SOME DECISION MAKING PROBLEMS IN INVENTORY CONTROL SYSTEM UNDER DIFFERENT ENVIRONMENTS

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BY

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This Thesis is dedicated to
my respected Sir

PROF. MANORANJAN MAITI

(Former Prof. of Vidyasagar University)

CERTIFICATE

This is to certify that the thesis entitled **"SOME DECISION MAKING PROBLEMS IN INVENTORY CONTROL SYSTEM UNDER DIFFERENT ENVIRONMENTS "** being submitted to the **VIDYASAGAR UNIVERSITY** by **Sri Nabakumar Chakraborty** for the award of degree of **DOCTOR OF PHILOSOPHY in Science** is a record of bona-fide research work carried out by him under our guidance and supervision. **Sri Chakraborty** has worked in the **Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University** as per the regulations of this University.

In our opinion, this thesis is of the standard required for the award of the degree of **DOCTOR OF PHILOSOPHY IN SCIENCE**.

The results, embodied in this thesis, have not been submitted to any University or Institution for the award of any degree or diploma.

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DECLARATION

I, Nabakumar Chakraborty, do hereby declare that, I have not submitted the results embodied in my thesis – "**SOME DECISION MAKING PROBLEMS IN INVENTORY CONTROL SYSTEM UNDER DIFFERENT ENVIRONMENTS**"or a part of it for any degree/ diploma or any other academic award anywhere before.

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List of Publications

A. List of Published Papers

- A deteriorating multi-item inventory model with price discount and variable demands via fuzzy logic under resource constraints. ***C*omputer & Industrial Engineering. 66 (2013) 976-987, Elsevier.**

- An EPQ Model for Deteriorating items under Random Planning Horizon with some Linguistic Relations between Demand, Selling Price and Trade Credit, Ordered Quantity. ***Journal of Mathematics and Informatics. 6 (2016) 73-92,*** **House of Scientific Research.**

B. List of Communicated Papers

- A Supply Chain Model with fuzzy logic under Random planning Horizon via Genetic Algorithm. Communicated to ***Fuzzy Information and Engineering. (Elsevier).***

- Optimum Production Policy for a Production Inventory Model in Random Time Horizon. Communicated to ***Journal of Intelligent Manufacturing. (Springer).***

- Two plant optimal production problem in random time horizon. Communicated to ***Applied Mathematical Modelling. (Elsevier).***

- Two layer supply chain model of deteriorating items with rework and two level credit period. Communicated to ***C*omputer and Operations Research. (Elsevier)**

- Note On: Partial trade credit policy of retailer in economic order quantity models for deteriorating items with expiration dates and price sensitive demand. Communicated to ***European Journal of Operational Research. (Elseveier).***

List of Acronyms

AUD	All Unit Discount
DRC	Due Raw-material Cost
DV	Decision Variable
EOQ	Economic Order Quantity
EPL	Economic Production Lot size
EPQ	Economic Production Quantity
FEMOGA	Fast and Elitist Multi-Objective Genetic Algorithm
GA	Genetic Algorithm
GMIR	Graded Mean Integration Representation
GRG	Generalized Reduced Gradient
H	High
IQD	Incremental Quantity Discount
L	Low
LFN	Linear Fuzzy Number
M	Middle
MF	Membership Function
MOGA	Multi-Objective Genetic Algorithm
MOLP	Multi-Objective Linear Programming
MONLP	Multi-Objective Non-Linear Programming
MOPP	Multi-Objective Programming Problem
MRP	Maximum Retail Price
PFN	Parabolic Fuzzy Number
SCM	Supply Chain Model
TC	Total Cost
TFN	Triangular Fuzzy Number
TP	Total Profit
TPBVP	Two Point Bounded Value Problem
TrFN	Trapezoidal Fuzzy Number
VH	Very High
VL	Very Low
VMP	Vector Minimum Problem

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Introduction and Solution Methodologies

Chapter 1

Introduction

1.1 Definition and History of Operations Research

Operations Research (OR) is a brunch of applied mathematics which encompasses a wide range of problem-solving techniques and methods applied in the pursuit of improved decision-making and efficiency, such as simulation, mathematical optimization, queuing theory, different stochastic-process models, econometric methods, data envelopment analysis, neural networks, expert systems, decision analysis, analytic hierarchy process etc. In 1957 Churchman *et al.* [69] defined OR as the application of scientific methods, techniques and tools to decision making problems (DMP) involving the operations of systems so as to provide these in the control of the operations with optimum solutions to the problem.

The requirement of the methods of OR was firstly felt during the Second World-War $(1939 - 45)$. A team of investigators and scientist were employed by the Allies (principally Britain, the Soviet Union, and the US) to use the limited resources (foods, weapons, medicines etc.) properly and to send these resources to different battle fields safely within a limited budget and man power from different service centers. i.e. the objective of the team was to formulate specific proposals and plans for aiding the military commanders to arrive at the decisions on optimal utilization of limited military logistical and armament supports and also to implement the decisions effectively.

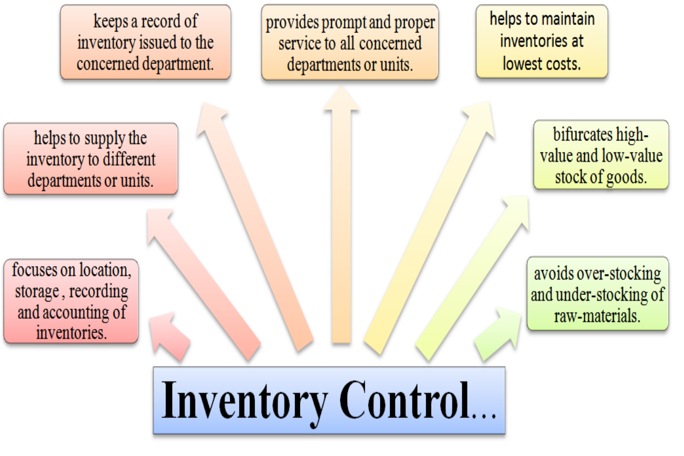
In the decades after the second world war, the techniques were more widely applied to problems in business, industry and society. Since that time, operational research has expanded into a field and widely used in industries ranging from petrochemicals to airlines, finance, logistics, and government, moving to a focus on the development of mathematical models that can be used to analyze and optimize complex systems, and has become an area of active academic and industrial research. In India, operations research came into existence with the opening of an OR unit in 1949 at the Regional Research Laboratory at Hyderabad.

An OR unit under Professor P. C. Mahalonobis was established in 1953 in the Indian Statistical Institute, Calcutta to apply OR methods in national planning and survey. He made the first important application of OR in India in preparing the draft of the Second Five Year Plan. The draft plan frame is still considered to be the most scientifically formulated plan of massive economic development of India.

1.1.1 Different Fields of Application of OR

The techniques of OR has been used successfully in various field of operation. Some of our interested fields are given by the following.

• **Inventory Control:** Inventory control involves all the approaches and techniques to control the inventory situation for different management systems. Some basic functions of inventory control are:



1. Deciding inventory replenishment decisions. There are some important replenishment decisions.
   (a) When is it necessary to place an order (or produce) to replenish inventory?
   (b) How much is to be ordered (or produced) in each replenishment?
2. To take a proper decision about maximum retail price (MRP).
3. Making right decision about the preservation of stock.

• **Supply Chain Management:** Supply chain is a sequence of processes involved in the production and distribution of a commodity. Supply chain management is a set of approaches utilized to efficiently coordinate and integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right locations and at the right time, in order to minimize system-wide costs while satisfying service level requirements.

• **Scheduling:** Scheduling is the method by which work specified by some means is assigned to resources that complete the work. Some of the scheduling process which can be handled by OR are scheduling of aircrews and the fleet for airlines, vehicles scheduling in supply chains, scheduling of orders in a factory and scheduling of operating theaters in a hospital.

• **Facility planning:** Facility planning primarily involves managing the planning, programming, designing, construction move-in, operation and maintenance of facilities to enable an organization to achieve its goals. Computer simulations of airports for the rapid and safe processing of travelers, improving appointment systems for medical practitioners etc. are some field where facility planning have a great importance.

• **Planning and forecasting:** Planning is a basic management function involving formulation of one or more detailed plans to achieve optimum balance of needs or demands with the available resources. The planning process

1. identifies the goals or objectives to be achieved,
2. formulates strategies to achieve them,
3. arranges or creates the means required,
4. implements, directs, and monitors all steps in their proper sequence.

Forecasting is a common statistical task in business, where it helps to inform decisions about the scheduling of production, transportation and personnel, and provides a guide to long-term strategic planning. Thus planning and forecasting helps in identifying possible future developments in telecommunications, deciding how much capacity is needed in a holiday business.

• **Yield management:** Yield management is a variable pricing strategy, based on understanding, anticipating and influencing consumer behavior in order to maximize revenue or profits from a fixed, time-limited resource (such as airline seats or hotel room reservations or advertising inventory).

• **Credit scoring:** A credit scoring is a process of deciding which customers offer the best prospects for credit companies.

• **Marketing:** Marketing is the action or business of promoting and selling products or services, including market research and advertising. It involves evaluation of the value of sale promotions, developing customer profiles and computing the life-time value of a customer.

• **Simulation:** Simulation is the imitation of the operation of a real-world process or system over time. Simulation is used in many contexts, such as simulation of technology for performance optimization, safety engineering, testing, training, education, and video games.

• **Queueing theory:** Queueing theory is the mathematical study of waiting lines, or queues. In queueing theory, a model is constructed so that queue lengths and waiting time can be predicted. Queueing theory is generally considered a branch of operations research because the results are often used when making business decisions about the resources needed to provide a service. The ideas have since seen applications including telecommunication, traffic engineering, computing and the design of factories, shops, offices and hospitals.

• **Defence and peace keeping:** It is a process of finding ways to deploy troops rapidly. OR techniques are deployed in defence operations (viz. administration, intelligence, training etc.) of the air force, army and navy in order to arrive at an optimum strategy to achieve consistent goals.

1.2 Basic Concepts and Terminologies

The definitions and concepts about some familiar terms used to represent the parameters in study of inventory control are given by the following. [Reader may refer to follow Hadley and Whitin [101], Naddor [191] for brief discussion about the parameters].

1.2.1 Definitions and Terminologies

Manufacturer : One who makes products through a process involving raw materials, components, or assemblies, usually on a large scale with different operations divided among different workers. Manufacturer is also known as producer.

Wholesaler : Wholesaler is a person or firm that buys large quantity of goods from manufacturers/ producer/ supplier. Thus a wholesaler takes the inventory in bulk and delivers a bundle of related product to retailers. Wholesaler is also known as distributor. A distributor is typically an organization that takes ownership of significant inventories of products. A wholesaler is a middleman between a manufacturer and retailers of the product. The wholesaler makes money by buying the product(s) from the manufacturer at a lower price- usually through discounts based on volume buying.

Demand: Demand refers to the quantity of a commodity required at a given time. It usually depends upon the decisions of people outside the organization which has the inventory problem. The size, rate and pattern can classify the demand into following categories. In

Table 1.1: Different type of demand

Deterministic demand	Random demand	Imprecise demand
• fixed or constant • dependent on stock • dependent on price • dependent on trade credit • dependent on time • etc.	• known distribution • unknown distribution • etc.	• Fuzzy demand • Fuzzy-random demand • etc.

some cases, demand may be represented by vague, imprecise and uncertain data. This type of demand is termed as fuzzy demand. Demand also can be treated as fuzzy-stochastic in nature.

Retailer : One who sells goods or commodities directly to consumers is known as retailer. A retailer purchased items from the manufacturer or wholesaler and sold to the end user at a marked up price. Retailers stock inventory and sell in smaller quantities to the general public.

Supply chain : A supply chain is a system of organizations, people, activities, information, and resources involved in moving a product or service from supplier to customer. Supply chain activities transform natural resources, raw materials, and components into a finished product that is delivered to the end customer. In sophisticated supply chain systems, used products may re-enter the supply chain at any point where residual value is recyclable.

Replenishment/Supply: Replenishment can be categorized according to size, pattern and lead time. Replenishment size refers to the quantity or size of the order to be received into inventory. The size may be constant or variable, depending on the type of inventory system. Replenishment patterns refer to how much amount of inventory is added to the inventory stock. The replenishment patterns are usually instantaneous or uniform. Normally, replenishment are made either in once or batch-wise.

Business/Time Horizon: The time period over which the inventory level will be controlled is called the time horizon. It may be finite or infinite depending upon the nature of the inventory system of the commodity.

Constraints: Constraints in inventory system deal with various properties that some way place limitations imposed on the inventory system. Constraints may be imposed on the amount of investment, available space, resources and finance, the amount of inventory held, average inventory expenditure, number of orders, etc. These constraints can also be fuzzy, random and fuzzy-random in nature.

Fully Back-logged/Partially Back-logged Shortages: During stock-out period, the sales and/or goodwill may be lost either by a delay or complete refusal in meeting the demand of the customers. In the case where the unfulfilled demand for the goods can be satisfied completely at a later date, then it is a case of fully back-logged shortages, i.e., it is assumed that no customer walk away during this period and the demand of all these waiting customers is met at the beginning of the next period. Again, it is normally observed that during the stock-out period, some of the customers wait for the product and others walk away. Such a phenomenon is called partially backlogged shortages.

Production Cost: The costs relevant to produce a finished good are called production cost. Unit production cost is also production dependent. For example, if one worker is needed to tend the machine, then as more units are produced per unit time, the wages of the worker spread over more units. More elaborately it can be split into the following costs.

- Purchasing Cost
- Labor Cost
- Wear and tear Cost
- Environment protection Cost

Purchase Cost: It is the purchase price of raw materials for manufacturer (or, of finished goods for wholesaler/retailer) to obtain the item from an external source. For manufacturer It may also depend upon the demand when production is done in large quantities as it results in reduction of production cost per unit. Also, when quantity discounts are allowed for bulk orders, purchasing price is reduced and depends on the quantity purchased or ordered.

Wear and tear Cost: In calculating the cost of production one should have to include the wear and tears of the instrument as well as that of the labors.

Labor Cost: The cost to employ the labors the root level of production.

Environment protection cost: To reduce the effect of global worming production firms have to follow different environment protocols. For this purpose a manufacturer must have to spend some extra money, which is called the environment protection cost.

Other Inventory Cost: There are some other costs relevant to inventory decision making, namely

- Ordering or Setup Cost
- Holding or Carrying Cost
- Shortage or Penalty Cost
- Transportation Cost

Ordering or Set-up Cost: It is the cost associated with the expense of issuing a purchase order to an out-side supplier or setting up machines before internal production. These costs also include clerical and administrative costs, telephone charges, telegram, transportation costs, loading and unloading costs, watch and ward costs, etc. Generally, this cost is assumed to be independent of the quantity ordered for or produced. In the costs like transportation cost, etc., some part of it may be quantity dependent.

Holding or Carrying Cost: It is the cost associated with the storage of the inventory until its use or sale. It is directly proportional to the quantity in inventory and the time for which the stocks are held. This cost generally includes the costs such as rent for storage space, interest on the money locked-up, insurance, taxes, handling, etc.

Shortage or Stock-out Cost or Penalty Cost: It is the penalty incurred when the stock proves inadequate to meet the demand of the customers. This cost parameter does not depend upon the source of replenishment of stock but upon the amount of inventory not supplied to the customer.

Transportation Cost: The expenditure for transporting products to different destination or for availing items from different sources is called transportation cost.

Advertisement : The non-personal communication of information which is usually paid for and usually persuasive in nature about products, services or ideas by identified sponsors through the various media, is regarded as Advertising.

Defective Product : A product is in a defective condition, unreasonably dangerous to the user, when it has a propensity or tendency for causing physical harm beyond that which would be contemplated by the ordinary user, having ordinary knowledge of the product's characteristics commonly known to the foreseeable class of persons who would normally use the product.

Recycling : Recycling is a process by which materials (waste) are change into new products to prevent waste of potentially useful materials, reduce the consumption of fresh raw materials, reduce energy usage, reduce air pollution (from incineration) and water pollution (from land filling) by reducing the need for "conventional" waste disposal, and lower greenhouse gas emissions. Recycling is a key component of modern waste reduction. Recyclable materials include many kinds of glass, paper, metal, plastic, textiles, and electronics. Materials to be recycled are either brought to a collection center or picked up from the curb side, then sorted, cleaned, and reprocessed into new materials.

Trade Credit : In recent competitive market, manufacturer /wholesaler /retailer frequently offers delay period for settling the account on purchasing amount of units (greater than or equal to a certain amount fixed by the wholesaler/manufacturer). This is termed as trade credit period. Depending upon the credit period, demand of an item increases or decreases. If credit period is offered to the retailers only by the supplier, it is called one level trade credit. On the other hand if both the supplier and the retailer offer credit period to his/her retailer and customers respectively, it is called two level trade credit. Again, if credit period is offered depending upon some conditions (like amount of purchase should exceed some label, frequency of order etc.), it is called conditionally delay in payment or conditional credit period.

Inflation and time value of money: Inflation is a present increase in the level of consumer price or a persistent decline in the purchasing power of money, caused by an increase in available currency and credit beyond the proportion of available goods and service. It is the rate at which the general prices for goods and services are rising and subsequently, purchasing power is falling. With the increase of inflation rate, more amount of money is to be paid for the same quantity of commodity. As for example, if the inflation rate is 1%, a \$ 5 of pen will cost \$ 5.05 in a year. Mathematically, Buzacott [27] assumed that cost at time t, $\phi(t)$, becomes $\phi(t+\delta t) = \phi(t) + i\,\phi(t)\delta t$ at time $(t+\delta t)$ (where δt is sufficiently small) when a constant inflation rate i (\$/unit) exists in the market, i.e.,

$$\begin{aligned}
& \phi(t+\delta t) = \phi(t) + i\,\phi(t)\,\delta t \text{ as } \delta t \to 0 \\
\Rightarrow \quad & \frac{\phi(t+\delta t) - \phi(t)}{\delta t} \to i\,\phi(t) \text{ as } \delta t \to 0 \\
\Rightarrow \quad & \frac{d\phi(t)}{dt} = i\,\phi(t) \qquad \Rightarrow \qquad \frac{d\phi(t)}{\phi(t)} = i\,dt
\end{aligned}$$

which yields a solution as $\phi(t) = \phi(0)e^{i\,t}$, where $\phi(0)$ is the cost at time $t = 0$.

On the other hand, time value of money is one of the basic concept of finance. We know that if we deposit money in a bank we will receive interest. For example, \$ 1 today invested for one year at 7% return would be worth \$1.07 in a year. Because of this, we prefer to receive money today rather than the same amount in the future. Money we receive today is more valuable to us than money received in the future by the amount of interest we can earn with the money. It is the changes in purchasing power of money over time.

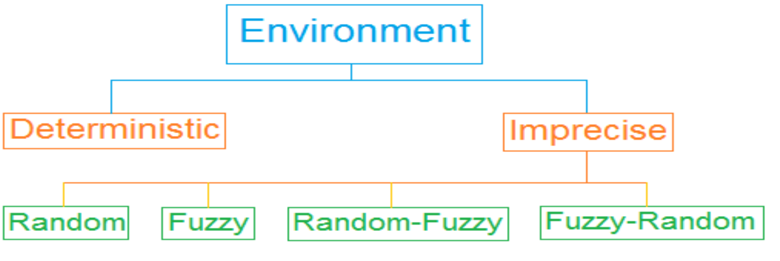
So, if $i\%$ and $r\%$ are the annual inflation and interest rate respectively, resultant effect of inflation and time value of money (i.e., increased rate of cost) on purchasing a unit of item in future is $(i-r)\%$. So if $\phi(0)$ is the cost of an item at time $t = 0$, its cost at time t, $\phi(t) = \phi(0)e^{-R\,t}$, where $R = (r-i)$ is called discount rate of cash flow.

Selling Price : Selling price is the price at which something is offered for sale. Generally, it is not fixed for long time for a particular item. Moreover it can vary with different parameters such as purchase cost, purchase amount, product quality, product availability etc.

Product Quality : Product quality means to incorporate features that have a capacity to meet consumer needs (wants) and gives customer satisfaction by improving products (goods) and making them free from any deficiencies or defects.

1.2.2 Different Environments

The inventory parameters, such as time horizon, demand, production cost, different inventory costs (viz., purchasing cost, set-up cost, holding cost, shortage cost, transportation cost etc.), advertisement cost, lead time, quantity, available resources, goals, etc., involved in the inventory system may be deterministic (crisp/precise) or some of them may be non-deterministic (i.e., imprecise like fuzzy, random, fuzzy-random, random-fuzzy etc.). Thus the environments in which inventory models are developed can be classified as follows:



Deterministic Environment : In deterministic environment all the system parameters including time horizon are crisp or deterministic in nature and all the logical expressions are of truth value 0 or 1.

Random Environment : In this environment some of the parameters like business plane, lead time, demand, resources, different inventory costs, etc., are random in nature and specified by some known or unknown probability distributions. Probability distribution of some of the random parameters may be obtained from previous experience.

