Synopsis of Thesis Entitled

A Comprehensive Study on Transportation Problems Under Multi-choice Environment

submitted to the

Vidyasagar University, Midnapore

For award of the degree

of

Doctor of Philosophy
in Science

by

Gurupada Maity

Under the guidance of

Dr. Sankar Kumar Roy
Associate Professor

Department of Applied Mathematics with

Oceanology and Computer Programming

Vidyasagar University

Midnapore, West Bengal

INDIA

Synopsis of Ph. D. Work

Chapter 1

1 Introduction

Operations Research (OR) is a discipline which encompasses a wide range of real-life problem involving solution techniques and methods applied in pursuit of improved decision-making and efficiency such as mathematical optimization, econometric methods, simulation, neural networks, data envelopment analysis, decision analysis and the analytic hierarchy process, etc. The modern ground of OR arose during World War II. In the World War II, OR was defined as a scientific method for providing executive departments with a quantitative basis for decisions regarding the operations under the control. The term "optimization" is the root of the study of OR. The optimization is used in different areas of study like Mathematical optimization, Engineering optimization, Economics and business, Information technology, etc.
A mathematical problem is an Optimization Problem (OP) where the objective function is maximized or minimized with or without some prescribed set of constraints. Requirements in real-life decision making situations enlarge the area of Mathematical optimization problems in different fields like Multi-Objective Optimization (MOO) problem, Multi-Choice Optimization Problem (MCOP), Multi-Modal Optimization Problem (MMOP), Optimization under uncertainty, Transportation Problem (TP), etc.

1.1 Optimization problem

Optimization is the mathematical discipline which is concerned with finding the maximum and minimum of functions with or without constraints. In the study of optimization, basically we need to optimize a real function of n variables $f(x_1, x_2, \ldots, x_n)$ with or without constraints. In an Optimization Problem (OP) for modeling a physical system, if there be only one objective function, and the task is to find the optimal solution, then it is called a single-objective optimization problem. The general form of single-objective optimization problem is as follows:

$$\begin{aligned} \text{minimize/maximize} \quad & f(x_1, x_2, \ldots, x_n) \\ \text{subject to} \quad & g(x_1, x_2, \ldots, x_n) \leq 0, \\ & h(x_1, x_2, \ldots, x_n) \geq 0, \\ & l(x_1, x_2, \ldots, x_n) = 0, \\ & \forall\ (x_1, x_2, \ldots, x_n) \in\ F \in R^n,\ \text{F is the feasible region.} \end{aligned}$$

Furthermore, single objective OP can be broadly divided into two different types of problem, namely, linear OP and non-linear OP. If the objective function or a constraint or a set of constraints or both be of non-linear type, then the OP is a non-linear OP otherwise it is a linear OP. Again, according to real-life situations, OP may be deterministic or fuzzy or interval order relation or multi-choice programming depending on parameter space.
Many real-world OPs cannot be formulated by a single objective function. When an OP is used for modeling a real-life problem which involves more than one objective function, the task of finding the optimal solution is called the MOO problem. It has been observed that the parameters which form a parameter space may be multiple types in which only one is to be selected which optimizes the objective functions. The most general mathematical model of the MOO problem is as follows:

$$\text{minimize/maximize} \quad f = f(f_1, f_2, \ldots, f_n) \tag{1.1}$$

$$\text{subject to} \quad g(x_1, x_2, \ldots, x_n) \leq 0, \tag{1.2}$$

$$h(x_1, x_2, \ldots, x_n) \geq 0, \tag{1.3}$$

$$l(x_1, x_2, \ldots, x_n) = 0, \tag{1.4}$$

$$\forall\ (x_1, x_2, \ldots, x_n) \in\ F, \tag{1.5}$$

where $f_1, f_2, \ldots, f_n$ are the objective functions of the decision variables $x_1, x_2, \ldots, x_n$ are called decision variables. Here, MOO problem is also studied in different environments.

1.2 Transportation problem

The TP is a kind decision making problem which may be considered as the central nerve system to keep the balance in economical world from ancient day to till today. It can be delineated as a special case of a Linear Programming Problem (LPP). The classical sense of TP determines how many units of a commodity are to be shipped from each point of origin to various destinations, satisfying source availabilities and destination demands, while minimizing the total cost of transportation along with cutting down the costs per

unit of items for the purchasers.
The mathematical model of transportation problem is as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \\
& x_{ij} \geq 0 \;\; \forall \;\; i \text{ and } \; j,
\end{aligned}$$

where x_{ij} is the decision variable which represents how much amount of goods delivered from the i-th origin to the j-th destination. C_{ij} is the transportation cost per unit commodity. a_i and b_j are supply and demand at the i-th origin and the j-th destination respectively and $\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j$ is the feasibility condition.

