Synopsis of the thesis entitled

m-polar Fuzzy Graphs and Their Applications

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1 Introduction

The origin of graph theory started with the problem of Konigsberg bridge, in 1735. This problem lead to the concept of Eulerian graph. Euler studied the problem of Konigsberg bridge and constructed a structure that solves the problem called Eulerian graph. In 1840, Mobious gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. Now a days, graph theoretical concepts are highly utilized by computer science applications. Especially in research areas of computer science including datamining, image segmentation, clustering, and networking. The introduction of fuzzy sets by Zadeh [29] in 1965 changed the face of science and technology to a great extent. Fuzzy set paved the way for a new philosophical thinking of 'Fuzzy Logic' which now, is an essential concept in artificial intelligence. The most important feature of a fuzzy set is that it consists of a class of objects that satisfy a certain (or several) property.

In 1994, Zhang [32, 33] initiated the concept of bipolar fuzzy sets. Juanjuan Chen et al. [7] introduced the notion of m-polar fuzzy set as a generalization of bipolar fuzzy sets. The first definition of fuzzy graphs was proposed by Kafmann [10], from Zadeh's fuzzy relations [29, 30, 31]. But Rosenfeld [16] introduced another elaborated definition including fuzzy vertex, fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. McAllister [12] characterized fuzzy intersection graphs. After that, the operation of union, join, Cartesian product and composition on two fuzzy graphs was defined by Mordeson and Peng [11]. Mordeson and Nair [13] defined the complement of fuzzy graph. Sunitha and Kumar [26] further studied the other properties of fuzzy graphs. The concept of weak isomorphism, co-weak isomorphism and isomorphism between fuzzy graphs was introduced by Bhutani in [4]. After that several researchers are working on fuzzy graphs like in [5, 6, 8, 9, 14, 15]. Samanta and Pal studied fuzzy tolerance graphs [19], fuzzy threshold graphs [20], fuzzy k-competition graphs and m step fuzzy competition graphs [23, 24], fuzzy planar graphs [25].

In 2011, Akram [1, 2, 3] introduced and investigated many properties of bipolar fuzzy graphs using the concepts of bipolar fuzzy sets. Rashmanlou et al. [17, 18] studied many properties bipolar fuzzy graphs. Some more work on bipolar fuzzy graphs may be found on [21, 22, 27].

In 2014, Chen et al. [7] introduced the notion of m-polar fuzzy set as a generalization of bipolar fuzzy set and showed that bipolar fuzzy sets and 2-polar fuzzy sets are cryptomorphic mathematical notions and that we can obtain concisely one from the corresponding one. The idea behind this is that "multipolar information" (not just bipolar information which correspond to two-valued logic) exists because data of real world problems are sometimes come from multiple agents. For example, the exact degree of telecommunication safety of mankind is a point in $[0,1]^n$ ($n \approx 7 \times 10^9$) because different persons have been monitored different times. There are many other examples such as truth degrees of a logic formula which are based on n logic implication operators ($n \geq 2$), similarity degrees of two logic formulas which are based on n logic implication operators ($n \geq 2$), ordering results of a magazine, ordering results of a university, and inclusion degrees (accuracy measures, rough measures, approximation qualities, fuzziness measures, and decision preformation evaluations) of a rough set. Chen et al. [7] first defined m-polar fuzzy graphs in 2014.

1.1 Preliminaries

Definition 1.1. (Fuzzy set [29]) *A fuzzy set A on a set X is characterized by a mapping $\mu : X \to [0,1]$, called the membership function. We shall denote a fuzzy set as $A = (X, \mu)$.*

Definition 1.2. (Fuzzy graph [13]) *A fuzzy graph $\xi = (V, \mu, \rho)$ is a non-empty set V together with a pair of functions $\mu : V \to [0,1]$ and $\rho : V \times V \to [0,1]$ such that for all $x, y \in V, \rho(x,y) \leq \mu(x) \wedge \mu(y)$ where $\mu(x)$ and $\rho(x,y)$ represent the membership values of the vertex x and of the edge (x, y) in ξ respectively.*

Definition 1.3. (Bipolar fuzzy set [32]) *Let X be a non-empty set. A bipolar fuzzy set B in X is an object having the form $B = \{((x, \mu_B^P(x), \mu_B^N(x)) : x \in X\}$, where $\mu_B^P : X \to [0,1]$ and $\mu_B^N : X \to [-1,0]$ are mappings.*