Fuzzy Environment : It is an environment in which all or some inventory parameters, resources and/or goal(s) of objective(s), etc., are imprecise and vague (i.e., inexact due to human perception process). It is uncertain in non-stochastic sense and called fuzzy. The fuzzy parameters or quantities are characterized by membership functions. In this case any real number between 0 and 1 may be the truth value of logical expressions with fuzzy parameters.

Fuzzy-Random/Random-Fuzzy Environment : It is an combination of both Random and fuzzy environments. Here, some of the model parameters are fuzzy and some others are random or, some of the model parameters are of fuzzy-random /random-fuzzy type. For example, in an inventory control problem, holding cost may be imprecise and demand as random. Again in the statement 'the probability of having large demand of football world cup ticket' contains both impreciseness and randomness together. Here large is fuzzy and 'probability' represents randomness.

1.3 Historical Review on Inventory Control System

The study of inventory control begins from earlier twentieth century by the research of Ford Harris [105] of Westinghouse Corporation, USA in 1915 and he derived the classical lot size formula. Later in 1934, R. H. Wilson [260] deduce the same formula independently. After that, the formula is named as Harris-Wilson formula or Wilson's formula. From that time till now various researchers investigated different type inventory models in different environment. Also there are some full length books about inventory control system written by several authors [5, 101, 191, 206, 235, 258]. Some of the existing literature for different context are represented by the following according to our investigation.

1.3.1 Models with credit period

Trade credit or, delay in payment becomes one of the most useful tool in last two decades to attract customers and used by different supply chain members (supplier, manufacturer, retailer) at different level. The concept of trade credit was first introduced by Haley and Higgins [103] in 1973 and the concept was first applied to an EOQ model by Goyal [92]. Goyal's EOQ model was formulated with a constant demand rate under the condition of permissible delay in payments. Chand and Ward [40] analyzed Goyal's problem under the assumptions of the classical economic EOQ model and presented some results different result. Shah [230] included delay in payment in a exponentially decaying inventory model in 1993. Next, in 1995 Aggarwal and Jaggi [2] generalized the EOQ model from non-deteriorating items to deteriorating items. In 1996 Hwang and Shinn [112] worked on delay in payment in a lot-sizing model for exponentially deteriorating items. On the same year Khouja [132] deduced an optimal inventory policy under different supplier credits. On next year Jamal *et al.* [120] generalized EOQ model for deteriorating items to allow for shortages and Shinn [232] lighted on Khouja's model from a different angle determine optimal retail price and lot size. Some notable researches are also done by Chu *et al.* [59] in 1998 and by K. J. Chung [61], Jamal *et al.* [121], Sarker *et al.* [223] in 2000. Thereafter, in 2002, Teng [241] amended an inventory model by using selling price to calculate the revenue instead of unit cost, and obtained an easy analytically closed-form solution. In 2003, Chang *et al.* [44] dealt with the problem of determining the EOQ for exponentially deteriorating items under permissible delay in payment depending on the ordering quantity. Chang [45] extended this issue to include inflation and finite time horizon in 2004. Afterwards, in 2007, Huang [115, 116] proposed two levels of trade credit policy where the supplier would offer the retailer a delay period for payment and the retailer also adopts the trade credit policy to stimulate his/her customer demand. Furthermore, he also assumed that the retailer's trade credit period offered by supplier, M is not shorter than the customer's trade credit period offered by the retailer, N ($M \geq N$). Then in 2008 Liao [148] further generalized Huang's model to an EPQ model for deteriorating items. Subsequently, Teng [242] established optimal ordering policies for a retailer to deal with bad credit customers as well as good credit customers in 2009. Lately, in 2013 Ouyang *et al.* [196] considered two-level trade credit link to order quantity. In the same year, Seifert *et al.* [229] organized a review of trade credit literature and provided a detailed agenda for future research. Recently (in 2014), Chen *et al.* [54] discussed the retailer's optimal EOQ / EPQ when the up-stream trade credit is linked to order quantity or when the down-stream trade credit is only a fraction of the purchase amount. In the same time, Chung *et al.* [67] established a new EPQ inventory model for deteriorating items under two levels of trade credit, in which the supplier offers to the retailer a permissible delay period and simultaneously the retailer in turn provides a maximal trade credit period to its customers in a supply chain system comprised of three stages. In the next year, Chung *et al.* [68] adopted the rigorous methods of mathematical analysis in order to develop the complete solution procedures to locate the optimal solution removing shortcomings in the earlier investigation by Ouyang *et al.* [195]. Recently, Ouyang *et al.* [197] proposed an integrated inventory model with capacity constraint and a permissible delay payment period that is order-size dependent.

1.3.2 Models with different types of demand

In study of different and realistic inventory models, different type of demand functions [c.f. Table-1.1] are considered by several researchers from the beginning of the study of inventory control. The literature review, that we have made are represented according to different types of demand and presented as follows.

Stock dependent demand: In reality, we often see that decoratively displayed stock of items in shops named "psychic stock" attract customers and stimulate more sales of some retail item(s). So in the context of present competitive market, a business man always tries to attract the customers by advertising about "psychic stock" through different media. In 1968, Wolfe [261] conducted an empirical analysis of the retail sales of woman's dress and sports wear and found that the unit sales of each style are proportional to the displayed inventory level. Baker and Urban [9] formulated a EOQ model with stock dependent demand where the dependency is of simple form. Next, Bar-Lev *et al.* [12] produced a EOQ model with inventory level dependent demand rate and random yield. Mandal and Phaujder [174] studied on an inventory model with stock dependent demand where demand is linear function of existing stock. In 1999 Mandal and Maiti [175] considered the demand function as $D = dq^{\beta}$ in their research and Maiti and Maiti [170] proceeds under the same consideration. There are also some recent researches [39, 124, 247, 267] in the existing literature under the same assumption about demand.

Time dependent demand: There are many decision making problem in business management where demand varies with time. For example, demand of worm cloth increases as winter comes and take its pick at the middle of winter session. Also the demand decreases as the weather temperature goes down. Silver and Meal [234] published a lot size model taking time varying demand. Dave and Patel [76] developed an inventory model with (T, S_i) Policy for deteriorating items with time proportional demand. In 1995 Benkherouf [18] studied the effect of time varying demand rate on a inventory model. Hariga [104] discussed an optimal EOQ model for deteriorating items with time varyinng demand. Yang *et al.* [265] published a note on Hariga's study discussed two important aspects concerning the uniqueness of the critical points for the inventory model with complete backlogging in a finite planning horizon with some affirmative suggestions. Also Several researchers [22, 91, 151, 189, 190] have been investigated the effect of time varying demand on different inventory models. Recently, Chung [65] considered time-varying demand in a deteriorating inventory model with two-phase pricing and cost under trade credit financing for a finite horizon and optimized using particle swarm optimization.

There are also some researches in the existing literature with different type of demand. Such as Mondal *et al.* [182, 183] produces some inventory models of ameliorating items with price sensitive demand rate. Roy *et al.* [209] studied a multiple period inventory model with fuzzy cost and fuzzy demand. Donaldson [81] and Silver [233] investigated the effect of linear trend demand on inventory models. Maiti [171] solved an inventory model with credit-linked promotional demand for an imprecise planning horizon. Lian *et al.* [146] considered a perishable inventory model with Markovian renewal demands on the same year Bag *et al.* [8] studied a production inventory model with fuzzy random demand and with flexibility and reliability considerations.

1.3.3 Models on imperfect production process

Practically, production of imperfect goods is very common to every production house. Recently several researchers focused their study on such problems. In the early season of this study Rosenblatt and Lee [208] studied the effects of an imperfect production process on the optimal production run time by assuming that time to out-of-control state is exponentially distributed. Salameh and Jaber [215] and Lin [150] studied the EOQ/EPQ model for the items with imperfect quality and proposed discount sales for them. Hayek and Salameh [106] derived an optimal operating policy for the finite production model under the effect of reworking of imperfect quality items. They assumed that all defective units are repairable and allowed back-orders. Chiu [56] extended the work of Hayek and Salameh [106] and examined an EPQ model with defective items reworking the repairable units immediately. Sana [218] presented an EPL model with random imperfect production process and defective units were repaired immediately when they were produced. Barzoki *et al.* [13] investigated the effects of imperfect production on the works in process inventory and evaluated the optimum lot size for the minimum total cost. Here, some imperfect products were reworked and others were sold at a reduced price. Krishnamoorthi and Panayappan [139] have studied an EPQ model that incorporated imperfect production quality, not screening out proportion of defects and thereby passing them on to customers and resulting in sales returns. Not all of the defective units are repairable, a portion of them are scrap and discarded beforehand. Chen *et al.* [48] developed an alternative optimization solution process to determine the optimal replenishment lot size considering imperfect rework and multiple shipments. Recently Chen [53] investigated a problem with production preventive maintenance, inspection and inventory for an imperfect production process. Pal *et al.* [199] formulated an EPQ model with imperfect production process and stochastic demand. Cárdenas-Barrón *et al.* [35] presented an easy method for the results of Chen *et al.* [48] and Chiu *et al.* [57] deriving the optimal number of replenishment and shipments jointly. Krishnamoorthi and Panayappan [140] evaluated optimal lot size minimizing the total cost for an EPQ model without and with shortages allowing imperfect production system and immediate rework of the imperfect units. Rad *et al.* [204] developed a model of an integrated vendor-buyer supply chain with imperfect production and shortages. In very Recent, an extended research was done by Taleizadeh and Wee [240] for a multi-product single machine manufacturing system with manufacturing capacity limitation and immediate reworking of imperfect products allowing partial back-ordering.

1.3.4 Models on multi-item problems

There are may factors which influences business owners to make business with muli-item from year to years. There are some business where multiple items can be produced from a single raw material with some minimum cost such as petrochemical industries. Also in some business multiple items are used to push the sell. Multinational companies (for example, Big Bazar) are example of such business. Padmanabhan and Vrat [198] developed multi-item multi-objective inventory model of deteriorating items with stock-dependent demand. Ben-Daya and Raouf [17] discussed a multi-item inventory model with stochastic demand subject to the restrictions on available space and budget. Kar *et al.* [129] proposed a model that there are fresh and deteriorating items sold from the primary and the secondary shop respectively. Bhattacharya [20] developed a two-item model for deteriorating items, the demand of one item depended on others stock level. The demand of fresh items depends on selling price and stock level. However, in real life, many companies, enterprises or retailers deal with several items and stock them in their showroom/warehouse for sale. There is a restriction either on maximum capital investment in stock at any time, or the maximum warehouse space available for storage. Saha *et al.* [231] developed a multi-item inventory model with the break-ability rate and the demand rate both stock-dependent. Tsao [246] considered multi-echelon multi-item channels subject to supplier's credit period and retailer's promotional effort.

Models with deteriorating item

Deterioration of items is also a very common problem in inventory management. The earliest EOQ model studied by Ghare and Schrader [87] under the assumption of exponential decay. Thereafter, different inventory models [1, 2, 70, 120] are formulated and solved with different type of deterioration. Shah *et al.* [230] have studied a lot-size model for exponentially decaying inventory when delay in payments is permissible. Huang *et al.* [114] developed an integrated vendor-buyer model with defective items which are treated as a single batch and returned to the vendor after a 100% screening process. Again, Santosh and Pakkal [220], Chang et al. [44, 45] and Jaggi et al. [117] considered deteriorating items in their inventory models. Wee et al. [251] formulated a multi-objective joint replenishment inventory model of deteriorated items in a fuzzy environment. Mishra and Mishra [180] worked on fuzzified deterioration under cobweb phenomenon and permissible delay in payments. A delayed deteriorating item is considered in the inventory model by Musa and San [188]. After that Taleizadeh *et al.* [238] enriched their inventory model for a deteriorating item with back-ordering and temporary price discount.Subsequently, Wu *et al.* [262] deduces the optimal credit period and lot size for a model with deteriorating items and expiration dates under two-level trade-credit financing. Maihami and Kamalabadi [163] investigated an optimum pricing policy and replenishment policy for non-instantaneous deteriorating items with stochastic demand and promotional efforts.

1.3.5 Models with inflation and time value of money

Inflation and time value of money plays an important roll in the present stock market dependent business world. Considering this effect on inventory costs, Buzacott [27] is the first one, who investigated the effect of inflation on EOQ models. Among others, Beirman and Thomas [24], Datta and Pal [74], Ray and Chaudhuri [205] studied some EOQ models with linear time-varying demand taking inflation and time value of money into account. Moon and Lee [184] presented an EOQ model with inflation and time value of money. Wee and Law [250] addressed an inventory problem with finite replacement rate of deteriorating items incorporating the effect of inflation and time value of money. In the same year, Chang [45] proposed an inventory model for deteriorating items with trade credit under inflation. In recent years, Jaggi *et al.* [117], Maiti [166, 167], Sana [217, 218] and Sarkar *et al.* [225, 226] and others presented inventory models in this direction.

1.3.6 Models with Uncertainty (Randomness and Impreciseness)

From the ancient time different researches have been made by several researchers to find the logic and proof (theoretical or, practical) about the unknown matters, topics, events etc. This behavioral tendency and requirements in practical life motivate different fellows to enter into the world of uncertainty. In inventory control uncertainty take places mainly in two from- Randomness (or, stochastic) and Impreciseness.

There are several full length books [85, 186, 257] available in the literature to get brief idea about randomness. One can follow some earlier researches [26, 102, 202], for the history of development of inventory models with randomness. Among others, Kalpakam and Sapan [126], Kodama [136], Hill *et al.* [108], etc. have developed their models with probabilistic lead time or probabilistic time scheduling or uncertain quantity receiving or random supplying. Bookbinder and Cakanyildirim [25] developed a continuous review inventory model under random lead-time. Cakanyildirim *et al.* [29] extended the model to a continuous review inventory model under lot-size dependent random lead-time. Das *et al.* [71] considered a stochastic inventory problem with fuzzy storage cost. The life time of seasonal products (such as fruits, jam and jelly, crops etc.) cannot be guessed previously due to the environmental effect. Moon and Yun [185] first lighted on EOQ models in which the time horizon is Random nature. After that large number of research papers have been published incorporating this assumption [184, 211, 213].

In 1965, a newer branch of uncertainty named impreciseness is opened by Prof. L. Zadeh and in this regard he published his valuable research [274] with the definitions and properties of fuzzy set theory. After that extensive research works have been done in this area [41, 82, 153, 200]. But applications of fuzzy sets in inventory control problems are around 25-30 years. Among these works one can refer the works of Park [200], Roy and Maiti [210], Mandal and Maiti [176], Alonso-Ayuso *et al.* [3], Wee *et al.* [251], etc. Lee and Yao [144] developed an EPQ model considering fuzzy demand and fuzzy production quantity. After one year, Yao and Lee [268] presented an inventory model with and without back order in fuzzy situations considering the fuzzy numbers. Katagiri and Ishii [130] developed their inventory model under fuzzy shortage cost. Ouyang and Chang [193] developed an inventory model with fuzzy lost sales. Dey and Maiti [79] presented an EOQ model with fuzzy lead-time under inflation and time-value of money. Generally, fuzzy inventory models are developed considering some of the inventory parameters as fuzzy in nature [19, 41, 144, 170, 214, 251]. To reduce the objective function, they defuzzified the fuzzy parameters to a crisp one by either defuzzification methods or following possibility/necessity measure of fuzzy events. Finally they solved the reduced crisp model to determine decision for the Decision Maker (DM). In the existing literature, little attention has been paid on fuzzy demand and fuzzy production rate. Wee *et al.* [251] developed a multi-objective joint replenishment inventory model of deteriorating items, where demand is stock dependent and fuzzy in nature. They solved the corresponding crisp model and fuzzified the total profit and return on inventory investment for optimal decisions. Recently, Sarkar and Chakraborti [227] developed an EPQ model, where demand is considered as time dependent fuzzy number and followed FDE approach to formulate the model. They found the α-cut of total variable cost and formulate the problem as multi-objective minimization problem by considering the two components of α-cut of total variable cost as two objectives.

1.3.7 Models with fuzzy logic (or, fuzzy inference)

Due to the vagueness and impreciseness of human language, it very difficult to handle any logical expression involving human language (such as low, high, huge, much, medium etc.). But in reality, to attract customers, a business man always have to judge human requirements on basis of their speech. Fuzzy logic (or, fuzzy inference) is the tool by which one can measure the truth value of the relations linguistic variable commonly used by human being. The logical relations are commonly expressed as-

”**If** premise (antecedent) **Then** conclusion (consequent)”.

To measure the truth value of these type of linguistic relations, commonly two type of fuzzy inference techniques (Mamdani type and Sugeno type) are used. Mamdani's method is based on the idea of Bellman and Zadeh's paper [15] about making decision in fuzzy environment. It was proposed in 1975 by Ebrahim Mamdani [173] as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Sugeno or Takagi-Sugeno-Kang method was introduced in 1985 [236]. It is similar to the Mamdani method in many respects. The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator, are exactly the same. The main difference between Mamdani and Sugeno is that the mamdani output is a fuzzy set with different type of membership functions whereas the Sugeno output membership functions are either linear or constant.

The technique of fuzzy inference is used by several researchers [16, 221, 222, 248] mainly for control system. There are very few researches of inventory control with fuzzy logic in the existing literature. Axsater [6] gave some concepts about control theory in production and inventory control. Towill [244] drew some common foundations between control engineering and manufacturing and management. GrubbstroK *et al.* [94] developed some inventory control policies in terms of control theory. An application of fuzzy logic to inventory control models was made by Gen *et al.* [86]. A Fuzzy pre-compensated PID controllers is represented by Kim *et al.* [134]. Lee [143] presented some methods to improve the performance of PI-type fuzzy logic controller. Samanta and Al-Araimi [216] developed an inventory control model using fuzzy logic. Recently, Sylvanus *et al.* [237] formulate an intelligent inventory model using the concept of fuzzy logic.

1.4 Motivation and Objective of the Thesis

1.4.1 Motivation of the Thesis

Study of inventory control began during the early twentieth century and was continuously supported by several researchers according to the growing necessity in reality and this process is still going on.

Uncertainty is observed in the real life inventory / production-inventory problems and this type of uncertainty may occur in the form of random [25, 29, 45, 56], fuzzy [80, 97, 144, 209], fuzzy-random [8, 39, 52], etc. In the literature, there are several models on credit period formulated with one level trade credit only i.e. suppliers offers credit period to its retailers. But in practice, retailer too might offer a credit period to his / her customers to stimulate the market demand. Beside this, at the present volatile economic conditions through out the world, one cannot ignore the effect of inflation and time value of money specially in developing countries like India, Bangladesh etc. [24, 27, 226].

In-spite of several studies on the above mentioned inventory control systems, there are some gaps which inspired us to develop and solve some inventory / production-inventory models in different environments.

In chapter-3 an inventory model is considered under some real life assumptions which were not considered all together included in previous studies.

- **i) The market demand (D) is depends on the customer's retail price which is of the form $D(s) = as^{-b}$ where, $a > 0,\ b > 1$ and s is the customer's retail price.**
- **ii) The items are deteriorating with a date of expiration and the rate of deterioration is a increasing function of time. Thus the rate of deterioration gets its maximum value (1) at the time of expiration, i.e. when the product is spoiled / becomes obsolete.**
- **iii) Retailer shares a part of his /her credit length to the customers to attract more customer.**

Though this problem was initially investigated by Mahata [161], but there were several mistakes in the formulation and evaluation. This promoted us to make an investigation of such a real-life problem correctly (chapter-3).

In manufacturing systems, production of defective / imperfect units is almost a natural phenomenon. This imperfect units are produced after the passage of some time from the commencement of production. Now-a-days, due to competitive market, offer of trade credit is very common amongst the traders. Normally, both supplier and retailer offer the trade credits to their respective customers and the amount of credit period depends on their respective stocks. **These real life practices promoted us to investigate a problem of a two layer supply chain for imperfect items with rework and two-level trade credits** (**Chapter-4**). **Chapter-4 is formulated on basis of the following considerations.**

i) The production system undergo an out-of-control (starting point of imperfect production) state from an in-control state. A mixture of perfect and imperfect quality items are produced by the manufacturer.

ii) The rework process is one of common criterion to control the amount of imperfect units. In this study, rework process is applied and some non-repairable items are sold at a reduced price where the demand of the customer is considered as stock dependent.

iii) Two level credit sharing is considered in this supply chain, where full payment of the customers indicates the end of a replenishment cycle.

The business period of seasonal products is limited, varies from year to year. This time period can be fitted to a probability distribution i.e. may be considered as random. Demand of these items normally increases with time. For example, at the beginning of winter seasons, demand of warm garments is less and increases with time as the cold increases. Thus the production houses of seasonal products also varies with time and it is adjusted against the trended demand. **This physical phenomenon prompted us to take up the problem of a production house having two production plants for the production of a above mentioned seasonal product in Chapter-5. This study is different from the existing literature in the following aspects.**

i) In this EPQ model, two production houses under a single management are considered to control the stock out situation under a random environment.

ii) The randomness of the different parameters of the model are removed using chance constraint method and taking expected value.

iii) The production rates are unknown function of time and the demand rates are known functions of time.

iv) None has investigated a two plant production-inventory models with dynamic production rate, trended demand, random shortage (occurs only at one production house) under a random planning horizon.