1.2.1 Multi-choice transportation problem:

Due to presence of multiple routes of transportation or fluctuation of the market, the transportation parameters like cost, supply and demand may not fixed always. Keeping the points of view, if we consider all or few of cost, supply and demand parameters as multi-choice nature, then the TP becomes a multi-choice TP. In the atmosphere of multi-choice transportation parameters, the mathematical model of the Multi-Choice Transportation Problem (MCTP) is defined as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij}^1 \text{ or } C_{ij}^2 \text{ or } \ldots \text{ or } C_{ij}^r) x_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \ldots \text{ or } a_i^p) \;\; (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \ldots \text{ or } b_j^q) \;\; (j = 1, 2, \ldots, n), \\
& x_{ij} \geq 0 \;\; \forall \;\; i \text{ and } \; j.
\end{aligned}$$

1.2.2 Multi-objective transportation problem:

Single objective transportation problem is not enough to formulate all the real-life transportation problems. The transportation problem with multiple objective functions are considered as Multi-Objective Transportation Problem (MOTP). However, we deal with those kind of objective functions, which are conflicting and non commensurable to each other involving TP. If there be more than one objective function in a TP, then it becomes a MOTP, whose mathematical model is as follows:

$$\begin{aligned}
\text{minimize/maximize} \quad & Z^t = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^t x_{ij} \quad (t = 1, 2, \ldots, K) \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \\
& x_{ij} \geq 0 \ \ \forall \ \ i \text{ and } \ j.
\end{aligned}$$

1.2.3 Multi-choice multi-objective transportation problem:

In multi-choice environment, the mathematical model of MOTP, i.e., the mathematical model of Multi-choice Multi-Objective Transportation Problem (MCMOTP) takes the following form:

$$\begin{aligned}
\text{minimize/maximize} \quad & Z^t = \sum_{i=1}^{m}\sum_{j=1}^{n} (C_{ij}^{t1} \text{ or } C_{ij}^{t2} \text{ or } \ \ldots \ \text{ or } C_{ij}^{tr}) x_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \ \ldots \ \text{ or } a_i^p) \quad (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \ \ldots \ \text{ or } b_j^q) \quad (j = 1, 2, \ldots, n), \\
& x_{ij} \geq 0, \ \ \forall \ \ i \text{ and } \ j.
\end{aligned}$$

1.2.4 Interval-valued transportation problem:

Considering the unstable situation of the market or weather condition, the transportation parameters like cost, supply and demand, may not be taken as crisp values. Keeping the points of view, if at least one of the transportation parameters is considered as interval valued then the TP becomes an interval valued TP.

Multi-choice interval-valued transportation problem:

Involvement of multi-choices in the interval valued TP, the transportation problem reduces to a multi-choice interval valued transportation problem. Mathematical model of

MCITP is as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij}^{1} \text{ or } C_{ij}^{2} \text{ or } \ldots \text{or } C_{ij}^{k}) x_{ij} \quad (k = 1, 2, \ldots, K) \\
\text{subject to} \quad & \sum_{j=1}^{n} x_{ij} \leq (a_i^1 \text{ or } a_i^2 \text{ or } \ldots \text{or } a_i^p) \quad (i = 1, 2, \ldots, m) && (1.6) \\
& \sum_{i=1}^{m} x_{ij} \geq (b_j^1 \text{ or } b_j^2 \text{ or } \ldots \text{or } b_j^q) \quad (j = 1, 2, \ldots, n), && (1.7) \\
& x_{ij} \geq 0, \quad \forall \; i \text{ and } \; j. && (1.8)
\end{aligned}$$

Here, multi-choice parameters C_{ij}^k, a_i^p and b_j^q are interval numbers and these are defined as $C_{ij}^k = [C_{ij}^{k^l}, C_{ij}^{k^u}]$, $a_i^p = [a_i^{p^l}, a_i^{p^u}]$, $b_j^q = [b_j^{q^l}, b_j^{q^u}]$. The feasibility condition in this case is

$$\sum_{i=1}^{m} \max_{p^u}(a_i^{1^u}, a_i^{2^u}, \ldots, a_i^{p^u}) \geq \sum_{j=1}^{n} \min_{q^l}(b_j^{1^l}, b_j^{2^l}, \ldots, b_j^{q^l}).$$