Definition 1.4. (Bipolar fuzzy graph [1, 27]) *A bipolar fuzzy graph of a graph $G^* = (V, E)$ is a pair $G = (V, A, B)$ where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on $\widetilde{V^2}$ such that $\mu_B^P(xy) \leq min\{\mu_A^P(x), \mu_A^P(y)\}$, $\mu_B^N(xy) \geq max\{\mu_A^N(x), \mu_A^N(y)\}$ for all $xy \in \widetilde{V^2}$ and $\mu_B^P(xy) = \mu_B^N(xy) = 0$ for all $xy \in (\widetilde{V^2} - E)$.*

Definition 1.5. (m-polar fuzzy set [7]) *An m-polar fuzzy set (or a $[0,1]^m$-set) on X is a mapping $A : X \to [0,1]^m$. The set of all m-polar fuzzy sets on X is denoted by $m(X)$.*

Here $[0,1]^m$ (m-power of [0,1]) is considered to be a poset with point-wise order $\leq$, where m is an natural number. $\leq$ is defined by $x \leq y \Leftrightarrow$ for each $i = 1, 2, \ldots, m$; $p_i(x) \leq p_i(y)$ where $x, y \in [0,1]^m$ and $p_i : [0,1]^m \to [0,1]$ is the i-th projection mapping.

Definition 1.6. *Let A and B are two m-polar fuzzy sets in X. Then $A \cup B$ and $A \cap B$ are also m-polar fuzzy sets in X defined by: for $i = 1, 2, \ldots, m$ and $x \in X$,*

$p_i \circ (A \cup B)(x) = max\{p_i \circ A(x), p_i \circ B(x)\}$ *and* $p_i \circ (A \cap B)(x) = min\{p_i \circ A(x), p_i \circ B(x)\}$.
$A \subseteq B$ *if and only if* $p_i \circ A(x) \leq p_i \circ B(x)$ *and* $A = B$ *if and only if* $p_i \circ A(x) = p_i \circ B(x)$.

Definition 1.7. *Let A be an m-polar fuzzy set on a set X. An m-polar fuzzy relation on A is an m-polar fuzzy set B of $X \times X$ such that $B(x, y) \leq min\{A(x), A(y)\}$ for all $x, y \in X$ i.e, for each $i = 1, 2, \ldots, m$, for all $x, y \in X$, $p_i \circ B(x, y) \leq min\{p_i \circ A(x), p_i \circ A(y)\}$. An m-polar fuzzy relation B on X is called symmetric if $B(x, y) = B(y, x)$ for all $x, y \in X$.*

2 Organization of the Thesis

The thesis is organized into 10 chapters. The brief contents of all chapters are given below.

Chapter 1

Introduction

Chapter 1 presents the introduction of the thesis. In this chapter some definitions are given which are used throughout the thesis. The definition of fuzzy sets and fuzzy graphs, bipolar fuzzy graphs, m-polar fuzzy sets and m-polar fuzzy graphs are given. This chapter presents a review of the previous works and also the motivation of the work.

Chapter 2

Fundamentals of m-polar fuzzy graphs

In this chapter, generalized m-polar fuzzy graphs (mFG) are defined after the definition by Chen et al. [7]. Four operations such as Cartesian product, composition, union and join have been defined on mFGs. Some useful properties of strong mFGs, self-complementary mFGs and self-complementary strong mFGs are discussed.

Chapter 3

Operations and degrees of m-polar Fuzzy Graphs

In this chapter, three new operations, viz. direct product, semi-strong product and strong product are defined on mFGs. A subclass of m-polar fuzzy graphs called product m-polar fuzzy graph is defined and many properties of them are discussed here. The degree of a vertex in mFG are introduced from two given mFGs G_1 and G_2 using the operations of Cartesian product, composition, direct product, semi-strong product and strong product. At the end, an application of 3-polar fuzzy influence graph is given.

Chapter 4

Density of m-polar Fuzzy Graphs

In this chapter, the notions of density of an mFG and balanced mFGs are defined. Some characterizations of balanced mFGs are given.