A production problem with shortages for a seasonal product is also considered in **Chapter-6**. In addition to random business period, sometimes both production and demand of a seasonal product depend on the current stock level of the product in the go-down. Normally, at initial stages, the set-up cost is sometimes more and decreases at later stages as the production cycle increases. Learning effects in different cycles may be introduced in set-up cost. Again, with the passage of times, laborers are trained and the unit production cost also decreases with the increase of cycles. **These natural phenomena in production systems prompted us to take a production problem with shortages with above mentioned features. Chapter-6 includes the following assumptions which are not considered in the previous studies.**

- **i) The present production-inventory model involves stock dependent production rate and stock dependent demand rate over a random planning horizon.**
- **ii) The present model is developed with and without shortages and the unit production rate is also considered as crisp, random, fuzzy and fuzzy-random in nature.**
- **iii) Learning effect with the increase of cycles is introduced in set-up cost and unit production cost.**

In supply chain business of seasonal products, supplier makes stocks of a product for wholesaler who purchases the item in a lot. A supplier decides the amount of stock depending upon the demand and selling price fixed by himself. All these relations are normally defined by fuzzy words such as low, medium, high, very high and very low. Similarly, from supplier sides, his / her purchasing price varies with his / her purchased amount. This relation is also expressed by verbal words- low, high and medium which are imprecise in nature. **Such a normal supplier-wholesaler business phenomenon prompted us to take a supply chain model with two level and single level fuzzy logic, i.e. fuzzy inference in Chapter-7. As the computation of these problems becomes cumbersome, here an appropriate Genetic Algorithm has been developed for solution. As usual, random planning horizon with normal distribution for seasonal products is incorporated in this model (Chapter-7).**

It is usually seen in reality that a single business man, say wholesaler, he/she makes the stocks of different items at a go-down and sells the items at different showrooms. These items may be seasonal products which deteriorates also. It is obvious that demand of an item depends on its price. It is very difficult to express this relation in a functional form. Rather, very often, it is expressed in linguistic terms using 'low', 'high', 'medium'. Again, selling price is subjected to discounts depending upon the purchased amount in the forms of AUD, IQD and combination of AUD and IQD (IQD in AUD). For multi-items, some constraints such as space and budget constraints are imposed and these may be imprecise (fuzzy) in nature. **These physical considerations forced us to investigate a model of deteriorating multi-item with linguistically price dependent demand allowing different price discounts under imprecise resource constraints. Thus Chapter-8 contains an inventory model which is unique in the following considerations.**

- **i) There are very few supply-chain models for deteriorating items with fuzzy inference expressed verbally using 'words'.**

ii) **Till now, none has used three types of price discount (AUD, IQD, AUD in IQD) in a supply-chain model connecting through fuzzy inferences and sharing the part of the commission with customers.**

iii) **No supply-chain is available with MRP and commission on this following fuzzy rules.**

iv) **Use of random planning horizon is very limited and none has used it in connection with fuzzy inferences.**

v) **Appropriate GA is developed connecting random planning horizon, fuzzy logic and price discount.**

vi) **For the first time, surprise function, possibility for resource constraints are used in a supply-chain model.**

Seasonal products do have several characteristics such as random business period etc. as mentioned earlier. For an item, relations between price and demand, quantity purchased and trade credits are well known. i.e. if price is more, demand is less and vice-verse, if purchased quantity is high, trade credit period is more, etc. These relations are normally expressed linguistically in business systems. These are mathematically expressed by fuzzy inferences. The difficulty lies in expressing these parameters by fuzzy membership functions. In reality, all available data or expert's opinions are deterministic. From past experiences, these collected crisp data are used to form the fuzzy membership functions. **All the above facts influenced us to formulate and solve an EPQ model for deteriorating items under random planning horizon with some linguistic relations between demand, selling price, trade credit and ordered quantity forming fuzzy membership functions from available raw data and using the appropriate developed Genetic Algorithm for solution (Chapter-9). Thus the basic contributions of this chapter are as follows:**

i) **Normally, in the inventory models with trade credit, amount of trade credit is given deterministically (a numerical value) depending on the ordered quantity. A relation is presented by a mathematical expression in crisp way. Similarly, a deterministic mathematical expression may be available connecting price and demand. In practice, often these relations are expressed by "words" linguistically. Here, linguistic relations between (price, demand) and (ordered quantity, credit period) are considered.**

ii) **A new method of payment of dues of retailer to supplier is presented and a lemma is presented which assures the validity of the new method. A comparative study has been provided with the conventional method.**

iii) **The business period of the seasonal products are finite and varies every year. Thus the time period of these products are assumed as random having a probability distribution.**

iv) **The construction of membership function (MF) from the market / business data is very important for the model with fuzzy inferences. Here, a methodology is presented for the construction of MF from the marketing experts' opinions.**

v) **GA is very appropriate for the solution of inventory models with fuzzy logic. Here an appropriate GA has been developed for this purpose.**

1.4.2 Objective of the Thesis

The main objectives of the thesis are

i) to develop some inventory / production-inventory model (s) in different types of environments (deterministic, random, fuzzy, fuzzy-random, etc.) from realistic point of view.

ii) to develop / modify some solution techniques (GA, MOGA, GRG etc.) and to apply these methods for the solution of above mentioned inventory / production-inventory models.

iii) to convert the uncertain models into the corresponding deterministic single or multi-objective problems by using different appropriate techniques (Fuzzy Inference, Chance constraint technique, Possibility / Necessity measures etc.).

iv) to formulate and solve some realistic supply chain models having some linguistic relations (fuzzy rules) among the parameters / variables of supply chain partners.

v) to show different effects or relations of the models' parameters and decision variables through some numerical examples and to perform their sensitivity analyses.

1.5 Organization of the thesis

In the proposed thesis, some real life uncertain inventory problems are considered and solved. The proposed thesis is divided into following six parts and seven chapters.

Part-I : Introduction and Solution Methodologies

Chapter-1

- **Introduction**

This Chapter contains an introduction giving an overview of the development on inventory control system in crisp, fuzzy, random and fuzzy-random environments.

Chapter-2

- **Solution Methodologies**

  In this Chapter, preliminary ideas on crisp set, fuzzy set, etc. are given. The following techniques/methods have been developed /modified and used to solve the proposed inventory models in uncertain environments.

1. Generalized Reduced Gradient Technique (GRG).

2. Genetic Algorithm with Varying Population (GAVP) .

3. Multi-objective Genetic Algorithm (MOGA).

Part-II: Inventory Models in Crisp Environment

Chapter-3

- **Note On: Partial trade credit policy of retailer in economic order quantity models for deteriorating items with expiration dates and price sensitive demand.**

  In this supply chain, a credit-worthy retailer frequently receives a permissible delay on the entire purchase amount without collateral deposits from his/her supplier (i.e., an up-stream full trade credit). By contrast, a retailer usually requests his/her credit-risk customers to pay a fraction of the purchase amount at the time of placing an order, and then grants a permissible delay on the remaining balance (i.e., a down-stream partial trade credit). Also, in selecting an item for use, the selling price of that item is one of the decisive factors to the customers. It is well known that the higher selling price of item decreases the demand rate of that item where the lesser price has the reverse effect. Hence, the demand rate of an item is dependent on the selling price of that item. In addition, many products such as sessional fruits and vegetables, pharmaceutical, volatile liquids etc. are not only deteriorate continuously but also have their expiration dates. However, only a few researchers take the expiration date of a deteriorating item into consideration. For the objective function sufficient conditions for the existence and uniqueness of the optimal solution are provided with an object to maximize the profit of the retailer. The GA process has been applied to determine the optimal pricing and inventory policies for the retailer. Finally, numerical examples are presented to illustrate the proposed model and the effect of key parameters on optimal solution is examined.

Part-III: Inventory Models in Random, Fuzzy and Fuzzy-Random Environment

Chapter-4

- **Two layer supply chain model of deteriorating items with rework and two level credit period.**

In this chapter, a single period SCM model is considered for a manufacturer with multiple retailer. A mixture of perfect and imperfect quality items are produced by the manufacturer and supplies them to the retailer's. Every Retailer achieves an up-stream credit and shares it partially to his / her customers. The defective rate is random in nature which follows an uniform distribution and certain percentage of defective items are reworked as a perfect quality items. Set-up cost of the manufacturer is linearly dependent on the production rate. Also the open end customer demand at each of the retailers depends on the displayed stock of the retailer. The whole model is formulated as profit maximization problem to maximize the SCM profit. The model is solved numerically using GRG method.

Chapter-5

- **Two plant optimal production problem in random time horizon.**

This chapter presents a time-dependent production policy for a single item which are produced from two plants situated at different location under a single management. The rate of demand and the rate of production at these plants are different. Demands of the item are primarily met locally from the respective plants but if a stock-out situation occurs in a plant, immediately some stock from the other plant (if available) is rushed to the stock-out plant. So the shortage is allowed only at one plant and inventory stock situation occurs at another. The demand at the different plants are known function of time whereas the production rate at the plants are unknown functions of time and taken as control variable. The planning horizon, the production cost, holding cost, shortage cost, and transportation costs all are random in nature. The model is formulated as a cost minimization problem in the form of an integral. The optimum results are obtained using Kuhn-Tucker conditions and Generalized Reduced Gradient (GRG) technique. The model is illustrated by numerically and optimal results are presented in both tabular and graphical form.

Chapter-6

- **Optimum Production Policy for a Production Inventory Model in Random Time Horizon.**

A profit maximization production inventory model with linearly stock dependent demand is developed in random time horizon under inflation and time value of money. The time period is random and follows exponential distribution with known mean. The unit production rate is partly stock dependent and decreases as the stock increases. At the end of the time period excess amount of stock (if any) is sold at a reduced price. This model is developed with and without shortages under the assumption that the shortage is fully backlogged. The unit production cost is considered as crisp, random and fuzzy in nature and different models are formulated for different types of unit production costs. Optimal result of the different type of models are obtained using a gradient based non-linear optimization technique- Generalized Reduced Gradient(GRG) method. All the models are illustrated with some numeric data and some sensitivity analysis are also presented.

Part-IV: Inventory Models with Fuzzy Logic

Chapter-7

- **A Supply Chain Model with Fuzzy Logic under Random Planning Horizon via Genetic Algorithm.**

In this model fuzzy logic is introduced in a single management Supply Chain Model (SCM) with m suppliers and one wholesaler having n showrooms for sale, to make the dealings among the chain members more meaningful and profitable in a random business period which follows normal distribution with known mean and variance. The suppliers with limited capacity offer some fuzzy cost discount depending on the ordered amount following one parameter fuzzy inference. The target of the wholesaler is to purchase the required amount of the item from the suppliers achieving maximum possible cost discounts. The quantity supplied by the wholesaler to showrooms for sale depends not only on the respective demand at that place but also on the selling price (i.e. mark-up) of the units following some two parameters fuzzy rules. The chance constraint technique is used to express the randomness of the planning horizon. The above mentioned SCM is formulated as a profit maximization problem with respect to the wholesaler using fuzzy logic(FL) and optimized using a real coded genetic algorithm (GA). Finally, to illustrate the model, a practical example is considered. Raw data from a rice selling merchant are collected and represented as imprecise numbers. The linguistic relations are derived and linguistic terms are quantified. Then the model is formulated for maximum profit with respect to the said merchant and solved using FL and GA. The behavior of profit and required quantity are plotted against selling price (mark-up).

Chapter-8

- **A Deteriorating Multi-item Inventory Model with Price Discount and Variable Demands via Fuzzy Logic under Resource Constraints.**

An inventory model of deteriorating seasonal products with Maximum Retail Price (MRP) for a wholesaler having showrooms at different places under a single management system is considered under random business periods with fuzzy resource constraints. The wholesaler replenishes the products instantaneously and earns commissions on MRP which vary with the ordered quantities following All Unit Discount (AUD), Incremental Quantity Discount (IQD) or IQD in AUD policy. Demand at showrooms are imprecise and related to selling prices by verbal words following fuzzy logic. The wholesaler shares a part of commission with customers. The business periods follows normal distribution and converted to deterministic ones through chance constraint technique. The fuzzy space and budget constraints and fuzzy relations are defuzzified using possibility measures, surprise function and Mumdani fuzzy inference technique. The model is formulated as profit maximization for the wholesaler and solved using a real coded Genetic Algorithm (GA) and illustrated through some numerical examples and some sensitivity analysis. A real-life problem of a developing country is presented, solved using the above mentioned procedures and an appropriate inventory policy is suggested.

Chapter-9

- **An EPQ Model for Deteriorating items under Random Planning Horizon with some Linguistic Relations between Demand, Selling Price and Trade Credit, Ordered Quantity.**

An environment friendly Economic Production Quantity (EPQ) model of a single item is presented in this chapter in which the business in each cycle starts with shortage and ends with the end of stock. The whole problem is formulated to maximize profit of the manufacturer with random business period and the randomness is removed by chance constrained method. This model involves selling price dependent demand and purchased raw material dependent credit period which are described by two sets of linguistic relations under fuzzy logic. In addition, after the end of credit period, due raw-material cost (DRC) is paid to the source as soon as it can be possible and a lemma is presented in support of this approach. A comparison is drawn between this approach and the old payment policy (i.e. DRC is paid to the end of the cycle). The model is optimized by a real coded genetic algorithm (GA) developed for this purpose with tournament selection, arithmetic crossover and polynomial mutation. The model is illustrated with different sets of numerical examples for different scenarios. A practical application has also been demonstrated with real world data. Some sensitivity analysis are presented graphically.

Part-V : Summary and Future Extension

Chapter-10

- **Summary and Future Extension**

Part-VI: Bibliography and Indices

- **Bibliography**
- **Indices**

Chapter 2

Solution Methodology

2.1 Mathematical prerequisites

2.1.1 Crisp Set Theory

Crisp Set: By crisp one means dichotomous, that is, yes or no type rather than more-or-less type. In conventional dual logic, for instance, a statement can be true or false- and nothing in between. In set theory, an element can either belongs to a set or not; and in optimization, a solution is either feasible or not. A classical set, X, is defined by crisp boundaries, i.e., there is no uncertainty in the prescription of the elements of the set. Normally it is defined as a well defined collection of elements or objects, $x \in X$, where X may be countable or uncountable.

Convex Set: A subset $S \subset \Re^n$ is said to be convex, if for any two points x_1, x_2 in S, the line segment joining the points x_1 and x_2 is also contained in S. In other words, a subset $S \subset \Re^n$ is convex, if and only if

$$x_1, x_2 \in S \Rightarrow \lambda x_1 + (1-\lambda)x_2 \in S; \quad 0 \leq \lambda \leq 1.$$

Convex Combination: Given a set of vectors $\{x_1, x_2, \cdots, x_n\}$, a linear combination $x = \lambda_1 x_1 + \lambda_2 x_2 + \cdots + \lambda_n x_n$ is called a convex combination of the given vectors, if $\lambda_1, \lambda_2, \cdots, \lambda_n \geq 0$ and $\sum\limits_{i=1}^{n} \lambda_i = 1$

Convex function: The function $f : S \to \Re$ is said to be convex if for any $x_1, x_2 \in S$ and $0 \leq \lambda \leq 1$, implies that

$$f\{(1-\lambda)x_1 + \lambda x_2\} \leq (1-\lambda)f(x_1) + \lambda f(x_2).$$

The definition of convex functions can be modified for concave functions by replacing $' \leq'$ by $' \geq'$.

2.1.2 Random Set Theory

Probability Space: An order tuple (S, Ω, P) is said to be Probability Space if

(I) S is a non-empty set of outcomes of a random experiment E,

(II) Ω is a set of all events (i.e., subsets of S), which is a σ-field, i.e., satisfies the following properties: (i) $\emptyset \in \Omega$ and (ii) $A \in \Omega \Rightarrow A^c \in \Omega$, where A^c is the complement of A in Ω,

(III) $A_1, A_2, \cdots \in \Omega \Rightarrow A = \bigcup\limits_{i=1}^{\infty} A_i \in \Omega$.

(IV) P is a probability function for the events, i.e., $P : \Omega \to [0, 1]$ and $P(\{x_i\}) = p_i$, $0 \leq p_i \leq 1, \forall x_i \in S(i = 1, 2, 3, \cdots)$, $\sum\limits_{i=1}^{\infty} p_i = 1$.

Random Variable: Let S be a sample space of of some given random experiment. It has been observed that the outcomes (i.e. sample points of S) are not always numbers. We may however assign a real numbers to each sample point according to some definite rule. Such an assignment gives us a "function defined on the sample space S". This function is called a random variable (or stochastic variable).
A random variable which assumes a finite number or countably infinite number of values is called a discrete random variable. If the random variable assumes an uncountably infinite number of values, it is called a continuous random variable.

Discrete Probability Distribution: Let X be a discrete random variable which can assume the values $x_1, x_2, x_3, \cdots$ (arranged in an increasing order of magnitude) with probabilities $p_1, p_2, p_3, \cdots$ respectively. The specification of the set of values x_i together with their probabilities p_i $(i = 1, 2, 3, \cdots)$ defines the discrete probability distribution of X, provided,

$$(i)\ P\{X = x_i\} = p_i \geq 0 \text{ and } (ii) \sum_i p_i = 1.$$

Continuous Probability Distribution: The distribution of random variable X is said to be continuous if the distribution function $F(x)$ is continuous and it's derivative $F^{'}(x)$ is piecewise continuous everywhere. In this case, for, $b > a$,

$$P(a < \hat{X} \leq b) = F(b) - F(a) = \int_b^a F^{'}(x)\, dx$$

$$\text{or, } P(a < \hat{X} \leq b) = \int_b^a f(x)\, dx$$

where $f(x) = F^{'}(x)$. The function $f(x)$ is called probability density function of the random variable x. Provided,

$$(i) f(x) \geq 0 \text{ and } (ii) \int_{-\infty}^{\infty} f(x)dx = 1$$

Table 2.1: Probability Distributions

Discrete distribution	Continuous distribution
Discrete uniform distribution	Uniform (or rectangular) distribution
Binomial distribution	Normal (or Gaussian) distribution
Geometric distribution	Gamma distribution
Multi modular distribution	Exponential distribution
Poisson distribution	Laplace distribution
Hypergeometric distribution	Weibull distribution
Negative binomial	Rayleigh distribution
or Pascal's distribution	Beta distribution

In any physical problem, one chooses a particular type of probability distribution depending on (i) the nature of the problem, (ii) the underlying assumptions associated with the distribution of the parameters, (iii) the shape of the graph between the probability density function $f(x)$ (or distribution function $F(x)$) and x obtained after plotting the available data and (iv) the convenience and simplicity afforded by the distribution. Some continuous probability distributions are presented here. In this thesis, Uniform Distribution, Exponential distribution and Normal distribution have been used for stochastic models.

Uniform Distribution or rectangular Distribution:
A continuous random variable X, is said to have a uniform distribution, if its probability density function $f(x)$ (cf. Figure-2.1) is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$$

where a and b are two parameters of the distribution.

Exponential Distribution:
A continuous random variable X, is said to have an exponential distribution, if its probability density function $f(x)$ (cf. Figure-2.2) is of the form:

$$f(x) = \lambda \exp\{-\lambda(x-\theta)\}, \; \theta \leq x\,, \; \lambda > 0$$

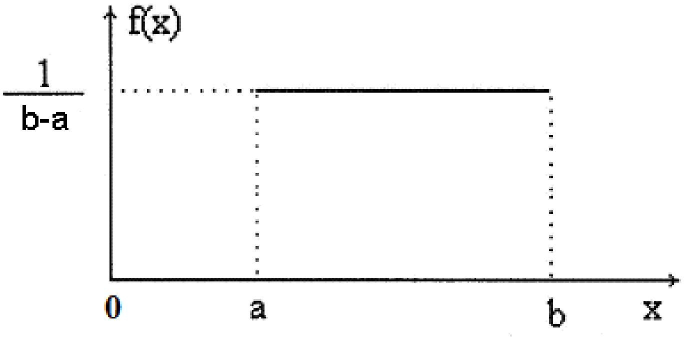


Figure 2.1: p.d.f. of uniform distribution

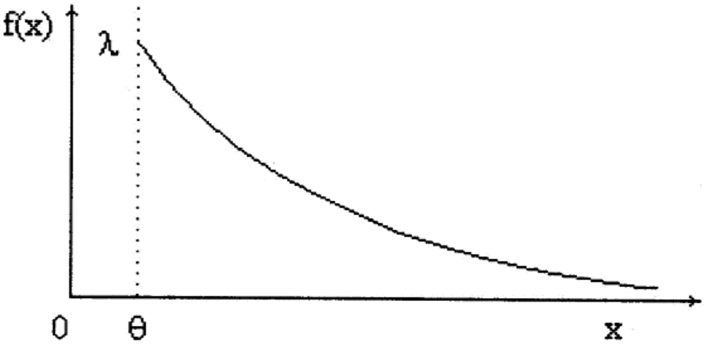


Figure 2.2: p.d.f. of exponential distribution

where λ and θ are two parameters of the distribution. When $\theta = 0$, the density function $f(x)$ reduces to the following.