1.2.5 Fuzzy transportation problem

Many real-life decision making problems, there may occur some cases where we need to optimize the objective function (Z) according to the decision maker's preferences. In this case, the decision variables (x_{ij}) of transportation problem are considered as real variables and the crisp solutions are obtained. In our daily life, many situations occur, where it is not applicable to fit a mathematical model using real variables and if we consider the decision variable in a TP as unknown fuzzy number (x_{ij}), then transportation problem becomes a Fuzzy Transportation Problem (FTP). The mathematical model of FTP can be written as follows:

$$\begin{aligned}
\text{minimize} \quad & Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \tilde{x}_{ij} \\
\text{subject to} \quad & \sum_{j=1}^{n} \tilde{x}_{ij} \leq a_i \quad (i = 1, 2, \ldots, m), \\
& \sum_{i=1}^{m} \tilde{x}_{ij} \geq b_j \quad (j = 1, 2, \ldots, n), \\
& \tilde{x}_{ij} \geq 0 \;\; \forall \;\; i \text{ and } \; j.
\end{aligned}$$

Using the concept of FTP in MOTP with considering the decision variable as fuzzy variable, it is easy to formulate multi-objective FTP.

1.2.6 Two-stage grey transportation problem

In the classical sense of transportation problem, there are two types of node, one is source node from which the goods are delivered and other is destination node in which the transported goods are gathered. According to the real-life situations, in a TP, sometimes it is also to be considered that before transporting the goods in the destinations from the sources, goods are to be stored at warehouses from the sources and thereafter they are delivered to the destinations. So, the Decision Maker (DM) utilizes the concept for managing Two-Stage transportation in his position which maximizes the profit. A TP is called a Two-Stage transportation problem, if it consists of transporting the goods by two stages, namely, One Stage transportation problem and Another Stage transportation problem. The TP which is considered in two stages of collecting the goods in warehouses is called an One Stage TP; the TP considered in Another Stage of distributing the goods is referred to as an Another Stage TP. Here, we introduce a new class of TP in which a single commodity of goods is transported to the destination using two times of transportation, and so, it is considered as Two-Stage TP. Considering multiple objective functions in Two-Stage TP, it reduces multi-objective Two-Stage TP. To accommodate the reality, the grey goals are considered into objective functions which make the TP a multi-objective Two-Stage grey transportation problem.

1.2.7 Multi-modal transportation problem

Treating reality in the decision making problem, transportation problem presents the situation such that, there may have origins/destinations in different levels to fulfill the requirements in the final destination points of a transportation network. Due to the factor of multiple routes or multi-modes of transportation in a TP, the TP becomes a Multi-Modal Transportation Problem (MMTP).

Multi-modal transport which is also known as combined transport allows to transport the goods under a single contract, but it is performed with at least two modes of transport; the carrier is liable (in a legal sense) for the entire carriage, even though it is used by several different modes of transport such as sea, road, etc. The carrier does not have to possess all the means of transport, and in practice usually it does not valid. The carrier is often performed by sub-carrier which is referred to in legal language as “actual carriers”. The carrier responsible for the entire carriage is addressed to as a Multi-Modal Transport Operator (MMTO). In a transportation problem, if there be at least one origin except the ground and final origins which have both receiving and dispatching capacity of goods, then it is called a MMTP.

1.2.8 Transportation problem under cost reliability

In the TP, the completion time of transportation of amount of goods should be finished within the specified time, otherwise there may be created a damage of the items or storing problem and/or the customer may reject the ordered item. In that situation, the transportation cost or the profit may not be considered as crisp value. Then the selection of goals for the objective functions or the solution of the MOTP cannot be made in usual way. To overcome this difficulty for selecting the proper goals to the objective functions, here, we incorporate the concept of reliability for the cost parameters in the TP. In that situation, we introduce a new term "cost reliability" for the transportation cost in the proposed study. Generally, Reliability refers the probability of a machine operating its intended purpose adequately for the period of time desired under the operating conditions encountered. More precisely, reliability is the probability with which the devices will not fail to perform a required operation for a certain period of time. Taking advantage of the reliability function in the real-life decision making problem, we formulate the MOTP where the objective functions are connected with some multi-choice goals. The advantage of MOTP under cost reliability is illustrated broadly in the proposed thesis.

1.2.9 Integrated optimization in inventory transportation

Inventory is the stock of items or resources used in an organization. The study of inventory refers to know how much amount of goods have to be sold by decision maker and how much amount left after sold and how much amount need to order from suppliers to keep stock with enough product. In the classical sense of basic inventory optimization, inventory and transportation are carried out separately and total logistics cost is calculated by summing the separate outcome results. On the other hand, an integrated optimization in inventory transportation is a problem to optimize the combination of transportation cost and inventory cost under the prerequisite assumptions. So, the main objective is to reduce the total logistics cost and determine the transportation and inventory strategies of the system. Considering the uncertain situations in real-life problems, the supply and demand are considered as stochastic and multi-choice type respectively in the proposed study of IOIT.