Chapter 5

m-polar fuzzy planar graphs and its dual

In this chapter, our study describes the m-polar fuzzy multigraphs, m-polar fuzzy planar graphs, and a very important consequence of m-polar fuzzy planar graphs known as m-polar fuzzy dual graphs. The new parameter "degree of planarity" used in this chapter characterizes an mFG in many ways. The graphs such as m-polar fuzzy multigraph, m-polar fuzzy planar graph, and m-polar fuzzy dual graph are also defined. In crisp planar graph, no edge intersects each other. But, the edges of any m-polar fuzzy graph may be m-polar fuzzy weak or m-polar fuzzy strong. Using the concept of m-polar fuzzy weak edge, we define m-polar fuzzy planar graph in such a way that an edge may intersect other edges. But, this facility violates the definition of planarity of graph. Since the role of m-polar fuzzy weak edge is insignificant, the intersection between an m-polar fuzzy fuzzy weak edge with any edge is less important. Motivating from this idea, we allow the intersection of edges in m-polar fuzzy planar graph. It is well known that if the membership values of all edges become one, the graph becomes crisp graph. Keeping this idea in mind, we define a new term called degree of planarity of an m-polar fuzzy graph. If the degree of planarity of an mFG is $\mathbf{1} = (1, 1, \ldots, 1)$, then no edge crosses other. This leads to the crisp planar graph. Thus, the planarity value measures the degree of planarity of an mFG. This is a very interesting concept of m-polar fuzzy graph theory. Strong m-polar fuzzy planar graph has been exemplified. Another important term of planar graph is 'face' which is redefined in m-polar fuzzy planar graph. In this chapter, new theories have been investigated for m-polar fuzzy planar graph. The m-polar fuzzy dual graph is defined for the m-polar fuzzy planar graph whose degree of planarity is $\mathbf{1} = (1, 1, \ldots, 1)$. These theories will be helpful to improve algorithms in different fields including computer vision, image segmentation, etc.

Chapter 6

Isomorphic properties of m-polar Fuzzy Graphs with applications

In this chapter, the notion of weak self complement m-polar fuzzy graphs, order, size, busy vertices and free vertices of an m-polar fuzzy graphs are defined. Self complement m-polar fuzzy graphs have many important role in the theory of m-polar fuzzy graphs. If an m-polar fuzzy graph is not self complement, then also we can say that it is self complement in some weaker sense. We can establish some useful results with this graph. This motivates to define weak self complement m-polar fuzzy graphs in this chapter. A necessary condition is mentioned for an m-polar fuzzy graph to be weak self complement. Several properties of them are discussed. A relative study of complement and operations on m-polar fuzzy graphs have been made. Some real life problems have been modeled using the concepts of m-polar fuzzy graphs.

Chapter 7

Edge regularity of m-polar Fuzzy Graphs

This chapter deals with the concept of edge regular, strongly regular, biregular, partially edge regular and fully edge regular m-polar fuzzy graphs. Some properties of them are studied. Finally, we introduced the notion of strongly edge irregular and strongly edge totally irregular m-polar fuzzy graphs. Some properties of them are also studied to characterize strongly edge irregular and strongly edge totally irregular m-polar fuzzy graphs.

Chapter 8

Morphism of m-polar Fuzzy Graphs

In this chapter, we generalized the usual concept of isomorphism in m-polar fuzzy graphs which we call as m-polar ψ-morphism. The action of m-polar ψ-morphism on m-polar fuzzy graphs are discussed. Then, d_2 degree, total d_2 degree of a vertex, $(2, \overline{k})$-regularity and totally $(2, \overline{l})$-regularity are defined in m-polar fuzzy graphs. A real life situation of a company has been modeled in terms of 4-polar fuzzy graphs as an application.

Chapter 9

Generalized regular bipolar fuzzy graphs and product bipolar fuzzy line graphs

In this chapter, generalized regular bipolar fuzzy graphs are introduced. A subclass of bipolar fuzzy graphs namely product bipolar fuzzy graph is defined. Then the notion of product bipolar fuzzy line graph is introduced and investigated some of its properties. A necessary and sufficient condition is given for a product bipolar fuzzy graph to be isomorphic to its corresponding product bipolar fuzzy line graph. It is also examined when an isomorphism between two product bipolar fuzzy graphs follows from an isomorphism of their corresponding fuzzy line graphs.

Chapter 10

Conclusion

In this chapter, we made some conclusions about the work presented in the thesis.

The natural extension of these work are

1. m-polar fuzzy soft graphs,
2. m-polar fuzzy soft planar graphs,
3. m-polar fuzzy soft hypergraphs,
4. m-polar fuzzy soft competition graphs,
5. m-polar fuzzy rough graphs,
6. Applications of m-polar fuzzy soft graphs on decision making problems, etc.

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