$f(x) = \lambda \exp\{-\lambda x\}, \ 0 \leq x < \infty$

Normal Distribution or Gaussian Distribution:
The best known and most widely used probability distribution is the Normal distribution. The density function of this is a bell-shaped symmetrical curve about mean and its probability density function with parameters μ and $\sigma(> 0)$ is defined as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\} \qquad -\infty < x < \infty$$

where,$=\mu$ and $=\sigma^2$ be the mean and variance of the distribution respectively.
The notation N(μ,σ) is usually used to represent this distribution with mean μ and standard deviation σ and its density function is a bell-shaped symmetrical curve about μ (cf. Figure-2.3).

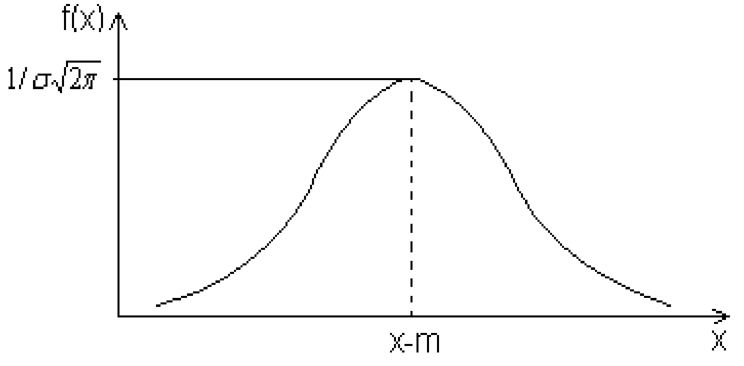


Figure 2.3: p.d.f. of normal distribution

Beta Distribution:
A continuous random variable X, is said to have a beta distribution [c.f. Figure-2.4], if its probability density function $f(x)$ is given by

$$f(x) = \frac{x^{l-1}(1-x)^{m-1}}{\beta(l,m)}, 0 \leq x \leq 1\,; l, m > 0$$

where,

$$\beta(l,m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)} = \int_0^1 x^{l-1}(1-x)^{m-1}dx$$

l and m being two positive parameters of the distribution.

Weibull Distribution:
A random variable X has a Weibull distribution if there exist three parameters c (> 0), d (> 0) and μ such that

$$Y = \left(\frac{X-\mu}{d}\right)^c$$

which has the exponential distribution with the probability density function

$$f_Y(y) = e^y \qquad (0 < y)$$

Then the probability density function of X denoted by $f(x)$ (cf. Figure-2.5) is given by

$$f_(x) = cd^{-1}\left(\frac{x-\mu}{d}\right)^{c-1} exp\left[\left\{\frac{-(x-\mu)}{d}\right\}^c\right]$$

where $\mu < x$. Now, the standard Weibull distribution is obtained by putting $d = 1$ and $\mu = 0$. When $\mu = 0$, then

$$f_(x) = cd^{-c}x^{c-1}exp\left\{-\left(\frac{x}{d}\right)^c\right\}$$

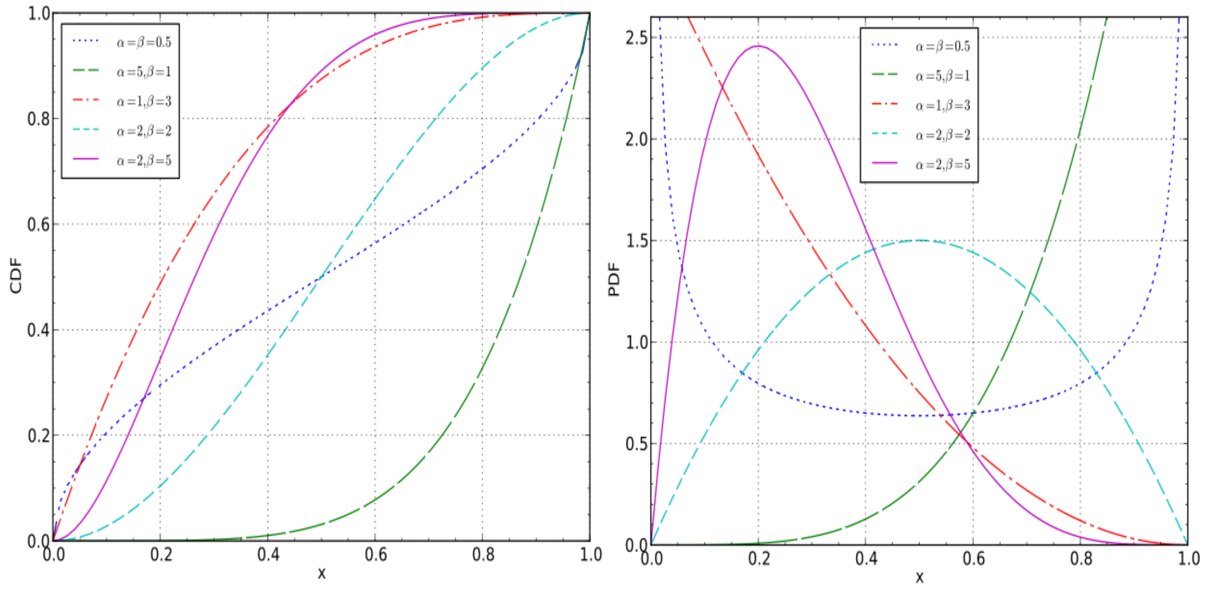


Figure 2.4: c.d.f. and p.d.f. of Beta distribution of first kind

Taking $\alpha = d^{-c}$ and $\beta = c$, we have

$$f_(x) = \alpha\beta x^{\beta-1} exp\{ - \alpha x^{\beta}\}$$

which is the form of two parameters Weibull distribution.

2.1.3 Chance Constraint Method

Chance-constrained programming is one of the techniques of stochastic programming which deals with a situation where some or all parameters of the problem are described by random variables. Chance-constrained technique is used in the models to remove the randomness of random parameters which follows normal distribution. The chance constraint is taken in the form-

$$Prob(|T - \hat{H}| \leq \beta) \geq pr \tag{2.1}$$

with $\hat{H}$ as the random variable which follows normal distribution with mean $m_{\hat{H}}$ and variance $\sigma_{\hat{H}}$. T is the deterministic form of $\hat{H}$. Then the equation (2.1) can be rewrite as

$$\begin{aligned} Prob(T - \beta \leq \hat{H}) &\geq pr \\ and\ Prob(\hat{H} - T \leq \beta) &\geq pr\,, \end{aligned}$$

From the first inequality

$$Prob(\frac{T - \beta - m_{\hat{H}}}{\sigma_{\hat{H}}} \leq \frac{\hat{H} - m_{\hat{H}}}{\sigma_{\hat{H}}}) \quad \geq \quad pr$$

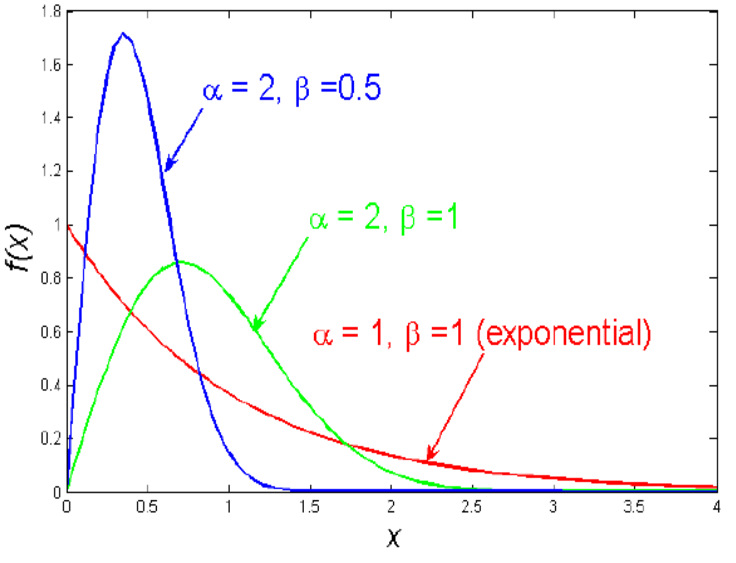


Figure 2.5: p.d.f of Weibull distribution

Now $\frac{\hat{H}-m_{\hat{H}}}{\sigma_{\hat{H}}}$ represents the standard normal variate with mean 0 and variance 1. i.e.

$$Prob(T-\beta \leq \hat{H}) = 1 - F(\frac{T-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}})$$

Where $F(x)$ represents the continuous distribution function of standard normal distribution. Let ϵ be the standard normal value such that

$$F(\epsilon) = pr = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\epsilon} e^{-\frac{t^2}{2}} dt.$$

Then the statement $Prob(T-\beta \leq \hat{H}) \geq pr$ is true if and only if

$$\begin{aligned} \frac{T-\beta-m_{\hat{H}}}{\sigma_{\hat{H}}} &\leq -\epsilon \\ T &\leq m_{\hat{H}} + \beta - \epsilon.\sigma_{\hat{H}} \end{aligned}$$

Similarly from the 2nd inequality it can be reduce that

$$m_{\hat{H}} - \beta - \epsilon.\sigma_{\hat{H}} \leq T$$

Thus the chance constraint (2.1) is reduces to

$$m_{\hat{H}} - \beta - \epsilon.\sigma_{\hat{H}} \leq T \leq m_{\hat{H}} + \beta - \epsilon.\sigma_{\hat{H}}$$

where, ϵ is a real number satisfying the equation (2.2).

2.1.4 Fuzzy Set Theory

The concept of fuzzy set was initialized by Zadeh [274] in 1965. Fuzzy set theory has been well developed and applied in a wide variety of real problems including inventory control problems. It was developed to define and solve the complex system with sources of uncertainty or impreciseness which are non-stochastic in nature. The term "*FUZZY*" was proposed by Prof. L, A. Zadeh in 1962 [273]. A short delineation of the fuzzy set theory is given below.

2.1.2.1 Fuzzy Set

Fuzzy sets deal with objects that are 'matter of degree', with all possible grades of truth between yes or no. So a fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Let X be a collection of objects and x be an element of X, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))/x \in X\}$, where $\mu_{\widetilde{A}}(x)$ is called the membership function or grade of membership of x in $\tilde{A}$ which maps X to the membership space M which is considered as the closed interval [0,u], where $0 < u \leq 1$.

Note: When M consists of only two points 0 and 1, $\widetilde{A}$ becomes a non-fuzzy set (or Crisp set) and $\mu_{\widetilde{A}}(x)$ reduces to the characteristic function of the non-fuzzy set (or crisp set).

- **Equality:** Two fuzzy sets $\widetilde{A}$ and $\widetilde{B}$ in X are said to be equal if and only if $\mu_{\widetilde{A}}(x) = \mu_{\widetilde{B}}(x), \forall x \in X$.

- **Containment:** A fuzzy set $\tilde{A}$ in X is contained in or is a subset of another fuzzy set $\widetilde{B}$ in X, written as $\tilde{A} \subset \tilde{B}$ if and only if $\mu_{\widetilde{A}}(x) \leq \mu_{\widetilde{B}}(x), \forall x \in X$.

- **Support:** The support of a fuzzy set $\widetilde{A}$ is a crisp set, denoted by $S(\widetilde{A})$, and defined as $S(\widetilde{A}) = \{x \,|\, \mu_{\widetilde{A}}(x) > 0\}$.

- **Height:** The height of a fuzzy set $\widetilde{A}$ is the maximum membership grade value of $\widetilde{A}$ and denoted by $h(\widetilde{A}) = \sup_{x \in X} \mu_{\widetilde{A}}(x)$, where X is universal set.

- **Normal fuzzy set:** A fuzzy set $\widetilde{A}$ is called normal if its height is 1, i.e., if $h(\widetilde{A}) = \sup_{x \in X} \mu_{\widetilde{A}}(x) = 1$

- **Core:** The core of a fuzzy set $\widetilde{A}$ is a set of all points with unit membership degree in $\widetilde{A}$ denoted by $Core(\widetilde{A})$, and defined as $Core(\widetilde{A}) = \{x \in X \,|\, \mu_{\widetilde{A}}(x) = 1\}$.

- **Convexity:** A fuzzy set $\widetilde{A}$ in X is said to be convex if and only if for any $x_1, x_2 \in X$, the membership function of $\widetilde{A}$ satisfies the inequality $\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$ for $0 \leq \lambda \leq 1$.

2.1.2.2 Fuzzy Number

A fuzzy number is a convex, normal fuzzy set defined on the real line. Here some definitions of fuzzy numbers are presented below.

A general shape of a fuzzy number following the above definition may be shown pictorially as in Fig. 2.6. Here, a_1, a_2, a_3 and a_4 are real numbers. A fuzzy number $\widetilde{A}$ in X is said to be discrete or continuous according as its membership function $\mu_{\widetilde{A}}(x)$ is discrete or continuous. Linear Fuzzy Number (LFN), Triangular Fuzzy Number (TFN), Parabolic Fuzzy Number (PFN) and Trapezoidal Fuzzy Number (TrFN), are special classes of continuous fuzzy numbers.

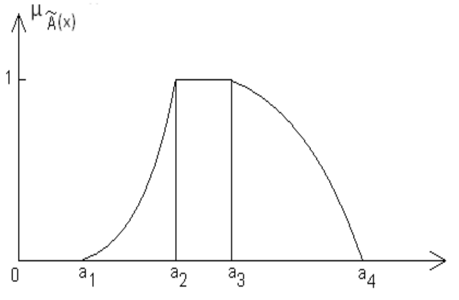


Figure 2.6: Membership function of general fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$

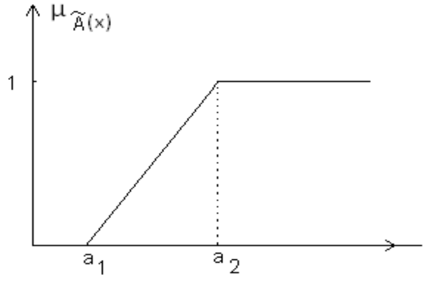


Figure 2.7: Membership function of LFN

Definition 2.1. Linear Fuzzy Number (LFN): *A LFN $\widetilde{A}$ is specified by two parameters (a_1, a_2) and is defined by its continuous membership function $\mu_{\widetilde{A}}(x) : X \to [0, 1]$ as follows (cf. Fig. 2.7):*

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ \dfrac{a_2 - x}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x \geq a_2 \end{cases}$$

Definition 2.2. Triangular Fuzzy Number (TFN): *A TFN $\widetilde{A}$ is specified by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\widetilde{A}}(x) : X \to [0, 1]$ as follows (cf. Fig. 2.8):*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

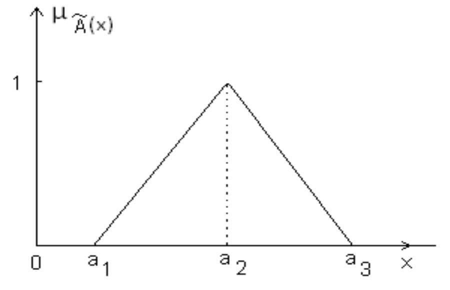


Figure 2.8: Membership function of TFN

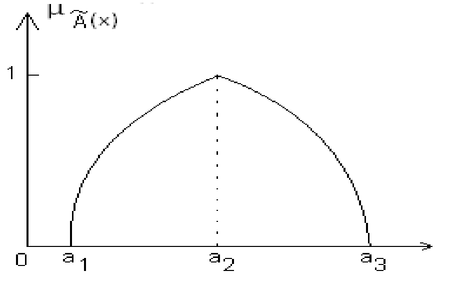


Figure 2.9: Membership function of PFN

Definition 2.3. Parabolic Fuzzy Number (PFN): *A PFN $\widetilde{A}$ is also specified by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\widetilde{A}}(x) : X \rightarrow [0, 1]$ as follows (cf. Fig. 2.9):*

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - (\dfrac{a_2 - x}{a_2 - a_1})^2 & \text{for } a_1 \leq x \leq a_2 \\ 1 - (\dfrac{x - a_2}{a_3 - a_2})^2 & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.4. Trapezoidal Fuzzy Number (TrFN): *A TrFN $\widetilde{A}$ is specified by four parameters (a_1, a_2, a_3, a_4) and is defined by its continuous membership function $\mu_{\widetilde{A}}(x) : X \rightarrow [0, 1]$ as follows (cf. Fig. 2.10):*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ 1 & \text{for } a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{for } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

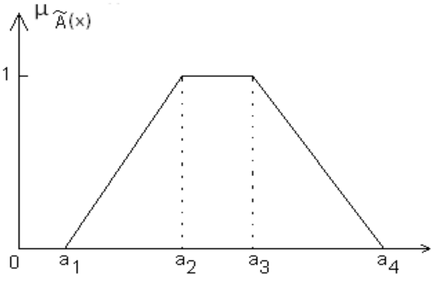


Figure 2.10: Membership function of TrFN

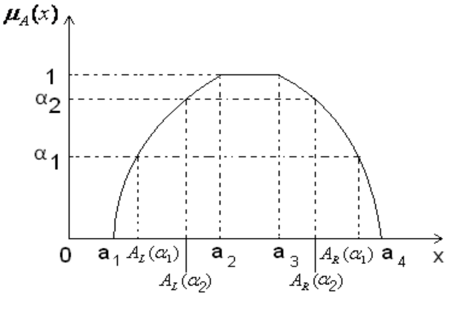


Figure 2.11: α-cut of general fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$

Definition 2.5. **α - Cut of a fuzzy number:** *α- cut of a fuzzy number $\tilde{A}$ in X is denoted by $A[\alpha]$ and is defined as the following crisp set (cf. Fig. 2.11):*

$$A[\alpha] = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X\} \text{ where } \alpha \in [0, 1]$$

$A[\alpha]$ is a non-empty bounded closed interval contained in X and it can be denoted by $A[\alpha] = [A_L(\alpha), A_R(\alpha)]$. $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval respectively. Fig. 2.11 represents a fuzzy number $\tilde{A}$ with α-cuts $A[\alpha_1] = [A_L(\alpha_1), A_R(\alpha_1)], A[\alpha_2] = [A_L(\alpha_2), A_R(\alpha_2)]$. It shows that if $\alpha_2 \geq \alpha_1$ then $A_L(\alpha_2) \geq A_L(\alpha_1)$ and $A_R(\alpha_1) \geq A_R(\alpha_2)$. Here, $A'[\alpha] = \{x \in X | \mu_{\tilde{A}}(x) > \alpha\}$ is called 'strong α-level set'

Definition 2.6. **α-cut of a function:** *Let $\widetilde{F}(X)$ be the space of all compact and convex fuzzy sets on X. If $f : \Re^n \rightarrow \Re$ is a continuous function, then $\tilde{f} : \widetilde{F}(\Re^n) \rightarrow \widetilde{F}(\Re)$ is well defined function and its α-cut $\tilde{f}(u)[\alpha]$ is given by (cf. Roman-Flores et al. [207])*

$$\tilde{f}(u)[\alpha] = f(u[\alpha]), \ \forall\alpha \in [0, 1], \forall\tilde{u} \in \widetilde{F}(\Re^n) \tag{2.2}$$

where $f(A) = \{f(a)/a \in A\}$.

Definition 2.7. **Fuzzy Extension Principle [275]:** *If $\tilde{a}$, $\tilde{b} \subseteq \Re$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$, where $f : \Re \times \Re \rightarrow \Re$ is a binary operation, membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is defined as (cf. page 53 of Zimmermann [278], second revised version)*

$$\text{For each } z \in \Re, \ \ \mu_{\tilde{c}}(z) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), \ x, y \in \Re \text{ and } z = f(x, y)\} \tag{2.3}$$

2.1.2.3 Defuzzification Method

In the literature of fuzzy mathematics, several approaches are available to convert a fuzzy number into its equivalent crisp number [50, 51, 95, 269]. Each method has some merits and demerits over the others. In this thesis, used defuzzification methods are discussed below.

A. Graded Mean Integration Representation (GMIR) of Fuzzy Number: Chen and Hsieh [50, 51] introduced GMIR method based on the integral value of graded mean α-level of a generalized fuzzy number. The graded mean α-level value of generalized fuzzy number $\tilde{A} = (A_1, A_2, A_3, A_4)$ is $\alpha[\frac{A_L(\alpha)+A_R(\alpha)}{2}]$, $\alpha \in [0, 1]$. Then the GMIR of a general fuzzy number $\tilde{A}$ is

$$P(\tilde{A}) = \int_0^1 \alpha[\frac{A_L(\alpha) + A_R(\alpha)}{2}]d\alpha / \int_0^1 \alpha d\alpha. = \frac{1}{6}[A_1 + 2A_2 + 2A_3 + A_4] \tag{2.4}$$

Here equal weightage has been given to the left and right parts of the membership function. The representation given by (2.4) can be generalized/modified by replacing $\frac{[A_L(\alpha)+A_R(\alpha)]}{2}$,

$\alpha \in [0,1]$ with $[kA_L(\alpha) + (1-k)A_R(\alpha)]$, $\alpha \in [0,1]$, where the value of k depends on the preference of the decision maker. Therefore, the modified form of Eq. (2.4) is

$$\begin{aligned} P_k(\tilde{A}) &= \int_0^1 \alpha[k\, A_L(\alpha) + (1-k)A_R(\alpha)]d\alpha \Big/ \int_0^1 \alpha d\alpha \\ &= \frac{1}{3}[k(A_1 + 2A_2) + (1-k)(2A_3 + A_4)]. \qquad (2.5) \end{aligned}$$

The method is also known as k-preference integration representation.