1.3 Fuzzy programming

In the real-life uncertain situations, the fuzzy set theory is an important topic to read. Usually, the fuzzy set theory is used in the field of OP as a tool for solving MOO problems. Nowadays, it is not only used as its classical sense but also plays an important role for accommodating real-life uncertain decision making problems. The fuzzy set theory has been applied in many fields, such as operations research, management science, artificial intelligence, human behavior, etc. The fuzzy mathematical programming has been applied to many disciplines such as advertising, assignment, budgeting, computer section, diet section, location media planing, networks, project selection, transportation, water resource management and many others.

Fuzzy set: A fuzzy set $\tilde{A}$ is a pair $(A, \mu_{\tilde{A}})$ where A is a crisp set belongs to the universal set X and $\mu_{\tilde{A}} : X \to [0, 1]$ is a function, called membership function.

Optimization under fuzziness: The ordinary optimization process generally is used to minimize or maximize the objective function subject to a set of constraints. In the fuzzy optimization, objective function and the constraints are both considered vague and they are expressed by fuzzy set. The process is to find the set of parameter values which maximizes the satisfaction of both the objective and the constraints at the same time. Thus the objective and the constraints perform the same purpose of defining the solution set.

The concept of fuzzy optimization is applied to the classical optimization algorithms, such as Fuzzy Linear Programming Problem (FLLP) problem and fuzzy dynamic programming. In the case of FLLP problem, transportation cost, supply and demand parameters are known only in fuzzy numbers and it determines the amount to be shipped between each demand and supply node.

Fuzzy decision variable: In an optimization problem, usually the unknown variables are considered as real variables. Sometimes there may occur some situations where the decisions are made by selecting fuzzy numbers among a set of fuzzy numbers, in that situations the decision variables are not considered as real and they are taken as fuzzy decision variable. The fuzzy decision variable in the Transportation problem creates a new field of transportation namely fuzzy transportation problem under fuzzy decision variable.

1.4 Goal programming

In an optimization problem, if the optimal solution is obtained according to the desired value (namely goal) of an objective function by the decision maker, then the optimization

problem becomes a Goal Programming (GP). Goal programming, an analytical approach is devised to address the decision making problem where targets have been assigned to all objective functions which are conflicting and non-commensurable to each other and DM interests to maximize the achievement level of the corresponding goals.
In long back, the main concept of GP was that to minimize the deviation between the achievement goals and the achievement levels. The mathematical model of Multi-Objective Decision Making (MODM) can be considered in the following form:
Model GP

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} w_i |Z^i(x) - g_i| \\ \text{subject to} \quad & x \in F, \end{aligned}$$

where F is the feasible set and w_i are the weights attached to the deviation of the achievement function. $Z^i(x)$ is the i-th objective function of the i-th goal and g_i is the aspiration level of the i-th goal. $|Z^i(x) - g_i|$ represents the deviation of the i-th goal. Later on, a modification on GP is provided and is denoted as Weighted Goal Programming (WGP) which can be displayed in the following form:
Model WGP

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{K} w_i(d_i^+ + d_i^-) \\ \text{subject to} \quad & Z^i(x) - d_i^+ + d_i^- = g_i, \\ & d_i^+ \geq 0, d_i^- \geq 0 \quad (i = 1, 2, \dots, K), \\ & x \in F, \end{aligned}$$

where d_i^+ and d_i^- are over and under achievements of the i-th goal respectively.
A vast studies has been developed in GP, but Chang [4] introduced the concept of Revised Multi-choice Goal Programming (RMCGP) for solving MODM, which is more effective than the GP or the WGP. The mathematical model of MODM using RMCGP is defined as follows:
Model RMCGP

$$\text{minimize} \quad \sum_{i=1}^{K} \left[w_i(d_i^+ + d_i^-) + \alpha_i(e_i^+ + e_i^-) \right]$$

$$\begin{aligned}
\text{subject to} \quad & Z^i(X) - d_i^+ + d_i^- = y_i \quad (i = 1, 2, \ldots, K), \\
& y_i - e_i^+ + e_i^- = g_{i,max} \ \text{ or } \ g_{i,min} \quad (i = 1, 2, \ldots, K), \\
& g_{i,min} \leq y_i \leq g_{i,max} \quad (i = 1, 2, \ldots, K), \\
& d_i^+, d_i^-, e_i^+, e_i^- \geq 0 \quad (i = 1, 2, \ldots, K), \\
& x \in F.
\end{aligned}$$

Here, F being the feasible set and y_i is the continuous variable associated with i-th goal which restricted between the upper ($g_{i,max}$) and lower ($g_{i,min}$) bounds and e_i^+ and e_i^- are positive and negative deviations attached to the i-th goal of $|y_i - g_{i,max}|$ and α_i is the weight attached to the sum of the deviations of $|y_i - g_{i,max}|$, other variables are defined as in WGP.

