalanos, John J. and Levin, Bruce R. (2006). Modeling the role of bacteriophage in the control of Cholera outbreak, Proceedings of the National Academy of Sciences USA, 103(12), $4652 - 4657$.

[6] Pal, D., Mahapatra, G.S. and Samanta, G.P. (2015). Stability and bionomic analysis of fuzzy parameter based prey-predator harvesting model using UFM, Nonlinear Dynamics, 79(3), $1939 - 1955$.

[7] Pal, S., Chatterjee, S., Das, K. and Chattopadhyay, J. (2009). Role of competition in phytoplankton population for the occurence and control of plankton bloom in the presence of environmental fluctuations, Ecological Modelling, 220(2), $96 - 110$.

[8] Sahoo, B. and Poria, S. (2014). Effects of supplying alternative food in a predator-prey model with harvesting, Applied Mathematics and Computation, 234, $150 - 166$.

[9] Tang, B. and Xiao, Y. (2015). Bifurcation analysis of a predator-prey model with anti-predator behaviour, Chaos, Solitons & Fractals, 70, 5868.

[10] Tapaswi, P.K., Ghosh, A.K. and Mukhopadhyay, B.B. (1995). Transmission of Japanese Encephalitis in a 3-population model, Ecological Modelling, 83(3), $295 - 309$.

[11] Wang, L., Teng, Z. and Zhang, T. (2013). Threshold dynamics of a Malaria transmission model in periodic environment, Communications in Nonlinear Science and Numerical Simulation, 18(5), $1288 - 1303$.

[12] Yunfei, L., Yongzhen, P., Shujing, G., and Changruo, L. (2010). Harvesting of a phytoplankton-zooplankton model, Nonlinear Analysis: Real World Applications, 11(5), $3608 - 3619$.