B. Possibility/Necessity Measure of Fuzzy Event

In order to measure a fuzzy event, Zadeh [275] proposed the concept of possibility measure in the year 1978. Considering the degree of membership $\mu_{\widetilde{F}}(u)$ of an element $\tilde{u}$ in a fuzzy set $\widetilde{F}$, defined on a referential U, one can find in the literature, three interpretations of this degree [83].

Degree of similarity: According to degree of similarity, $\mu_{\widetilde{F}}(u)$ is the degree of proximity of $\tilde{u}$ to prototype elements of $\widetilde{F}$. Historically, this is the oldest semantics of membership grades since Bellman *et al.* [14].

Degree of preference: According to degree of preference, $\widetilde{F}$ represents a set of more or less preferred objects (or values of a decision variable x) and $\mu_{\widetilde{F}}(u)$ represents an intensity of preference in favour of object $\tilde{u}$, or the feasibility of selecting $\tilde{u}$ as a value of x. Fuzzy sets then represent criteria or flexible constraints. This view is the one later put forward by Bellman and Zadeh [15], it has given birth to an abundant literature on fuzzy optimization, especially fuzzy linear programming and decision analysis.

Degree of uncertainty: This interpretation was proposed by Zadeh [275] when he introduced the possibility theory and developed his theory of approximate reasoning [276]. $\mu_{\widetilde{F}}(u)$ is then the degree of possibility that a parameter x has value $\tilde{u}$, given that all that is known about it is that "x is $\widetilde{F}$". Then the values encompasses by the support of the membership functions are mutually exclusive, and the membership degrees rank these values in terms of their respective plausibility. Set functions called possibility and necessity measures can be derived so as to rank-order events in terms of unsurprising-ness and acceptance respectively.

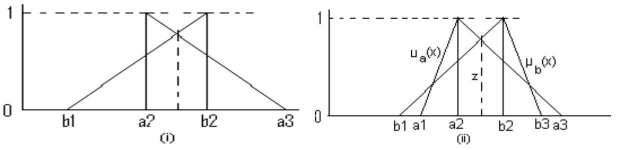


Figure 2.12: (i)Possibility; (ii)Measure of Pos($\widetilde{a} \geq \widetilde{b}$)

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$ respectively. Then according to Dubois and Prade [82], Liu and Iwamura [153], Zadeh [275]

$$pos(\tilde{a} * \tilde{b}) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re, x * y\}, \tag{2.6}$$

where pos represents possibility, $*$ is any one of the relations $>, <, =, \leq, \geq$ [c.f. Figure-2.12].

$$nes(\tilde{a} * \tilde{b}) = 1 - pos\overline{(\tilde{a} * \tilde{b})}, \tag{2.7}$$

where nes represents necessity.

Similarly, possibility and necessity measures of $\tilde{a}$ with respect to $\tilde{b}$ are denoted by $\Pi_{\tilde{b}}(\tilde{a})$ and $N_{\tilde{b}}(\tilde{a})$ respectively and are defined as

$$\Pi_{\tilde{b}}(\tilde{a}) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(x)), x \in \Re\} \tag{2.8}$$

$$N_{\tilde{b}}(\tilde{a}) = min\{sup(\mu_{\tilde{a}}(x), 1 - \mu_{\tilde{b}}(x)), x \in \Re\}. \tag{2.9}$$

According to the definitions of fuzzy numbers, following lemmas can easily be derived.

Lemma 2.1. *If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b is a crisp number, $pos(\tilde{a} > b) \geq \alpha$ iff $\dfrac{a_3 - b}{a_3 - a_2} \geq \alpha$.*

Lemma 2.2. *If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b is a crisp number, $nes(\tilde{a} > b) \geq \alpha$ iff $\dfrac{b - a_1}{a_2 - a_1} \leq 1 - \alpha$.*

Lemma 2.3. *If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b is a crisp number, $pos(\tilde{a} \leq b) \geq \alpha$ iff $\dfrac{b - a_1}{a_2 - a_1} \geq \alpha$.*

Lemma 2.4. *If $\tilde{b} = (b_1, b_2, b_3)$ and $\tilde{a} = (a_1, a_2, a_4)$ be TFNs with $0 < a_1$ $0 < b_1$, $pos(\tilde{b} \geq \tilde{a}) \geq \alpha$ iff $\frac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2} \geq \alpha$.*

Lemma 2.5. *If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN and b be a crisp number with $0 < a_1$ and $0 < b$,*

$$\Pi_{\tilde{a}}(b) = N_{\tilde{a}}(b) = \begin{cases} \frac{b - a_1}{a_2 - a_1} & for \;\; a_2 \geq b \geq a_1 \\ \frac{a_3 - b}{a_3 - a_2} & for \;\; a_3 \geq b \geq a_2 \\ 0 & otherwise. \end{cases}$$

Lemma 2.6. *[165]: If $\tilde{a} = (a_1, a_2, a_3, a_4)$ be a TrFN and b be a crisp number then*

$$pos(\tilde{a} \geq b) = \begin{cases} 1 & if \;\; a_3 \geq b \\ \frac{a_4 - b}{a_4 - a_3} & if \;\; a_3 \leq b \leq a_4 \\ 0 & otherwise \end{cases}$$

Lemma 2.7. *[165]: If $\tilde{a} = (a_1, a_2, a_3, a_4)$ be a TrFN and b be a crisp number then*

$$nes(\tilde{a} \geq b) = \begin{cases} 1 & if\ \ a_1 \geq b \\ \frac{a_2 - b}{a_2 - a_1} & if\ \ a_1 \leq b \leq a_2 \\ 0 & otherwise \end{cases}$$

C. Credibility Measure [156]: Let $\tilde{A}$ be any fuzzy number. Then credibility measure of $\tilde{A}$ is denoted by $cr(\tilde{A})$ and defined as

$$cr(\tilde{A}) = \frac{1}{2}[pos(\tilde{A}) + nes(\tilde{A})] \tag{2.10}$$

More generally, according to Maity [165] the above form can be considered as

$$cr(\tilde{A}) = [\rho pos(\tilde{A}) + (1 - \rho) nes(\tilde{A})] \ \ where\ \ 0 \leq \rho \leq 1.$$

D. Fuzzy Expectation [154]: Let $\tilde{X}$ be any normalized fuzzy variable. The expected value of the fuzzy variable $\tilde{X}$ is denoted by $E[\tilde{X}]$ and defined by

$$E[\tilde{X}] = \int_0^{\infty} cr(\tilde{X} \geq r)\, dr - \int_{-\infty}^{0} cr(\tilde{X} \leq r)\, dr \tag{2.11}$$

provided that at least one of the two integral is finite.

Lemma 2.8. *[165]: If $\tilde{A} = (a_1, a_2, a_3)$ be a TFN and r be a crisp number, expected value of $\tilde{A}$, $E(\tilde{A})$ is given by*

$$E[\tilde{A}] = \frac{1}{2}[(1 - \rho)a_1 + a_2 + \rho\, a_3]$$

where $\rho\ (0 \leq \rho \leq 1)$ is the degree of optimism/pessimism for DM.

Lemma 2.9. *[165]: If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a TrFN and r is a crisp number, then expected value of $\tilde{A}$, $E[\tilde{A}]$, is given by*

$$E[\tilde{A}] = \frac{1}{2}[(1 - \rho)(a_1 + a_2) + \rho(a_3 + a_4)]$$

where $\rho\ (0 \leq \rho \leq 1)$ is the degree of optimism/pessimism for DM.

2.1.5 Fuzzy Logic and Fuzzy Inference

fuzzy expression: An n-dimensional fuzzy expression function is a mapping from $[0, 1]^n$ to $[0, 1]$. i.e, $f : [0, 1]^n \to [0, 1]$.
fuzzy logic: The fuzzy logic is a logic represented by the fuzzy expression (formula) which satisfies the followings.

i) Truth values, 0 and 1, and variable x_i ($\in$ $[0.1]$, $i = 1, 2, .., n$) are fuzzy expressions.

ii) If f is a fuzzy expression, $\sim f$ is also a fuzzy expression.

iii) If f and g are fuzzy expressions, $f \wedge g$ and $f \vee g$ are also fuzzy expressions.

where $\sim$ $(negation)$, $\wedge (conjunction)$ *and* $\vee$ $(disjuction)$ are used as in the classical logic. Thus, for $a, b \in [0\,, 1]$, Negation; $\sim a = 1 - a$; Conjunction $a \wedge b = Min(a, b)$; Disjunction $a \vee b = Max(a, b)$.

Over view of fuzzy inference process:

The term "inference" refers to a process of obtaining new information by using existing knowledge and it is commonly referred to as IF-THEN rule-based form. It typically expresses an inference such that if we know a fact (premise, hypothesis, antecedent), then we can infer or derive another fact called a conclusion (consequent) i.e.

"If x is a Then y is b ".

Different steps of fuzzy inference process are

Fuzzification of input value: When a value of premise is given as an input, it must correspond to some one or more linguistic fuzzy sets with some membership values.

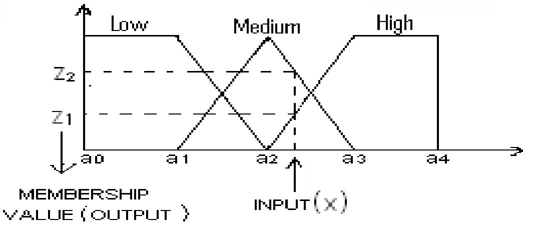


Figure 2.13: fuzzyfication of the inputs

Rule Strength Calculation: After the inputs are fuzzified, the degree to which each part of the antecedent is satisfied for each rule is known. The degree of a rule is the rule strength of the corresponding rule. If there are more than one antecedent then the rule strength is calculated by the standard min operator $\mu_{R_i} = \wedge\{\mu^{R_i}{}_{\widetilde{A}}(x)\,, \mu^{R_i}{}_{\widetilde{B}}(y), ...\}$, where $\mu^{R_i}{}_{\widetilde{A}}(x)\,, \mu^{R_i}{}_{\widetilde{B}}(y), ...$ are the membership values of the inputs $x, y...$ to the antecedents $\widetilde{A}$, $\widetilde{B}$ of the rule R_i. Thus the output is a single truth value for each rule and this is the rule strength of the corresponding rule lies between 0 and 1.

Fuzzy output: After calculation of rule strength for each rule, the fuzzy output implied by the rule is the area bounded by the line corresponding to the rule strength calculated by standard aggregation operator $\vee\{\mu_{\widetilde{A}}(x)\,, \mu_{\widetilde{B}}(y), ...\}$.

Defuzzification: The defuzzification process consists of fuzzy output and gives a crisp value as an output. Here centroid formula, which returns the center of area under the curve, is given by - output = $\frac{\int x\,\mu_(x)}{\int \mu_(x)}$

2.1.6 A Method for Construction of a Fuzzy Number

In this section, triangular fuzzy number corresponding to a market parameter [such as selling price, demand etc.] is constructed from a set of data collected from some market experts following Chang [46]. The membership function of a triangular fuzzy number $\widetilde{A}(m, a, b)$ is of the from.

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a}{m-a}, & for\ a \le x \le m \\ \dfrac{b-x}{b-m}, & for\ m < x \le b \\ 0, & elsewhere \end{cases}$$

Let $g_1, g_2, \cdots, g_n$ are the assertions made by n different experts for a particular parameter [like low demand, high selling price etc]. In estimation of the fuzzy numbers, the center around which the g_i gather is to be estimated by giving more importance to the g_i's lying closer to the center. In other words, the estimation of this center is a weighted average of the g_i. To approximate the center, a distance matrix $G = [d_{ij}]_{n,n}$ of the relative distances between g_i's is calculated, where $d_{ij} = |g_i - g_j|$ and $d_{ii} = 0$, $d_{ij} = d_{ji}$. Then the average relative distances corresponding to g_i is given by $\bar{d}_i = \sum_{j=1}^{n} d_{ij}/(n-1)$. In this method the degree of importance is determined by pair-wise comparisons between g_i's which is based on the average distances. If $P = [p_{ij}]_{n,n}$ be the pair -wise comparison matrix then $p_{ij} = \bar{d}_j/\bar{d}_i$, $p_{ii} = 1$, $p_{ij} = 1/p_{ji}$. Let w_i be the true degree of importance of g_i and $0 \le w_i \le 1$. As P is the matrix obtained from comparison of distances, it is perfectly consistent. Therefore, $p_{ij} = w_i/w_j\ \forall i, j$. If w be the column vector of w_i, then $Pw = nw$, which implies that n is an eigenvalue of P and w is the corresponding eigenvector with $\sum_{i=1}^{n} w_i = 1$, $w_j = 1/\sum_{i=1}^{n} p_{ij}$, $j = 1, 2, \cdots, n$. The importance degree w_i serves as the weight associated with g_i.

Thus the mode of the fuzzy number, $m = \sum_{i=1}^{n} w_i g_i$.

The mean deviation of the fuzzy number $\widetilde{A}(m, a, b)$ is defined as $\sigma = \frac{\int_a^b |x-m|.\mu_{\widetilde{A}}(x)dx}{\int_a^b \mu_{\widetilde{A}}(x)dx}$.

Rewriting this equation we have $\sigma = \frac{(m-a)^2+(b-m)^2}{3(b-a)}$.

Let ξ be the ratio of left spread to right spread, that is $\xi = \frac{m-a}{b-m}$. Using the expressions for σ and ξ we have,

$a = m - \frac{3(1+\xi)\xi.\sigma}{1+\xi^2}$. $\qquad b = m + \frac{3(1+\xi)\sigma}{1+\xi^2}$.

Now as the parameters σ, ξ are depends on a, b therefore σ, ξ are unknown before a, b known. So σ, ξ are approximated from the collected data as follows. σ is approximated by the average deviation where the average deviation is calculated from the collected data by the formula $\sigma = \sum_{i=1}^{n} w_i|g_i - m|$. For the approximation of ξ all g_i's are partitioned into two set such as

$$Let,\ \Delta = \{1, 2, \cdots, n\},\ \ A = \{i\,;\, g_i < m, i \in \Delta\},\ B = \{i\,;\, g_i \geq\ m, i \in \Delta\}$$

now calculate to values g^l, g^r defined as

$$g^l = \frac{\sum_{i\in A} w_i g_i}{\sum_{i\in A} w_i},\ g^r = \frac{\sum_{i\in B} w_i g_i}{\sum_{i\in B} w_i}$$

Thus the ratio of the left spread to the right spread approximated as, $\xi = \frac{m-g^l}{g^r-m}$. Then the lower end and the upper end of the fuzzy number $\widetilde{A}(m, a, b)$ are calculated.

2.1.7 Fuzzy-random variable and its properties

Definition 2.8. Fuzzy-random variable [203]: *Let R is the set of real numbers, $F_c(R)$ is set of all fuzzy variables and $G_c(R)$ is all of non-empty bounded close interval. In a given probability space (Ω, F, P), a mapping $\xi : \Omega \to F_c(R)$ is called a fuzzy random variable in (Ω, F, P), if $\forall\ \alpha \in$(0,1], the set-valued function $\xi_\alpha : \Omega \to G_c(R)$ defined by $\xi_\alpha(\omega) = (\xi(\omega))_\alpha = \{x|x \in R, \mu_{\xi(\omega)}(x) \geq \alpha\}, \forall\omega \in \Omega$, is F measurable. Different semantics of fuzzy-random variable are also presented by Xu and Zhou [?].*

Theorem 2.1. *Let $\tilde{\bar{\xi}}$ is LR fuzzy random variable, for any $\omega \in \Omega$, the membership function of $\tilde{\bar{\xi}}(\omega)$ is*

$$\mu_{\tilde{\bar{\xi}}(\omega)}(t) = \begin{cases} L\left(\frac{\bar{\xi}(\omega)-t}{\xi_L}\right) & for\ t \leq \bar{\xi}(\omega) \\ R\left(\frac{t-\bar{\xi}(\omega)}{\xi_R}\right) & for\ t \geq \bar{\xi}(\omega) \end{cases}$$

where the random variable $\bar{\xi}(\omega)$ is normally distributed with mean m_ξ and standard deviation σ_ξ and ξ_L, ξ_R are the left and right spreads of $\tilde{\bar{\xi}}(\omega)$. The reference functions L: [0,1]→ [0,1] and R : [0,1]→ [0,1] satisfy that L(1)=R(1)=0,L(0)=R(0)=1, and both are monotone functions. Then

$$\begin{cases} Pr[Pos\{\tilde{\bar{\xi}}(\omega) \geq t\} \geq \delta] \geq \gamma \\ Pr[Nec\{\tilde{\bar{\xi}}(\omega) \geq t\} \geq \delta] \geq \gamma \end{cases} \quad are\ equivalent\ to$$

$$t \leq \begin{cases} m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) + \xi_R R^{-1}(\delta) \\ m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) - \xi_L L^{-1}(1-\delta) \end{cases}$$

where Φ is standard normally distributed, $\delta, \gamma \in$[0,1] are predetermined confidence levels.

Proof. According to definition of possibility we get,

$$Pos[\tilde{\bar{\xi}}(\omega) \geq t] \geq \delta \ \Leftrightarrow\ R\left[\frac{t-\bar{\xi}(\omega)}{\xi_R}\right] \leq \delta \ \Leftrightarrow\ \bar{\xi}(\omega) \geq t - \xi_R R^{-1}(\delta)$$

So for predetermined level $\delta, \gamma \in [0, 1]$ we have,

$$Pr[Pos\{\tilde{\bar{\xi}}(\omega) \geq t\} \geq \delta] \geq \gamma$$
$$\Leftrightarrow Pr[\bar{\xi}(\omega) \geq t - \xi_R R^{-1}(\delta)] \geq \gamma$$
$$\Leftrightarrow Pr[\frac{\bar{\xi}(\omega) - m_\xi}{\sigma_\xi} \geq \frac{t - \xi_R R^{-1}(\delta) - m_\xi}{\sigma_\xi}] \geq \gamma$$
$$\Leftrightarrow \Phi\left(\frac{t - \xi_R R^{-1}(\delta) - m_\xi}{\sigma_\xi}\right) \leq 1 - \gamma$$
$$\Leftrightarrow t \leq m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) + \xi_R R^{-1}(\delta)$$

Similarly from the measure of necessity we have,

$$Nes[\tilde{\bar{\xi}}(\omega) \geq t] \geq \delta \;\Leftrightarrow\; L\left[\frac{\bar{\xi}(\omega) - t}{\xi_L}\right] \geq 1 - \delta \;\Leftrightarrow\; \bar{\xi}(\omega) \geq t + \xi_L L^{-1}(1-\delta)$$

So for predetermined level $\delta, \gamma \in [0, 1]$ we have,

$$Pr[Nes\{\tilde{\bar{\xi}}(\omega) \geq t\} \geq \delta] \geq \gamma$$
$$\Leftrightarrow t \leq m_\xi + \sigma_\xi \Phi^{-1}(1-\gamma) - \xi_L L^{-1}(1-\delta)$$

The proof is complete.

□

2.2 Single-Objective Optimization in Crisp Environment and Solution Techniques

2.2.1 Single-Objective Optimization Problem

The problem of optimization concerns with the maximization/minimization of an algebraic or a transcendental equation of one or more variables, known as objective function under some available resources which are represented as constraints. Such type of problem is known as Single-Objective Optimization Problem (SOOP). This can be formulated as:

$$\left.\begin{array}{l} \text{Find} \quad x = (x_1, x_2, ..., x_n)^T \\ \text{which maximizes/minimizes } f(x) \\ \text{subject to} \quad x \in X \\ \text{where } X = \left\{ x : \begin{array}{ll} g_j(x) \leq 0, & j = 1, 2, ..., l \\ h_k(x) = 0, & k = 1, 2, ..., m \\ x_i \geq 0, & i = 1, 2, ..., n \end{array} \right\} \end{array}\right\} \tag{2.12}$$

where, $f(x),\ g_j(x),\ \ j = 1, 2, ..., l$ and defined on n-dimensional set.

It is noted that, when both the objective function and $h_k(x),\ \ k = 1, 2, ..., m$ are functions constraints are linear, the above SOOP becomes a SOLOP. Otherwise, it is a SONLOP.

A decision variable vector x satisfying all the constraints is called a feasible solution to the problem. The collection of all such solutions forms a feasible region. The SOOP (2.12) is to find a feasible solution x^* such that for each feasible point $x, f(x) \leq f(x^*)$ for maximization problem and $f(x) \geq f(x^*)$ for minimization problem. Here, x^* is called an optimal solution or solution to the problem.

Local Minimum: $x^* \in X$ is said to be a local minima of (2.12) if there exists an $\epsilon > 0$ such that $f(x) \geq f(x^*), \ \forall x \in X \ : |x - x^*| < \epsilon,$.
Convex Function: A function $f(x_1, x_2, ..., x_n)$ is convex if the Hessian Matrix, given by $H(x_1, x_2, ..., x_n) = \left[\frac{\partial^2 f}{\partial x_i \partial x_j}\right]_{n\times n}$, is positive semi-definite/positive definite.
Global Minimum: $x^* \in X$ is said to be a global minima of (2.12) if $f(x) \geq f(x^*), \ \forall x \in X$. Otherwise, if the function $f(x)$ is convex then the local minimum solution $x \in X$ is global minimum.
Convex Programming Problem: The problem defined in (2.12) is to be called convex programming problem if the objective function $f(x_1, x_2, ..., x_n)$ and the constraint functions $g_j(x_1, x_2, ..., x_n), j = 1, 2, ..., m$ are convex.