1.5 Conic scalarization

In most of the cases for determining optimal solution of the MOO [equations (1.1) to (1.5)], the model is transformed to a scalar-valued optimization problem and on solving it, we obtain the compromise solution. In many research works such as Kim and Weck [26], Koski [28] using the weight w_t $(t = 1, 2, \ldots, K)$ for the t-th objective function, the MOO problem is reduced to the scalar problem as follows:

$$\text{minimize} \ \sum_{t=1}^{K} w_t f_t(x), \quad x \in F.$$

By solving the scalar problem for a variety of parameters, for instance, for different weights, several solutions of the MOO problem are generated. Based on much better computer performances, it is now possible to represent the whole efficient set those are not obtained through the old techniques.

Using the classical sense of cone of a set and efficient point, the Conic Scalarization approach is introduced, which is an effective technique to obtain proper efficient solutions for MOO problem. In one of the chapter of this thesis, we use the Conic Scalarization approach for solving MCMTP and to establish the effectiveness of the approach on comparing to the other scalar optimization techniques, like GP and RMCGP.

2 Literature survey

The TP was formalized in 1781 by the French mathematician Monge [41], which was considered as the earliest known anticipation of linear programming type of problems.

Major advances were made in the field during World War II by the Soviet/Russian mathematician and economist Kantorovich [24]. Consequently, the problem as it is now stated is sometimes known as the Monge-Kantorovich transportation problem. Kantorovich [24] published a research work on continuous version of the problem and later on Kantorovich [25] developed a study on planning and production in a transportation problem. Many scientific approaches incorporated toward analyzing real-life problems associated with the transportation problem, including operations research, engineering, economics, geographic information science and geography, etc. It is explored especially in the field of mathematical programming and engineering literatures. Sometimes it is referred to as the facility location and allocation problem, the transportation optimization problem can be modeled as a large-scale mixed integer Linear Programming Problem (LPP). The basic model of transportation problem was originally developed by Hitchcock [18]. He first considered the problem of minimizing the cost of distribution of product from several factories to a number of customers. Later on, Koopmans [27] presented an independent study on optimum utilization of the transportation system. Dantzig [7] proposed the simplex method for solving transportation problem which is known as the primal simplex transportation method.
The single objective transportation problem is not enough to handle real-life decision making problem due to our present competitive market scenario. To cover all the real-life situations on TP, we have to introduce here multi-objective TP. Charnes and Cooper [6] first discussed various approaches on the solution of managerial level problems involving multiple conflicting objective functions. Garfinkl and Rao [11] worked out the two objective problems by giving high and low priorities to the objective functions. Verma et al. [57] used fuzzy min operator approach to develop a compromise solution for the multi-objective transportation problem. Ringuest and Rinks [44] proposed two interactive algorithms for generating all non-dominated solutions and identified minimum cost solution as the best compromise solution. Waiel [58] developed a multi-objective transportation problem under fuzziness to get compromise solution. Ebrahimnejad [8] developed a new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers.

Linear programming problem is not sufficient to form all types of decision making problem in our real-life situations. As a result, non-linear programming problem has been incorporated in the multi-objective TP. In this regard, we present here three important research works on TP with non-linear cost. Shetty [55] discussed a method to solve non-linear transportation problem with non-linear cost. Florian [10] developed a study on non-linear cost network models in transportation analysis. A study on non-linear integer programming transportation models introduced by Yang et al. [60]. Recently, Maity and Roy [37] established a study on MOTP introducing a new concept on non-linear cost.

The concept of Grey Numbers emerged as an effective model for systems with partial information known. Liu [30] proposed a study on forming a new algebraic system of grey numbers. Palanci et al. [43] studied on uncertainty under grey goals in a cooperative game. A study on interval grey numbers to solve grey multi-attribute decision making problem was introduced by Honghua and Yong [19]. Liu et al. [31] proposed a study on interval grey number which suggested some knowledge on the degree of greyness of grey numbers. Maity and Roy [38] proposed an integrated study on inventory and transportation problem under multi-choice and stochastic environment.

Goal Programming (GP), an analytical approach is devised to address the decision making problem where targets have been assigned to all objective functions which are conflicting and non-commensurable to each other and DM interests to maximize the achievement level of the corresponding goals. Charnes et al. [5] introduced the concept of GP. The related research can be categorized into two broad classes: goal programming techniques which are proposed for crisp decision making problems and fuzzy goal programming models. The research papers by Lee [29], Ignizio [20], Romero [45], and Tamiz et al. [56] belong to this class. The second class contains the research papers by Narasimhan [42], Hannan [17], Wang et al. [59], and many others. Finally, a comprehensive overview of the state-of-the-art in goal programming can be found in [cf. Ignizio [21, 22]]. Recently, Maity and Roy [36] presented a study to solve MOTP using utility function approach under goal environment.

However, in the real world situations, decision making problems may arise in economics, industry, health care, transportation, agriculture, storing seeds, military purpose, and technology etc. with different structures which cannot be handled using standard decision making approaches. To deal with this type of problems, it is very much essential to develop new decision making models. To do this, Chang [3] proposed a Multi-Choice Goal Programming (MCGP) approach to deal with such problems. Chang [4] revised his approach to make it easier understanding and implementation of linear programming packages for solving such problems. Mahapatra et al. [34] formulated and solved multi-choice stochastic transportation problem involving extreme value distribution. Of late, Roy [48] described and solved multi-choice stochastic transportation problem involving

Weibull distribution which has added a new dimension on real-life TP. Recently, Maity and Roy [35] studied and discussed the solution procedure on multi-choice multi-objective transportation problem using utility function approach.

Fuzzy programming technique is a well known tool to solve multi-objective optimization problems occurred in real-life situations. Cadenas and Verdegay [2] solved multi-objective linear programming problem by fuzzy ranking function. Gupta and Kumar [15], Ebrahimnejad and Tavana [9] worked on uncertainty under different fuzzy environments. Jiménez and Verdegay [23] used the fuzzy uncertainty in solving solid transportation problem. Recently, Maity et al. [39] presented a study on uncertain TP under the consideration of cost reliability.

A study on Characterization over the Benson proper efficiency and scalarization in a non-convex optimization field proposed by Gasimov [12]. Later on, Gasimov [13] introduced a class of monotonically increasing sub-linear functions on partially ordered real normed spaces and showed without any convexity and bounded-ness assumptions that support points of a set obtained by these functions are properly minimal in the sense of Benson [1]. Thereafter, Gasimov and Ozturk [14] presented a study on separation via polyhedral conic functions. Of late, Roy et al. [51] established a study on MOTP using Conic Scalarization approach.

3 Objective and scope of the thesis

The main objective of the thesis has been defined after an extension literature survey based on transportation problem under multi-choice environment. The main objectives of the research work are as follows:

1: To incorporate the concept of extra cost in cost parameter of a TP which produces a non-linear TP and analyze in multi-objective ground of TP under multi-choice demands.

2: Applying the utility function approach to solve a MOTP under multi-choice environment, an effective solution is obtained in compare to the existing studies such as GP and RMCGP.

3: The concept of conic scalarization is implemented to solve MOTP in which each objective function has some goals and an extended study is given to justify the efficiency of the study in multi-choice environment.

4: In a transportation problem, transportation time is a key factor, so considering transportation time and transportation cost in a TP we have studied an bi-objective transportation problem and most importantly the bi-objective function is solved through single objective function. The study is developed by considering the multi-choice interval-valued transportation parameters.

5: Here, we have defined transportation problem under fuzzy decision variable. Assuming the expected allocations in the cells of a TP as multi-choice fuzzy numbers, a new technique using multi-choice goal programming is introduced and solve it to get better results in both single objective and multi-objective grounds.

6: Introducing the time in a TP, we have discussed the concept of cost reliability in MOTP. Again, according to the vague phenomenon of real market situations, the supply and demand constraints are treated as uncertain variables and transformed them through a uncertain measure concept. Numerical example is given to justify the proposed study.

7: In a transportation problem there may occur two types of transportation in warehouses where the goods are stored from markets and then delivered to other places. So, there are two stages involving in the transportation and to introduce as Two-Stage TP. Based on real-life situation, the study is formulated under grey environment and multi-objective ground. A new method using utility function approach is discussed to solve and to select the goal of the objective functions in the proposed problem.

8: Again, we formulate the mathematical model of a TP considering different modes of transportation which is termed a multi-modal TP. On solving the MMTP, the optimal solution and corresponding mode of transportation are presented.

9: Transportation and inventory are two different branches of study, here we made a connection between them and formulated a technique IOIT which gives better solution in compare to traditional inventory and transportation optimization procedure. The study is extended for multi-item of goods in uncertain environments which make it more realistic.