For solution of SONLOP by any available NLP method, local optimal solutions are guaranteed. Also, it is known that, a local minimum/maximum solution is a global minimum/maximum for a convex/concave optimization (i.e., a NLP problem to minimize a convex function or to maximize a concave function) problem.

Lot of mathematical techniques based on linearization, gradient based techniques, evolutionary algorithms, stochastic search algorithms, etc., are available in the literature to solve such type of SONLOP. Here, few methods are illustrated, which have been used in this thesis to solve the inventory problems, non-linear in nature.

2.2.2 Gradient Based Solution Techniques for Single-Objective Optimization

Necessary Condition for Optimality: If a function $f(x)$ is defined for all $x \in X$ and has a relative minimum at $x = x^*$, where $x^* \in X$ and all the partial derivatives $\frac{\partial f(x)}{\partial x_r}$ for $r = 1, 2, ..., n$ are exists at $x = x^*$, then $\frac{\partial f(x^*)}{\partial x_r} = 0$.
Sufficient Condition for Optimality: The sufficient condition for a stationary point x^* to be an extreme point is that the matrix of second partial derivatives (Hessian Matrix) of $f(x)$ evaluated at $x = x^*$ is (i) positive definite when x^* is a relative minimum point, and (ii) negative definite when x^* is a relatively maximum point.

2.2.2.1 Generalized Reduced Gradient (GRG) Technique

The GRG technique is a method for solving NLP problems for handling equality as well as inequality constraints. Consider the NLP problem:

$$\left.\begin{array}{ll} \text{Find} & x = (x_1, x_2, ...x_n)^T \\ \text{which maximizes} & f(x) \\ \text{subject to} & x \in X \\ \text{where } X = & \left\{ x : \begin{array}{ll} g_j(x) \leq 0, & j = 1, 2, ..., l \\ h_k(x) = 0, & k = 1, 2, ..., m \\ x_i \geq 0, & i = 1, 2, ..., n \end{array} \right\} \end{array}\right\} \tag{2.13}$$

By adding a non-negative slack variable $s_j\,(\geq\ 0),\ \ j\ =\ 1,2,...,l$ to each of the above inequality constraints, the problem (2.13) can be stated as,

$$\left.\begin{array}{ll}\text{Maximize} & f(x)\\ \text{subject to} & x\in X\\ \text{where } X= & \left\{x:\ \begin{array}{ll} x=(x_1,x_2,...x_n)^T & \\ g_j(x)+s_j=0, & j=1,2,...,l\\ h_k(x)=0, & k=1,2,...,m\\ x_i\geq 0 & i=1,2,...n\\ s_j\geq 0, & j=1,2,...l\end{array}\right\}\end{array}\right\} \tag{2.14}$$

where the lower and upper bounds on the slack variables, $s_j,\ \ j=1,2,...,l$ are taken as a zero and a large number (infinity) respectively.

Denoting s_j by x_{j+n}, $g_j(x)+s_j$ by ξ_j, $h_k(x)$ by ξ_{l+k}, the above problem can be rewritten as,

$$\left.\begin{array}{ll}\text{Maximize} & f(x)\\ \text{subject to} & x\in X\\ \text{where } X= & \left\{x:\ \begin{array}{ll} x=(x_1,x_2,...x_{n+l})^T & \\ \xi_j(x)=0, & j=1,2,...l+m\\ x_i\geq 0 & i=1,2,...n+l\end{array}\right\}\end{array}\right\} \tag{2.15}$$

This GRG technique is based on the idea of elimination of variables using the equality constraints. Theoretically, $(l+m)$ variables (dependent variables) can be expressed in terms of remaining $(n-m)$ variables (independent variables). Thus one can divide the $(n+l)$ decision variables arbitrarily into two sets as

$$x=(y,z)^T$$

where, y is $(n-m)$ design or independent variables and z is $(l+m)$ state or dependent variables and

$$\begin{array}{rcl} y &=& (y_1,y_2,...,y_{n-m})^T\\ z &=& (z_1,z_2,...,z_{l+m})^T\end{array}$$

Here, the design variables are completely independent and the state variables are dependent on the design variables used to satisfy the constraints

$$\xi_j(x)=0,\ (j=1,2,...,l+m).$$

Consider the first variations of the objective and constraint functions:

$$df(x)=\sum_{i=1}^{n-m}\frac{\partial f}{\partial y_i}dy_i+\sum_{i=1}^{l+m}\frac{\partial f}{\partial z_i}dz_i=\nabla_y^T f\,dy+\nabla_z^T f\,dz \tag{2.16}$$

$$d\xi_j(x) = \sum_{i=1}^{n-m} \frac{\partial \xi_j}{\partial y_i} dy_i + \sum_{i=1}^{l+m} \frac{\partial \xi_j}{\partial z_i} dz_i$$

$$\text{or} \quad d\xi = C\,dy + D\,dz \tag{2.17}$$

$$\text{where} \quad \nabla_y^T f = \left(\frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial y_2}, ..., \frac{\partial f}{\partial y_{n-m}} \right)$$

$$\text{and} \quad \nabla_z^T f = \left(\frac{\partial f}{\partial z_1}, \frac{\partial f}{\partial z_2}, ..., \frac{\partial f}{\partial z_{l+m}} \right)$$

$$C = \begin{bmatrix} \frac{\partial \xi_1}{\partial y_1} & \cdots\cdots\cdots & \frac{\partial \xi_1}{\partial y_{n-m}} \\ \frac{\partial \xi_2}{\partial y_1} & \cdots\cdots\cdots & \frac{\partial \xi_2}{\partial y_{n-m}} \\ \cdots & \cdots\cdots\cdots\cdots\cdots & \cdots \\ \cdots & \cdots\cdots\cdots\cdots\cdots & \cdots \\ \frac{\partial \xi_{l+m}}{\partial y_1} & \cdots\cdots\cdots & \frac{\partial \xi_{l+m}}{\partial y_{n-m}} \end{bmatrix}, \qquad D = \begin{bmatrix} \frac{\partial \xi_1}{\partial z_1} & \cdots\cdots\cdots & \frac{\partial \xi_1}{\partial z_{l+m}} \\ \frac{\partial \xi_2}{\partial z_1} & \cdots\cdots\cdots & \frac{\partial \xi_2}{\partial z_{l+m}} \\ \cdots & \cdots\cdots\cdots\cdots\cdots & \cdots \\ \cdots & \cdots\cdots\cdots\cdots\cdots & \cdots \\ \frac{\partial \xi_{l+m}}{\partial z_1} & \cdots\cdots\cdots & \frac{\partial \xi_{l+m}}{\partial z_{l+m}} \end{bmatrix},$$

$$dy = (dy_1,\, dy_2, ..., dy_{n-m})^T$$
$$\text{and} \quad dz = (dz_1, dz_2, ..., dz_{l+m})^T$$

Assuming that the constraints are originally satisfied at the vector x $(\xi(x) = 0)$, any change in the vector dx must correspond to $d\xi = 0$ to maintain feasibility at $x + dx$. Thus, Eq. (2.17) can be solved as

$$C dy + D dz = 0$$
$$\text{or} \quad dz = -D^{-1} C dy \tag{2.18}$$

The change in the objective function due to the change in x is given by the Eq. (2.16), which can be expressed, using Eq. (2.18) as

$$df(x) = (\nabla_y^T f - \nabla_z^T f D^{-1} C) dy$$

$$\text{or} \quad \frac{df(x)}{dy} = G_R$$
$$\text{where} \quad G_R = \nabla_y^T f - \nabla_z^T f D^{-1} C$$

is called the generalized reduced gradient. Geometrically, the reduced gradient can be described as a projection of the original $n-$dimensional gradient into the $(n - l)$

dimensional feasible region described by the design variables.

A necessary condition for the existence of minimum of an unconstrained function is that the components of the gradient vanish. Similarly, a constrained function assumes its minimum value when the appropriate components of the reduced gradient are zero. In fact, the reduced gradient G_R can be used to generate a search direction S to reduce the value of the constrained objective function. Similarly, to the gradient ∇f that can be used to generate a search direction S for an unconstrained function. A suitable step length λ is to be chosen to minimize the value of $f(x)$ along the search direction. For any specific value of λ, the dependent variable vector z is updated using Eq. (2.18). Noting that Eq. (2.17) is based on using a linear approximation to the original non-linear problem, so the constraints may not be exactly equal to zero at λ, i.e., $d\xi \neq 0$. Hence, when y is held fixed, in order to have

$$\xi_j(x) + d\xi_j(x) = 0, \;\; j = 1, 2, ..., l + m \tag{2.19}$$

following must be satisfied.

$$\xi(x) + d\xi(x) = 0 \tag{2.20}$$

Using Eq. (2.17) for $d\xi$ in Eq. (2.20), following is obtained

$$dz = D^{-1}(-\xi(x) - Cdy) \tag{2.21}$$

The value dz given by Eq. (2.21) is used to update the value of z as

$$z_{update} = z_{current} + dz \tag{2.22}$$

The constraints evaluated at the updated vector x and the procedure of finding dz using Eq. (2.22) is repeated until dz is sufficiently small.

2.2.2.2 Variational Principle (Unconstraint problems)

The variational principle or method deals with the problem in which the quantity to be optimized appears in an integral, i.e., it gives necessary condition for extreme value of the quantity appearing in the integral. Here we have to find out a path $y = y(x)$, $a \leq x \leq b$ which optimizes the functional

$$J = \int_a^b F(y, \frac{dy}{dx}, x)dx \tag{2.23}$$

The value of y is given at the end points $x = a$ and $x = b$. These are called boundary conditions.

Now for the optimum value of J, the necessary condition is known as Euler's equation and is given by

$$\frac{\partial F}{\partial y} - \frac{d}{dx}(\frac{\partial F}{\partial \dot{y}}) = 0, \quad \text{where } \dot{y} = \frac{dy}{dx} \tag{2.24}$$

Solving above equation, the path $y = y(x)$ can be obtained easily and for which the functional J can be optimized.

Transversality condition: In the case, if y is not prescribed at an end points say, $x = a$ then we need an additional condition $\frac{\partial F}{\partial \dot{y}} = 0$ at $x = a$. This condition is called transversality condition. Similar condition can be consider when y is not prescribed at $x = b$.

2.2.3 Optimal Control Theory (Pontryagin Maximum Principle)

Pontryagin Maximum Principle: Let, the dynamic behavior of the system be described by a vector differential equation subject to some specified initial conditions and

$$\frac{dx(t)}{dt} = f_1(x(t), u(t), t),\; x(0) = x_0 \tag{2.25}$$

$x(t)$ is a vector of dependent variable and $\dot{x}(t)$ is the rate of change of $x(t)$ w.r.t time t. $u(t)$ is a dependent variable vector and f is the vector value function for the dependent variable $u(t)$ and time t.

Initial condition: The value of the dependent variable vector $x(t_0)$ takes on a specified set of values, denoted by the vector x_0 with some specified time. The value of the dependent variable x(t) w. r. t. time, starting from the initial time t_0 up to the whole planning period T will depend on the values of the control variables $u(t)$.

Therefore, for a given set of values of $u(t)$ in the interval (t_0, T) and the above differential equation can be solved to determine the values of x(t) over time in (t_0, T) and subsequently the trajectory of the system can be obtained. The governing vector differential equation (2.25) yields solution in general. However the existence and uniqueness of the solution may exist under some vigorous mathematical criterion.

The decision marker is to choose a set of control functions such that the system can be moved to a desired terminal state in the whole planning period T and the problem is the realism of Control Theory. A system is said to be under control specified to a time t_0 if for any state $x(t_0)$ and any desired terminal state $x(t_1)$, there exit a finite time $t_1 > t_0$ and control function u(t) in t within (t_0, t_1) that moves $x(t_0)$ to $x(t_1)$ at t_1.

The optimal control problem and the problem in hand is to

$$\begin{aligned} \text{Maximize } J(u, x) &= \int_0^T F(x(t), u(t), t)dt + S(x(T)) \\ \text{subject to } \frac{dx(t)}{dt} &= f_1(x(t), u(t), t), \quad x(0) = x_0 \end{aligned}$$

where F stands for the scalar integrated function in the objective function and $S(x(t))$ is the salvage value function of the terminal state of the system.

The objective function J(u, x) is the control function being denoted as a function of u(t). The problem is to choose the control function $u(t),\ t\epsilon(0,t)$ which maximizes the objective function J subjected to the dynamics constraints of the system.

Generally $u(t)$ is taken as unconstrained. But in most practical cases of interest $u(t)$ is restricted to lie within the set of admissible functions and to satisfy some constraint equation represented by the vector equation $g(x(t), u(t), t) \leq b$.

The formulation of the problem now becomes

$$\begin{aligned} \text{Maximize } \; J(u,x) &= \int_0^T F(x(t),u(t),t)dt + S(x(T)) \\ \text{subject to } \; \frac{dx(t)}{dt} &= f_1(x(t),u(t),t), \quad x(0) = x_0 \end{aligned}$$

$u(t)$ is a piece-wise continuous function and $g(x(t), u(t), t) \leq b$

To solve the above optimal control problem we defined two function, viz (i) Hamitonion and (ii) Lagrangian. Hamiltonion (H) can be expressed as

$$H(x,u,q,t) = F(x,u,t) + q^T(t)f_1(x,u,t),$$

Here, $q(t)$ is a vector function of adjoint variables and are multipliers associated with the dynamic system constraint. Again Lagrangian function can be expressed as:

$L(x,u,q,\lambda,t) = H(x,u,q,t) + \lambda^T(g(x,u,t) - b), \lambda$ is the Lagrange multipliers associated with the constraints $g(x(t), u(t), t) \leq b$

The necessary conditions (not sufficient) to be satisfied for a control function to be optimal are as follows:

A) Hamiltonian Maximizing condition: $H(x^*, q^*, u^*, t) \geq H(x^*, u, q^*, t)$ for all admissible u.

B) System Dynamic Constraints:

$$\frac{dx^*(t)}{dt} = f_1(x^*(t), u^*(t), t), \; x^*(0) = x_0$$

C) Stationary of the gradient of the Lagrangian with respect to $u(t)$ at $(x^*, u^*, \lambda^*.t^*)$ is

$L_u(x^*, u^*, \lambda^*, \mu^*, t) = \phi$, a zero vector

D) Complementary slackness condition:

$$\lambda^*(t) \geq \phi, \; \lambda^*(t)^T.(g(x^*(t), u^*(t), t) - b) = 0$$

E) Adjoint equation:

$$\frac{dq(t)}{dt} = -L_x(x^*(t), u^*(t).q^*(t), \lambda^*(t), t)$$

where L_x is the vector of partial derivatives of the Lagrangian with respect to $x(t)$.
The terminal condition in $q(t)$ as $q^*(T) = S_x(x(T))$ is the Gradient of the Salvage value function S w.r.t. x(t) at t=T.
For $x^*(t), u^*(t), q^*(t), \mu^*(t)$, the above conditions are to be solved simultaneously.
The steps of solution is then.

CASE-2.2.3a: Direct Solution

1) Solve with the Hamiltonian maximizing condition to obtain $u^*(t)$ as a functions of $q(t), x(t)$ and t.
2) Solve the system dynamic constraint (2.25) equations with the initial condition $x(0) = x_0$ and the adjoint equations along with the boundary condition $q(T) = S_x(x(T))$ for trial values of $\mu^*(t)$ to get a two-point boundary valued problem (TPBVP) as x(t) is specified at $t = 0(x(0) = x_0)$ where as $q(t)$ is specified at $t = T(q(T) = S_x(x(T)))$.
3) Check the Lagrangian and Complementary Slackness condition by choosing appropriate values for $\mu^*(t)$.

CASE 2.2.3b: Numerical Solution

Start by using Hamiltonian maximizing condition, $u^*(t)$ can be solved in terms of $S(t)$ and $q(t)$ So solved values of $u^*(t)$ then can be plugged into: $\frac{dx^*(t)}{dt} = f_1^1(x^*, q^*, t)$ with $x^*(0) = x_0$

and $\frac{dq^*(t)}{dt} = -H_x(x^*, q^*, t)$ with $q^*(T) = S_x(x^*(t))$.

The (TPBVP) for the variables (x^*, q^*) can be solved either numerically or by trial and error method using the following steps.
Steps:
a) Guess a value of $q(0)$ which converts the (TPBVP) into an initial value problem (IVP).
b) Solve for $x^*(t)$ and $q^*(t)$ numerically, and
c) Check for $q^*(T) = S_x(x(T))$.
If this is not satisfied, we take better set of $q(0)$ and proceed accordingly.

2.2.3.1 Lagrange Function

Let us consider Maximize $J(u, x) = \int_0^T F(x(t), u(t), t)dt + S(x(T))$
subject to $\dot{x}(t) = f_1(x(t), u(t), t)$,
$g(x(t), u(t), t) \leq b$,

$x(0) = x_0$ and $u(t)$ is a piece-wise continuous function of t.
Again Lagrangian function can be expressed as:
$L(x, u, q, \lambda, t) = H(x, u, q, t) + \lambda^T(g(x, u, t) - b), \lambda$ is a Lagrange multipliers associated with the constraints $g(x(t), u(t), t) \leq b$ which also satisfy the complementary slackness condition.

2.2.3.2 Kuhn-Tucker Condition

Kuhn-Tucker's necessary and sufficient conditions for Optimality: Let us consider a maximization problem as

$$\text{Maximize } J(u, x) \tag{2.26}$$
$$\text{subject to } g_i(x(t), u(t)) \leq b_i, \;\; i = 1, 2, ..l \tag{2.27}$$

where $x(t) = x(x_1(t), x_2(t), ..., x_n(t))^T$ and $u(t) = u(u_1(t), u_2(t), ..., u_n(t))^T$

For (2.26)-(2.27), to determine optimal u* and x*,
Kuhn-Tucker's necessary conditions are

$$\frac{\partial J(u, x)}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i(x(t), u(t))}{\partial x_j} = 0, \;\; j = 1, 2, ...n \tag{2.28}$$
$$\frac{\partial J(u, x)}{\partial u_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i(x(t), u(t))}{\partial u_j} = 0, \;\; j = 1, 2, ...n \tag{2.29}$$
$$\lambda_i \left(g_i(x(t), u(t)) - b_i\right) = 0 \tag{2.30}$$
$$\text{and } g_i(x(t), u(t)) \leq b_i, \; \lambda_i \geq 0 \tag{2.31}$$

and **sufficient conditions** are :
$J(u, x)$ is concave and all $b_i - g_i(x, u)$ are convex functions of x, u.

2.2.4 Soft Computing Techniques for Optimization

Heuristic optimization provides a robust and efficient approach for solving complex real world problems. Recently, complicated inventory control problems are also solved using heuristic approaches by several researchers [155, 168]. Among basic heuristic algorithms GA and PSO are much used in different areas of science and technology [19, 142]. In this thesis, some soft computing techniques are developed/modified to solve different inventory control problems and are presented below.

2.2.4.1 Genetic Algorithm (GA)

Now-a-days Genetic Algorithm (GA) (Michalewicz [178]; Mondal and Maiti, [181]) is extensively used to solve complex decision making problems in different fields of science and technology. GA is an exclusive search algorithm based on the mechanics of natural selection and genesis which initially was developed by Holland [110], then Goldberg [**?**]. General structure of GA is presented below:

GA procedures

Representation: A n-dimensional real vector', $X_i = (x_{i1}, x_{i2}, ..., x_{in})$, is used to represent the i^{th} solution, where $x_{i1}, x_{i2}, ..., x_{in}$ represent n decision variables of the decision making problem under consideration. X_i is called i^{th} chromosome and x_{ij} is called j^{th} gene of i^{th} chromosome.
Initialization: N such solutions $X_i = (x_{i1}, x_{i2}, ..., x_{in})$, i = 1, 2, ...,N are randomly generated by random number generator within the boundaries of each variable $[B_{jl}, B_{jr}]$, j= 1, 2, ..., n. These bounds are calculated from the nature of the problem and previous experience. Initialized (P(1)) sub-function is used for this purpose.
Constraint Checking: For constrained optimization problems, at the time of generation of each individuals X_i of P(1), constraints are checked using a separate sub-function "check constraint(X_i)", which returns 1 if X_i satisfies the constraints otherwise returns 0. If check constraint (X_i) =1, then X_i is included in P(1) otherwise X_i is again generated and it continues until constraints are satisfied.
Diversity Preservation: At the time of generation of P(1), diversity is maintained using entropy originating from information theory. Following steps are used for this purpose.

(i) Probability, pr_{jk}, that the value of the i^{th} gene (variable) of the j^{th} chromosome which is different from the i^{th} gene of the k^{th} chromosome, is calculated using the formula $pr_{jk} = 1 - \frac{x_{ji}-x_{ki}}{B_{jr}-B_{jl}}$ where $[B_{jl}, B_{jr}]$ is the variation domain of the i^{th} gene.