4 Organization of the thesis

The whole thesis contains eleven chapters. A brief introduction related to the proposed research work is presented to Chapter-1. In Chapter-2, we develop a multi-objective transportation problem with non-linear cost and multi-choice demand. The Chapter-3 is devoted to solve multi-objective transportation problem using utility function approach. In Chapter-4, we extend the concept of Conic Scalarization approach and is used to solve multi-objective transportation problem. The Chapter-5, present the study of solving bi-objective optimization problem under the environment of multi-choice and interval valued transportation parameters. In Chapter-6, we provide the concept of transportation problem under fuzzy decision variable in both single objective and multi-objective cases. Chapter-7 is introduced the concept of cost reliability in multi-objective transportation problem under uncertain environment. In Chapter-8, we introduce the Two-Stage grey transportation problem using utility function approach. The Chapter-9 is devoted the study on multi-modal transportation problem. In Chapter-10, we introduce the study of integrated optimization in inventory and transportation problem. In the last Chapter, the conclusions and scope of future works are presented regarding our research work.

The chapter wise summary of the proposed research works is given below:
Chapter 1 introduces the study of optimization problem, especially in the field of transportation problem under several environments. A brief survey on optimization problem, transportation problem, fuzzy programming, goal programming, conic scalarization approach is furnished. We discuss in short the cost reliability, two-stage transportation problem, multi-modal transportation problem, integrated study on inventory transportation problem. Finally, we present the objective, scope and organization of the thesis.

In **Chapter** 2, we develop a mathematical model of multi-objective transportation problem with non-linear cost and multi-choice demand. The objective functions of the proposed transportation problem are non-commensurable and conflict with each other. Furthermore, the objective functions are non-linear type which are occurred due to extra cost for supplying the remaining goods from origin to destinations and demand parameters are treated as multi-choice type. Thus, the mathematical model is formulated by considering the non-linear cost and multi-choice demand. A general transformation technique is developed to tractable the multi-choice demand with the help of binary variables. Therefore, an equivalent multi-objective decision making model is established in order to find the optimal solution of the problem. The outcome from numerical example demonstrates the feasibility of the proposed method [A part of this chapter has been published in ***International Journal of Management Science and Engineering Management,*** Taylor & Francis, ESCI, 11(1), 62-70, (2016)].

Chapter 3 contains two parts, in first part, we present the study of Transportation Problem (TP) with interval goal under multi-objective environment. In most of the cases, Multi-Objective Transportation Problems (MOTPs) are solved by Goal Programming (GP) approach. Using GP, the solution of MOTP may not be satisfied always by the Decision Maker (DM) when the proposed problem contains interval-valued aspiration level. To overcome this difficulty, here we propose the approaches of Revised Multi-Choice Goal Programming (RMCGP) and utility function into the MOTP, and then compare the solutions. A real-life example is presented to justify and to test reality of the proposed concept. In second part, the study of first part has been extended in the multi-choice environment. An example is presented in multi-choice environment to justify the concept [First part of this chapter has been published in ***International Journal of Operational Research***, Inderscience, Scopus, 27(4), 513-529, (2016), and second part of this chapter has been published in ***Journal of Uncertainty Analysis and Applications***, Springer Open, 2:11, doi:10.1186/2195-5468-2-11, (2014)].

Chapter 4 explores the concept of Multi-Choice Multi-Objective Transportation Problem (MCMTP) under the light of Conic Scalarizing function. The MCMTP is a multi-objective transportation problem where the parameters such as cost, demand and supply are treated as multi-choice parameters. Most of the MOTPs are solved by goal programming approach, but the solution of MOTP may not be satisfied always by the decision maker when the objective functions of the proposed problem contains interval-valued aspiration levels. To remove this difficulty, here we propose the approaches of revised multi-choice goal programming and conic scalarizing function into the MOTP, and then we compare among the obtained solutions. Two numerical examples are presented to show the feasibility and usefulness of the discussion topic in the chapter [A part of this chapter has been published in ***Annals of Operations Research,*** **Springer, SCI, IF: 1.406,** DOI 10.1007/s10479-016-2283-4.]

In **Chapter** 5 we consider the study of Transportation Problem (TP) in the light of multi-Choice environment with interval analysis. The parameters of TP follow multi-choice interval valued type so this form of TP is called Multi-Choice Interval Transportation Problem (MCITP). Introduction of time is an important notion in TP of this chapter. Transportation time and cost, both are minimized through single objective function of TP, which is the main aim of this chapter. A procedure is shown for converting from MCITP to deterministic TP and then solve it. Finally, a case study is presented to illustrate the usefulness of the proposed study [A part of this chapter has appeared in **Journal of Intelligent & Fuzzy Systems**, IOS Press, SCIE, IF: 1.004].

Chapter 6 develops a study on TP under fuzzy decision variable. This chapter is divided into two parts. In first part, we consider the study of a single objective TP with fuzzy decision variable. Generally, the decision variable in a transportation problem is considered as real variable. But, here the decision variable in each node is chosen from a set of multi-choice fuzzy numbers. A new formulation of mathematical model of Fuzzy Transportation Problem (FTP) with fuzzy goal to the objective function is designed. After that, the solution technique of the proposed model is included through multi-choice goal programming approach. The proposed approach is not only improved the applicability of goal programming in real world situations but is also provided useful insight about the solution of a new class of the TP. Finally, a real-life example is incorporated to analyze the feasibility and usefulness of the study. The last part of this chapter considered the study of first part extending the concept of single objective TP to a MOTP. The usefulness is justified through the numerical examples [First part of this chapter is communicated in **International Journal**; and second part part of this chapter has accepted for publish in ***International Journal of Operations Research and Information Systems (IJORIS), IGI Global, Info-SCI,*** Vol 8, No. 2].

Chapter 7 initiates the study of cost reliability in the multi-objective transportation problem under uncertain environment. Assuming the uncertainty in real-life decision making problems, the concept of reliability is incorporated in the transportation cost and the effectiveness is justified through the proposed MOTP. Again, considering the real phenomenon in the MOTP, we consider the transportation parameters, like supply and demand as uncertain variables. Also, we consider the fuzzy multi-choice goals to the objective functions of the MOTP; and Fuzzy Multi-Choice Goal Programming (FMCGP) is used to select the proper goals to the objective functions of the proposed MOTP. A numerical example is presented to illustrate and to justify the proposed study [A part of this chapter has been published in ***International Journal of Computational Intelligence Systems (IJCIS)***, Atlantis Press and Taylor & Francis, SCIE, IF: 0.391, Vol. **9**, No. 5, pp. 839-849].

In **Chapter** 8 we define Multi-Objective Goal Programming (MOGP) is applied to solve problems in many application areas of real-life decision making problems. This chapter attempts to formulate the mathematical model of Two-Stage Multi-Objective Transportation Problem (MOTP) where we design the feasibility space based on the selection of goal values. Considering the uncertainty in real-life situations, we incorporate the interval grey parameters for supply and demands in the Two-Stage MOTP, and a procedure is applied to reduce the interval grey numbers into real numbers. Thereafter, we present a solution procedure to the proposed problem by introducing an algorithm and using the approach of Revised Multi-Choice Goal Programming (RMCGP). In the proposed algorithm, we introduce a utility function for selecting the goals of the objective functions. Finally, a case study is encountered to justify the reality and feasibility of the proposed study [A part of this chapter is submitted after revision in ***Central***

European Journal of Operations Research, **Springer, SCI, IF. 0.978**].

In **Chapter** 9, a new method is designed for solving Transportation Problem (TP) by considering the multi-modal transport systems. This new method is a combination of TP and multi-modal systems and here it is referred to as Multi-Modal Transportation Problem (MMTP). To analyze the proposed method a case study is included and solved which reveals a better impact for analyzing the real-life decision making problems [A part of this chapter is Communicated to **International Journal**].

In **Chapter** 10, we investigate the study of multi-item multi-choice transportation problem in the ground of inventory optimization. We study the basic inventory optimization and then we develop a methodology for Integrated Optimization in Inventory Transportation (IOIT) to reduce the logistic cost of a system. To accommodate the present status of real-life TP, the stochastic supply is taken into consideration in the study. We describe a technique to reduce stochastic supply constraint to deterministic supply constraint with the help of stochastic programming. An algorithm is presented to solve the proposed problem using MATLAB. Then the proposed problem is solved by well known optimization technique and the obtained solution is compared with the solution of basic inventory optimization method. An example is encountered to verify the effectiveness of the study in the chapter [A part of this chapter has been accepted for publication in **International Journal of Operational Research**, Inderscience, Scopus].

Finally, the concluding remarks on the work carried out in Chapters 2 to 10 are described in **Chapter 11**. Future scope of further research works on the presented topic is also discussed.

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List of Publications

1. Maity, G., Roy, S.K., (2014) Solving multi-choice multi-objective transportation problem: a utility function approach, *Journal of Uncertainty Analysis and Applications,* Springer Open, Vol. **2**, No. 11, doi:10.1186/2195-5468-2-11.

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The list of Communicated Papers in Journals

10. Maity, G., Roy, S.K., Transportation problem under fuzzy decision variable, Communicated to International Journal.
11. Maity, G., Roy, S.K., A new approach for solving transportation problem using multi-modal systems, Communicated to International Journal.