(ii) Entropy of the i^{th} gene, $E_i(M)$, i=1, 2, ..., n is calculated using the formula: $E_i(M) = \sum_{j=1}^{M-1}\sum_{k=j+1}^{M} -pr_{jk}log(pr_{jk})$, where M is the size of the current population.

(iii) Average entropy of the current population is calculated by the formula: $E(M) = \frac{1}{n}\sum_{i=1}^{n} E_i(M)$

(iv) Incorporating the above three steps, a separate sub-function "check diversity(X_i)" is developed. Every time a new chromosome X_i is generated, the entropy between this one and previously generated individuals is calculated. If this information quantity is higher than a threshold, E_T, fixed at the beginning, X_i is included in the population otherwise X_i is again generated until diversity exceeds the threshold, E_T . This method induces a good distribution of initial population.

Fitness Value: This fitness value is measured to check whether the initialised or generated chromosomes are suited for the consideration. Chromosome with higher fitness value receives larger probability of inheritance in subsequent generation, whereas chromosome with low fitness will more likely to be eliminated. In this thesis, the value of the objective function is taken as the fitness of the chromosome.

Algorithm 1: GA PSEUDOCODE

1. **Start**
2. Set iteration counter t=0, Maxsize=200, $\epsilon = 0.0001$ and $p_m(0)$ = 0.9.
3. Randomly generate **Initial** population P(t), where diversity in the population is maintained using entropy originating from information theory.
4. **Evaluate** initial population P(t).
5. Set Maxfit= Maximum fitness in P(t) and Avgfit=Average fitness of P(t).
6. **While** $(Maxfit - Avgfit \leq \epsilon)$ **do**
7.   t=t + 1.
8.   Increase age of each chromosome.
9.   **For** each pair of parents **do**
10   Determine probability of crossover $\tilde{p_c}$ for the selected pair of parents
11.   Perform crossover with probability $\tilde{p_c}$.
12.   **End** for
13.   **For** each offspring perform mutation with probability p_m **do**
14.   Store offsprings into offspring set.
15.   **End** for
16.   Evaluate P(t).
17.   Remove from P(t) all individuals with age greater than their lifetime.
18.   Select a percent of better offsprings from the offspring set and insert into P(t), such that maximum size of the population is less than Maxsize.
19.   Remove all offsprings from the offspring set.
20.   Reduce the value of the probability of mutation p_m.
21. **End While**
22. Output: Best chromosome of P(t).
23. **End** algorithm.

Crossover: For each pair of parent solutions X_i, X_j, a random number c is generated from the range [0, 1] and if c ¡ p_c, crossover operation is made on X_i, X_j. To make crossover operation on each pair of coupled solutions X_i, X_j a random number c_1 is generated from the range [0,1] and their offsprings Y_1 and Y_2 are determined by the formula: $Y_1 = c_1X_i + (1 - c_1)X_j, Y_2 = c_1X_j + (1 - c_1)X_i$.

For constrained optimization problems, if a child solution satisfies the constraints of the problem, then it is included in the offspring set otherwise it is not included in the offspring set.

Mutation:

(i) *Selection for mutation:* For each offspring generate a random number r from the range [0, 1]. If $r < p_m$ then the solution is taken for mutation, where p_m is the probability of mutation.

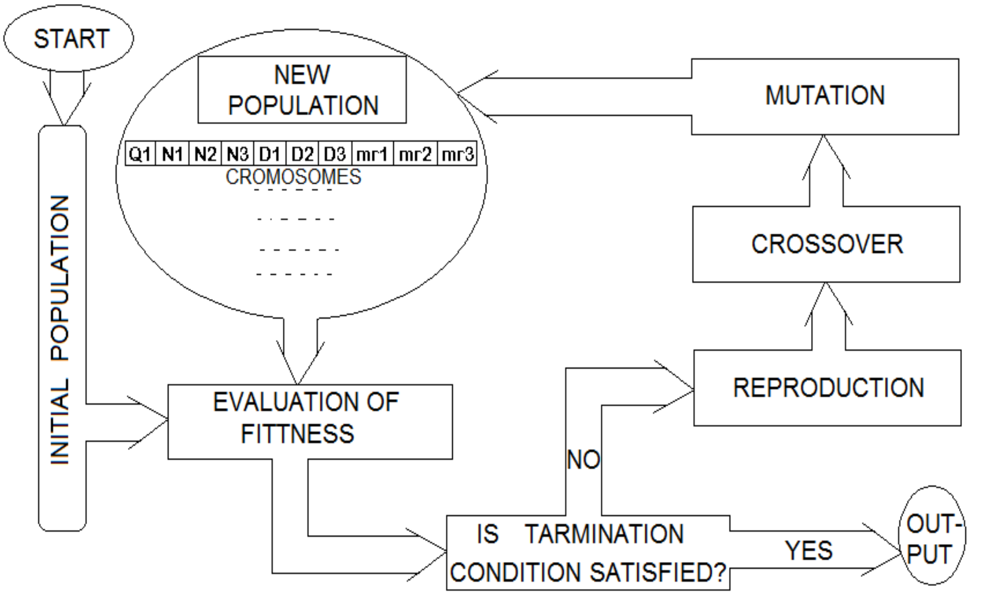


Figure 2.14: GA Process

(ii) *Mutation process:* To mutate a solution $X = (x_1, x_2, ..., x_n)$, a random integer I in the range [1,n] has to be selected. Then replace x_i by randomly generated value within the boundary $[B_{il}, B_{ir}]$ of i^{th} component of X. New solution (if satisfies constraints of the problem) replaces the parent solution. If child solution does not satisfy the constraint, then parent solution will not be replaced by child solution. Constraint checking of a child solution C_i is made using "check constraint (C_i)" function.

Reduction process of p_m: According to real world demand as generation increases, p_m will decrease smoothly since the search space was more wide initially and after some iterations, it should move towards the convergence. This concept lead us to reduce the value of p_m in each generation. Let $p_m(0)$ is the initial value of p_m. Then probability of mutation in T-th generation $p_m(T)$ is calculated by the formula $p_m(T) = p_m(0)\ exp(-T/\alpha_1)$, where α_1 is calculated so that the final value of p_m is small enough (10^{-2} in our case). So $\alpha_1 = Maxgen/log[\frac{p_m(0)}{10^{-2}}]$, where Maxgen is the expected number of generations that the GA can run for convergence.
Selection of offsprings: Maximum population growth in a generation is assumed as forty percent. So not all offsprings are taken into the parent set for next generation. At first offspring set is arranged in descending order in fitness. Then better solutions are selected and entered into parent set such that population size does not exceeds Maxsize.
Termination Condition: Algorithm terminates when difference between maximum fitness (Maxfit) of chromosome, i.e., fitness of the best solution of the population and average fitness (Avgfit) of the population becomes negligible.
Implementation: With the above functions and values, the algorithm is implemented using

C-programming language

2.3 Multi-Objective Optimization Problem

2.3.1 Multi-Objective Programming Problem

Development of single objective mathematical programming problems and methods for their solutions have been presented in the earlier section. But, the world has become more complex and almost every important real-world problem involves more than one objective. In such cases, decision makers find imperative to evaluate best possible approximate solution alternatives according to multiple criteria.

A general multi-objective programming problem (MOPP) is of the following form:

$$\left.\begin{array}{l} minimize \;\; f_m(X), \qquad m = 1, 2, ..., M; \\ subject\ to \;\; g_j(X) \geq 0, \quad j = 1, 2, ..., J; \\ \qquad\qquad x_i^L \leq x_i \leq x_i^U, \quad i = 1, 2, ..., n; \end{array}\right\} \tag{2.32}$$

where the solution X is a vector of n decision variables (DV). i.e. $X = (x_1, x_2, ..., x_n)^T$. The last set of constraints are called variable bounds, restricting each DV x_i to take a value within a lower x_i^L and an upper x_i^U bound. These bounds constitute the decision space. Here $f_1(x), f_2(x), ..., f_M(x)$ are $M\,(\geq 2)$ objectives. It is noted that, if the objectives of the original problem are: minimize $f_i(x)$, for $i = 1, 2, ..., m_0$ and maximize $f_i(x)$ for $i = m_0 + 1, m_0 + 2, ..., M$, then the objective in the mathematical formulation will be

$$\text{Min } F(x) = (f_1(x), f_2(x)...., f_{m_0}(x), -f_{m_0+1}(x), -f_{m_0+2}(x), ..., -f_M(x))^T.$$

subject to the same constraints as in (2.32).

If $f_i(x),\ (i = 1, 2, ..., M)\ and\ g_j(x),\ (j = 1, 2, ..., J)$ are linear, the corresponding problem is called Multi-Objective Linear Programming (MOLP) problem. When all or any one of the above functions is non-linear, it is referred as a Multi-Objective Non-linear Programming (MONLP) problem. Here, the problem is often referred to as a Vector Minimum Problem (VMP).

Convex and non-convex MOPP: The multi-objective optimization problem (2.32) is said to be convex if all the objective functions and the feasible region are convex, otherwise it is called non-convex.

Ideal Objective Vector: An objective vector minimizing each of the objective functions is called an ideal (or perfect) objective vector.

Complete optimal solution: x^* is said to be a complete optimal solution to the MONLP in (2.32) iff there exists $x^* \in X$ such that $f_i(x^*) \leq f_i(x),\ i = 1, 2, ..., k$ for all $x \in X$.

In general, the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist and so Pareto (or non dominated) optimality concept is introduced.

Pareto optimal solution: x^* is said to be a Pareto optimal solution to the MONLP iff there does not exist another $x \in X$ such that $f_i(x) \leq f_i(x^*)$ for all $i,\ i = 1, 2, ..., k$ and $f_j(x) < f_j(x^*)$ for at least one index $j,\ j = 1, 2, ..., k$.

An objective vector F^* is Pareto-optimal if there does not exist another objective vector $F(x)$ such that $f_i \leq f_i^*$ for all $i = 1, 2, \cdots, k$ and $f_j < f_j^*$ for at least one index j. Therefore, F^* is Pareto-optimal if the decision vector corresponding to it is Pareto optimal.

Unless an optimization problem is convex, only locally optimal solution is guaranteed using standard mathematical programming techniques. Therefore, the concept of Pareto-optimality needs to be modified to introduce the notion of a locally Pareto-optimal solution for a non-convex problem as defined by Geoffrion [**?**].

Locally Pareto optimal solution: $x^* \in X$ is said to be a locally Pareto optimal solution to the MONLP if and only if there exists an $r > 0$ such that x^* is Pareto optimal in $X \cap N(x^*, r)$, where $N(x^*, r)$ is a r-neighborhood of x^*, i.e, there does not exist another $x \in X \cap N(x^*, r)$ such that $f_i(x) \leq f_i(x^*)$.

Concept of Domination: Most evolutionary multi-objective optimization algorithms use the concept of domination. In these algorithms, two solutions are compared on the basis of whether one dominates the other solution or not. Let us use the operator $\sqsupseteq$ between two solutions i and j as $i \sqsupseteq j$ denotes that solution i is better than solution j on a particular objective. Similarly $i \sqsubseteq j$ for a particular objective implies that solution i is worse than solution j on this objective. With this assumption a solution x is said to dominate the other solution y, if both the following conditions hold.

- The solution x is not worse than the solution y in all the objectives.
- The solution x is strictly better than the solution y in at least one objective, i.e., $f_j(x) \sqsupseteq f_j(y)$ for at least one $j \in \{1, 2, ..k\}$

Now, let us introduce some non-linear programming techniques which have been used in this thesis to achieve at least local Pareto optimal solutions.

2.3.2 Solution Techniques for Multi-Objective Programming Problem in Crisp Environment

2.3.2.1 Multi-Objective Genetic Algorithm (MOGA) :

Genetic algorithm approach was first proposed by Holland [110]. Because of its generality and its several advantages over conventional optimization methods it has been successfully applied to many optimization problems. There are several approaches using genetic algorithms to deal with the multi-objective optimization problems. These algorithms can be classified into two types-(i) Non-Elitist MOGA and (ii) Elitist MOGA. A fast and elitist MOGA was developed following Deb *et al.* [78] and is named as Fast and Elitist Multi-objective Genetic Algorithm (FEMOGA).

2.3.2.2 Fast and Elitist Multi-Objective Genetic Algorithm

This multi-objective genetic algorithm has the following two important components.

(a) Division of a population of solutions into subsets having non-dominated solutions: Consider a problem having M objectives and take a population P of feasible solutions of the problem of size N. We like to partition P into subsets $F_1, F_2, \cdots, F_k$, such that every subset contains non-dominated solutions, but every solution of F_i is not dominated by any solution of F_{i+1}, for $i = 1, 2, ..k - 1$. To do this for each solution, x, of P, calculate the following two entities.

(i) Number of solutions of P which dominate x, let it be n_x.

(ii) Set of solutions of P that are dominated by x. Let it be S_x.

The above two steps require $O(MN^2)$ computations. Clearly F_1 contains every solution x having $n_x = 0$. Now for each solution $x \in F_1$, visit every member y of S_x and decrease n_y by 1. In doing so if for any member y, $n_y = 0$, then $y \in F_2$. In this way F_2 is constructed. The above process is continued to every member of F_2 and thus F_3 is obtained. This process is continued until all subsets are identified. For each solution x in the second or higher level of non-dominated subsets, n_x can be at most $N - 1$. So each solution x will be visited at most $N - 1$ times before n_x becomes zero. At this point, the solution is assigned a subset and will never be visited again. Since there is at most $N - 1$ such solutions, the total complexity is $O(N^2)$. So overall complexity of this component is $O(MN^2)$.

(b) Determine distance of a solution from other solutions of a subset: To determine distance of a solution from other solutions of a subset following steps are followed:

(i) First sort the subset according to each objective function values in ascending order of magnitude.

(ii) For each objective function, the boundary solutions are assigned an infinite distance value (a large value).

(iii) All other intermediate solutions are assigned a distance value for the objective, equal to the absolute normalized difference in the objective values of two adjacent solutions.

(iv) This calculation is continued with other objective functions.

(v) The overall distance of a solution from others is calculated as the sum of individual distance values corresponding to each objective. Since M independent sorting of at most N solutions (In case the subset contains all the solutions of the population) are involved, the above algorithm has $O(MNlogN)$ computational complexity.

Using the above two operations proposed multi-objective genetic algorithm takes the following form:

1. Set probability of crossover p_c and probability of mutation p_m.
2. Set iteration counter $T = 1$.
3. Generate initial population set of solution $P(T)$ of size N.
4. Select solution from $P(T)$ for crossover and mutation.
5. Made crossover and mutation on selected solution and get the child set $C(T)$.
6. Set $P_1 = P(T)UC(T)$ // Here U stands for union operation.
7. Divide P_1 into disjoint subsets having non-dominated solutions. Let these sets be $F_1, F_2, \cdots F_k$.
8. Select maximum integer n such that order of $P_2(= F_1UF_2U \ ... \ UF_n) \leq N$.
9. if $O(P_2) < N$ sort solutions of F_{n+1} in descending order of their distance from other solutions of the subset. Then select first $N - O(P_2)$ solutions from F_{n+1} and add with P_2, where $O(P_2)$ represents order of P_2.
10. Set $T = T + 1$ and $P(T) = P_2$.
11. If termination condition does not hold go to step-4.
12. Output: P(T)
13. End algorithm.

MOGAs that use non-dominated sorting and sharing are mainly criticized for their

- $O(MN^3)$ computational complexity
- non-elitism approach
- the need for specifying a sharing parameter to maintain diversity of solutions in the population.

In the above algorithm, these drawbacks are overcame. Since in the above algorithm computational complexity of step-7 is $O(MN^2)$, step-9 is $O(MNlogN)$ and other steps are $\leq O(N)$, so overall time complexity of the algorithm is $O(MN^2)$. Here selection of new population after crossover and mutation on old population, is done by creating a mating pool by combining the parent and offspring population and among them, best N solutions are taken as solutions of new population. By this way, elitism is introduced in the algorithm. When some solutions from a non-dominated set F_j (i.e., a subset of F_j) are selected for new population, those are accepted whose distance compared to others (which are not selected) are much i.e., isolated solutions are accepted. In this way taking some isolated solutions in the new population, diversity among the solutions is introduced in the algorithm, without using any sharing function. Since computational complexity of this algorithm $< O(MN^3)$ and elitism is introduced, this algorithm is named as FEMOGA.

Part II

Inventory Models in Deterministic Environment

Chapter 3

Note on: Partial Trade Credit Policy of Retailer in Economic Order Quantity Models for Deteriorating Items with Expiration Dates and Price Sensitive Demand

3.1 Introduction

In supply chain management, it is too difficult to preserve deteriorating items in all business sectors. Many products such as fruits, vegetables, medicines, high-tech products, pharmaceuticals, and volatile liquids not only deteriorate continuously due to evaporation, obsolescence and spoilage but also have their expiration dates, i.e., the product will have a maximum lifetime which is time bound. However, only a few researchers take the expiration date of a deteriorating item into consideration. In the present article, we consider the replenishment policies for inventory of the items which are subject to deteriorate continuously and also have their expiration dates.

On the other hand, to take the decision about procuring of an item, inventory management is generally influenced by pricing of that item. Again, in selecting an item for use, the selling price of that item is one of the decisive factors to the customers. It is well known that the higher selling price of an item decreases the demand rate of that item where the lesser price has the reverse effect. Hence, the demand rate of an item is dependent on the selling price of that item. Incorporating this effect, we investigate the dependency on pricing for deteriorating items with their expiration dates.

In daily life, the deterioration of goods is a frequent and common phenomenon. Incorporating this feature in model formulation, Chung and Huang [63] amended Huang [113] by developing two-warehouse inventory model for deteriorating items under trade credit financing. Min et al. [179] formulated an inventory model for deteriorating items under stock-dependent demand and two-level trade credit to study the retailer's optimal ordering policy. Liang and Zhou [147] developed a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. Guchhait et al. [96] developed a two storage inventory model of a deteriorating item with variable demand under partial trade credit period. Majumder et al. [172] developed an EPQ model for deteriorating items under two level partial trade credit policy with crisp and fuzzy demand.

Again, it is usually observed that customers pay reasonable prices of a commodity on the basis of its quality and longevity. Hence, pricing strategy becomes one of the most important aspect for business organizations to sell deteriorating inventory and enhance revenues. In this context, Thangam and Uthayakumar [243] presented two-echelon trade credit financing model for perishable items to derive optimal credit period, selling price and replenishment time with price and credit linked demand. Dye and Ouyang [84] established EOQ model for deteriorating items to determine optimal selling price, replenishment number and replenishment schedule with time and price dependent demand under two levels of trade credit policy. Other interesting articles can be found in Mahata and Goswami [158], Mahata and Mahata [160], Tsao [245], Chang et al. [47], Kreng and Tan [137, 138], Ho [109], Chung [66].

All the above mentioned research work did not consider the fact that deteriorating items have their expiration dates. In fact, the study of deteriorating items with expiration dates has received a relatively little attention in the literature. Currently, Bakker et al. [10] provided an excellent review of inventory systems with deterioration since 2001. Recently, Seifert et al. [229] presented an excellent review of trade credit financing. Some relevantly recent articles in trade credit financing were developed by Chern et al. [55], Taleizadeh [239], Yang et al. [266], Mahata [161] and Guchhait et al. [97].

In this chapter, we reformulate the supplier-retailer inventory system for deteriorating items with expiration dates and price sensitive demand of Mahata [161]. In this SCM the retailer receives an up-stream credit period offered by the supplier(s) and offers a down stream partial trade credit period to the end costumers. The demand of the items is a function of selling price which changes reversely with selling price. Items are deteriorated with an expiration date. Each buyer pays a part of his/her total purchase cost during the placing of order and the rest part is paid during the end of offered credit period. The whole model is formulated as a profit maximization problem. The model is also evaluated theoretically and numerically.

The rest of this chapter is organized as follows. Section-3.2 contains the notations which are required to formulate the model and the assumptions under which the model is developed. The formulation of the model is given in section-3.3. All the theoretical developments with supporting lemma are given in section-3.4. Section-3.5 provides a solution methodology to explore the model numerically and the results of the numerical experiment is given in section-3.6. Some discussions and conclusions about the model have been made in the section-3.7 and section-3.8 respectively.

3.2 Notations and Assumptions

Partial trade credit policy of retailer in Economic Order Quantity model for deteriorating items with expiration dates and price sensitive demand has been developed with the following notations and assumptions.

3.2.1 Notations:

A	ordering cost per order in dollars.
h	inventory holding cost in dollar per unit per year excluding interest charges.
c	unit purchasing cost in dollars.
s	unit selling price in dollars with ($s > c$).
M	retailer's trade credit period offered by the supplier in years.
N	customer's trade credit period offered by the retailer in years.
α	fraction of the purchase cost in which the customer must pay the retailer at the time of placing an order with $0 \leq \alpha \leq 1$.
$1 - \alpha$	portion of the purchase cost for which the retailer offers its customer a permissible delay of N periods.
T	time in years.
$I(t)$	stock level in units at any time t.
$\theta(t)$	time-varying deterioration rate at time t, where $0 \leq \theta(t) < 1$.
m	expiration date or maximum lifetime in years of the deteriorating item.
T	replenishment cycle time in years (a decision variable).
Q	order quantity.
I_e	interest earned per dollar per year.
I_c	interest charged per dollar in stocks per year.
$TP(s, T)$	annual total profit in dollars of inventory system, which is a function of s and T.
$D(s) = as^{-b}$	unit time demand, which is a decreasing function of the retail price s, where $a(> 0)$ is a scaling factor and $b(> 1)$ is a price elasticity coefficient. For simplicity of notations, $D(s)$ and D will be used interchangeably.

3.2.2 Assumptions:

i) The market demand for the item is assumed to be sensitive to the customer's retail prices and is defined as $D(s) = as^{-b}$.

ii) All deteriorating items have their expiration dates. The physical significance of the deterioration rate is the rate to be closed to 1 when time is approaching to the maximum lifetime m. The items deteriorates at a rate $\theta(t)$ which depends on time as follow:

$$\theta(t) = \frac{1}{1+m-t}, \quad 0 \leq t \leq T \leq m. \tag{3.1}$$

Note that it is clear from equation (3.1) that the replenishment cycle time T must be less than or equal to m, and the proposed deterioration rate is a general case for non-deteriorating items, in which $m \longrightarrow \infty$ and $\theta(t) \longrightarrow 0$.

iii) During the credit period offered by the supplier, the retailer uses the sales revenue to earn interest at a rate I_e. At the end of the permissible delay period, the retailer pays the purchasing cost to the supplier and pays interest charges at a rate of I_c for the items in stock or the items already sold but have not been paid for yet.

iv) Replenishment rate is instantaneous.

v) In today's time-based competition, we may assume that shortages are not allowed to occur.

vi) Time horizon is infinite.

3.3 Model Formulation

The retailer receives Q units at $t = 0$. Hence, the inventory starts with Q units at $t = 0$, and then gradually depletes to zero at $t = T$ due to the combined effect of demand and deterioration. Hence, the inventory level is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -D - \theta(t)I(t) \text{ for } \quad 0 \leq t \leq T \tag{3.2}$$

with boundary condition $I(T) = 0$ .Solving the above differential equation (3.2) we obtain the inventory level at time t as

$$I(t) = D(1+m-t)ln\left(\frac{1+m-t}{1+m-T}\right) \text{ for } \quad 0 \leq t \leq T \tag{3.3}$$

As a result, the retailer's order quantity is

$$Q = I(0) = D(1+m)ln\left(\frac{1+m}{1+m-T}\right) \tag{3.4}$$

The annual total relevant cost consists of the following elements:

1. Annual ordering cost is $\frac{A}{T}$.

2. Annual purchase cost per cycle is $\frac{c}{T}I(0) = \frac{cD(1+m)}{T} ln\left(\frac{1+m}{1+m-T}\right)$

3. Annual stock holding cost (AHC) (excluding interest charges)

$$\begin{aligned} AHC &= \frac{h}{T}\int_0^T I(t)dt \\ &= \frac{hD}{T}\left[\frac{(1+m)^2}{2} ln\left(\frac{1+m}{1+m-T}\right) + \frac{T^2}{4} - \frac{(1+m)T}{2}\right] \end{aligned}$$

3.3.1 Interest calculation for retailer:

Case 1: $N < M$
Based on the values of M (i.e., the time at which the retailer must pay the supplier to avoid interest charge), T (i.e., the replenishment cycle time), and $T+N$ (i.e., the time at which the retailer receives the payment from the last customer), we have to examine following three situations: (1) $0 < T+N \leq M$, (2) $T \leq M \leq T+N$, and (3) $M \leq T$.
Situation 1: $0 < T+N \leq M$
In this case, the retailer receives all returns from the customers before paying the purchase amount to the supplier. Consequently, the retailer does not incur interest charges in this case. On the other hand, interest earned per cycle per unit time is $\frac{I_e}{T}[IE_1 + IE_2]$ where Figure 3.1(a) for IE_1 and Figure 3.1(b) for IE_2.

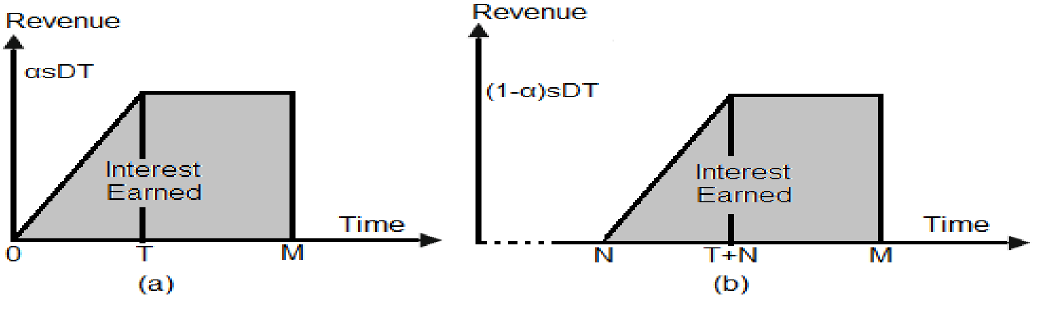


Figure 3.1: Earned Interest ($N \leq M, 0 \leq T+N \leq M$): (a) For Instant Payment; (b) For Delay payment;

$$\begin{aligned} IE_1 &= \text{interest earned due to instant payment of sold units in time (0,T]} \\ &= \int_0^T \{\alpha s D(M-t)\}dt \\ &= \alpha s D\left(MT - \frac{T^2}{2}\right) \end{aligned} \tag{3.5}$$

$$\begin{aligned} IE_2 &= \text{interest earned due to credit payment of sold units in time (0,T]} \\ &= \int_0^T \{(1-\alpha)sD(M-N-t)\}dt \\ &= (1-\alpha)sDT\left(M-N-\frac{T}{2}\right) \end{aligned} \tag{3.6}$$

Situation 2: $T \leq M \leq T+N$

In this case, the retailer's interest earned per cycle per unit time is $\frac{I_e}{T}[IE_1 + IE_2]$ where Figure 3.2(a) for IE_1 and Figure 3.2(b) for IE_2.

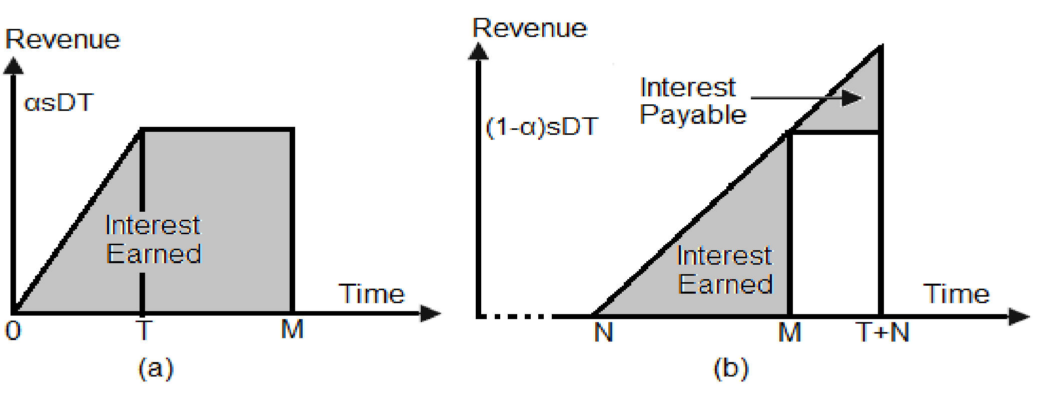


Figure 3.2: Earned Interest ($N \leq M, T \leq M \leq T+N$): (a) For Instant Payment; (b) For Delay payment;

$$\begin{aligned} IE_1 &= \text{interest earned due to instant payment of sold units in time (0,T]} \\ &= \int_0^T \{\alpha s D(M-t)\}dt \\ &= \alpha s D\left(MT - \frac{T^2}{2}\right) \end{aligned} \tag{3.7}$$

$$\begin{aligned} IE_2 &= \text{interest earned due to credit payment of sold units in time (0,M-N]} \\ &= \int_0^{M-N} \{(1-\alpha)sD(M-N-t)\}dt \\ &= (1-\alpha)sD\frac{(M-N)^2}{2} \end{aligned} \tag{3.8}$$

Since $M \leq T + N$, the retailer has to pay interest. Interest charged per cycle per unit time is $\frac{I_c}{T}[IC_1 + IC_2]$ where

$$\begin{aligned} IC_1 &= \text{interest charged due to credit payment of sold units in time (M-N,T]} \\ &= \int_{M-N}^{T} \{(1-\alpha)cD(t-M+N)\}dt \\ &= (1-\alpha)cD\frac{(T-M+N)^2}{2} \end{aligned} \tag{3.9}$$

$$\begin{aligned} IC_2 &= \text{interest charged due to deteriorated units} \\ &= cD\left[(1+m)ln\left(\frac{1+m}{1+m-T}\right) - T\right](T-M+N) \end{aligned} \tag{3.10}$$

Situation 3: $M \leq T$

In this case, the retailer's interest earned per cycle per unit time is $\frac{I_e}{T}[IE_1 + IE_2]$ where Figure 3.3(a) for IE_1 and Figure 3.3(b) for IE_2.

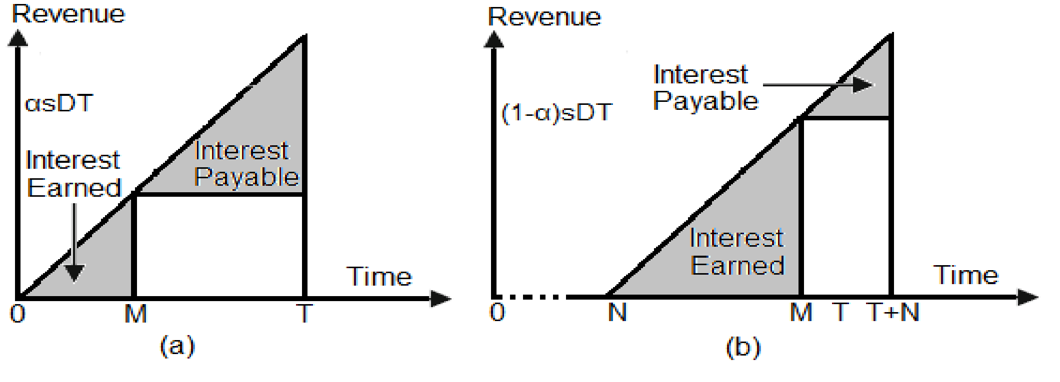


Figure 3.3: Earned Interest ($N \leq M$, $M \leq T$): (a) For Instant Payment; (b) For Delay payment;

$$
\begin{aligned}
IE_1 &= \text{interest earned due to instant payment of sold units in time (0,M]} \\
&= \int_0^M \{\alpha s D(M-t)\} dt \\
&= \alpha s D \frac{M^2}{2} \qquad (3.11)
\end{aligned}
$$

$$
\begin{aligned}
IE_2 &= \text{interest earned due to credit payment of sold units in time (0,M-N]} \\
&= \int_0^{M-N} \{(1-\alpha) s D(M-N-t)\} dt \\
&= (1-\alpha) s D \frac{(M-N)^2}{2} \qquad (3.12)
\end{aligned}
$$

Since $M \leq T$, the retailer has to pay interest. Interest charged per cycle per unit time is $\frac{I_c}{T}[IC_1 + IC_2 + IC_3 + IC_4 + IC_5]$ where

$$
\begin{aligned}
IC_1 &= \text{interest charged due to stock in time [M,T]} \\
&= c \int_M^T I(t) dt \\
&= cD\left[\frac{(1+m-M)^2}{4}\left\{2ln\left(\frac{1+m-M}{1+m-T}\right) - 1\right\} + \frac{(1+m-T)^2}{4}\right] \qquad (3.13)
\end{aligned}
$$

$$
\begin{aligned}
IC_2 &= \text{interest charged due to instant payment of sold units in time [M,T]} \\
&= \int_M^T \{(1-\alpha) cDN\} dt \\
&= (1-\alpha) cDN(T-M) \qquad (3.14)
\end{aligned}
$$

$$
\begin{aligned}
IC_3 &= \text{interest charged due to credit payment of sold units in time (M-N,M]} \\
&= \int_{M-N}^{M} \{(1-\alpha) cD(t-M+N)\} dt \\
&= (1-\alpha) cD \frac{N^2}{2} \qquad (3.15)
\end{aligned}
$$

$$\begin{aligned} IC_4 &= \text{interest charged due to deteriorated units during (0,M]} \\ &= c(T+N-M)\int_0^M \theta(t)I(t)dt \\ &= cD(T+N-M)\Bigg[(1+m)ln\Big(\frac{1+m}{1+m-M}\Big) \\ &\quad -M\Big\{1-ln\Big(\frac{1+m-M}{1+m-T}\Big)\Big\}\Bigg] \end{aligned} \tag{3.16}$$

$$\begin{aligned} IC_5 &= \text{interest charged due to deteriorated units during (M,T]} \\ &= c\int_M^T \theta(t)I(t)(T+N-t)dt \\ &= cD\Bigg[\frac{ln(1+m-T)}{2}\Big\{(T-M)(3T-M+2N)-(1+m-T)(3+m-T)\Big\} \\ &\quad +ln(1+m-M)\frac{(1+m-M)(3+m-M)}{2}-(T-M)\frac{(6+M-T)}{4}\Bigg] \end{aligned} \tag{3.17}$$

Therefore, the total profit per unit time for the retailer when $N \leq M$ is given by

$$TP_1(s,T) = \begin{cases} TP_{11}(s,T) & \text{if} \quad 0 < T \leq M-N \\ TP_{12}(s,T) & \text{if} \quad M-N \leq T \leq M \\ TP_{13}(s,T) & \text{if} \quad M \leq T \end{cases} \tag{3.18}$$

where,

$$\begin{aligned} TP_{11}(s,T) &= sD+\frac{sDI_e}{2}\Big(2M-2N-T+2N\alpha\Big) \\ &\quad -\frac{cD(1+m)}{T}ln\Big(\frac{1+m}{1+m-T}\Big)-\frac{A}{T} \\ &\quad -\frac{hD}{T}\Bigg[\frac{(1+m)^2}{2}ln\Big(\frac{1+m}{1+m-T}\Big)+\frac{T^2}{4}-\frac{(1+m)T}{2}\Bigg] \end{aligned} \tag{3.19}$$

$$\begin{aligned} TP_{12}(s,T) &= sD - \frac{cD(1+m)}{T} ln\left(\frac{1+m}{1+m-T}\right) - \frac{A}{T} \\ &-\frac{hD}{T}\left[\frac{(1+m)^2}{2} ln\left(\frac{1+m}{1+m-T}\right) + \frac{T^2}{4} - \frac{(1+m)T}{2}\right] \\ &+\frac{sDI_e}{2T}\left\{\alpha(2MT-T^2) + (1-\alpha)(M-N)^2\right\} \\ &-\frac{cDI_c}{T}\left[\frac{(1-\alpha)(T-M+N)}{2} + (1+m)ln\left(\frac{1+m}{1+m-T}\right)\right. \\ &\left.-T\right](T-M+N) \end{aligned} \tag{3.20}$$

$$\begin{aligned} TP_{13}(s,T) &= sD - \frac{cD(1+m)}{T} ln\left(\frac{1+m}{1+m-T}\right) - \frac{A}{T} \\ &-\frac{hD}{T}\left[\frac{(1+m)^2}{2} ln\left(\frac{1+m}{1+m-T}\right) + \frac{T^2}{4} - \frac{(1+m)T}{2}\right] \\ &+\frac{sDI_e}{2T}\left\{\alpha M^2 + (1-\alpha)(M-N)^2\right\} \\ &-\frac{cDI_c}{T}\left[(1+m)(T+N-M) ln\left(\frac{1+m}{1+m-T}\right)\right. \\ &-\frac{(T-M)(1+m+M)}{2} - \frac{N(T+M)}{2} \\ &\left.+(1-\alpha)N\left(T-M+\frac{N}{2}\right)\right] \end{aligned} \tag{3.21}$$

Case 2: $N \geq M$
Based on the values of M and T, we have to explore following two situations: (a) $T \leq M$, (b) $M \leq T$.
Situation a: $T \leq M$
In this case, the retailer's interest earned per cycle per unit time is $\frac{I_e}{T}[IE_1]$ where 3.4(a) for Instant Payment, 3.4(b) For Delay Payment.

$$\begin{aligned} IE_1 &= \text{interest earned due to instant payment of sold units in time (0,T]} \\ &= \int_0^T \{\alpha sD(M-t)\} dt \\ &= \alpha sD\left(MT - \frac{T^2}{2}\right) \end{aligned} \tag{3.22}$$

Since $M \leq T+N$, the retailer has to pay interest. Interest charged per cycle per unit time

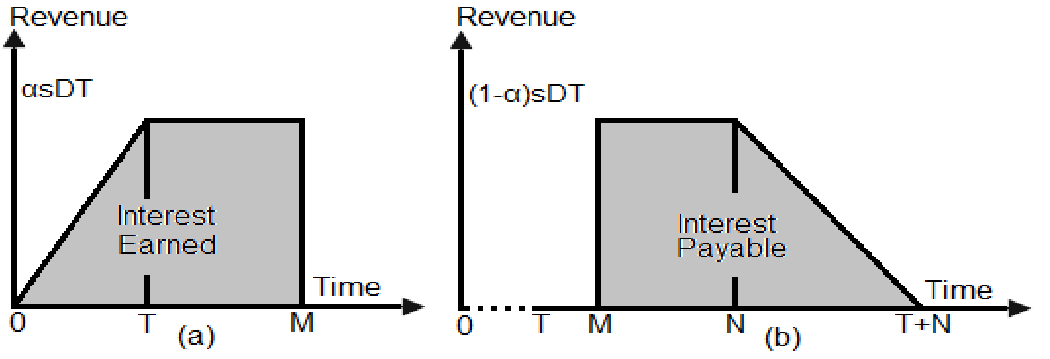


Figure 3.4: Case ($N \geq M, T \leq M$): (a) For Instant Payment; (b) For Delay payment;

is $\frac{I_c}{T}[IC_1 + IC_2]$ where

$$
\begin{aligned}
IC_1 &= \text{interest charged due to credit payment of sold units in time (0,T]} \\
&= \int_0^T \{(1-\alpha)cD(t+N-M)\}dt \\
&= \frac{(1-\alpha)cDT}{2}(T+2(N-M)) \qquad (3.23) \\
IC_2 &= \text{interest charged due to deteriorated units} \\
&= cD\left[(1+m)ln\left(\frac{1+m}{1+m-T}\right) - T\right](T+N-M) \qquad (3.24)
\end{aligned}
$$

Situation b: $M \leq T$

In this case, the retailer's interest earned per cycle per unit time is $\frac{I_e}{T}[IE_1]$ where Figure 3.5(a), 3.5(b).

$$
\begin{aligned}
IE_1 &= \text{interest earned due to instant payment of sold units in time (0,M]} \\
&= \int_0^M \{\alpha sD(M-t)\}dt \\
&= \alpha sD\frac{M^2}{2} \qquad (3.25)
\end{aligned}
$$

Since $M \leq T$, the retailer has to pay interest. Interest charged per cycle per unit time is

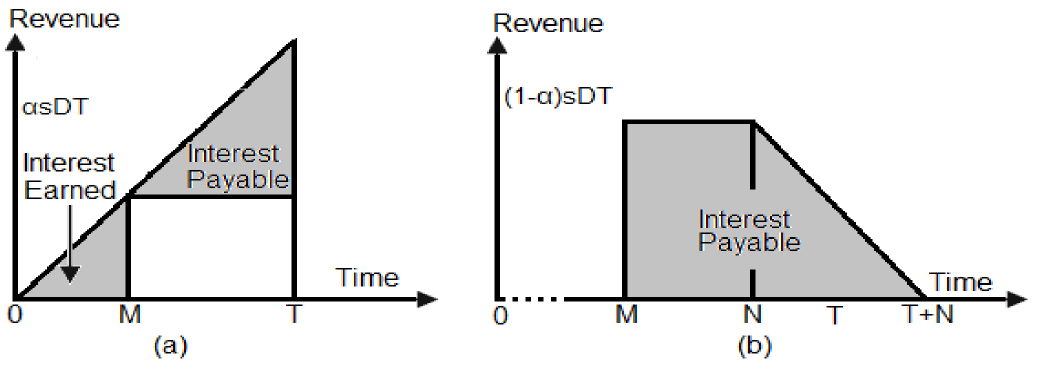


Figure 3.5: Case ($N \leq M, T \leq M$): (a) For Instant Payment; (b) For Delay payment;

$\frac{I_c}{T}[IC_1 + IC_2 + IC_3 + IC_4 + IC_5]$ where

$$\begin{aligned} IC_1 &= \text{interest charged due to stock in time [M,T]} \\ &= c\int_M^T I(t)dt \\ &= cD\left[\frac{(1+m-M)^2}{4}\left\{2ln\left(\frac{1+m-M}{1+m-T}\right)-1\right\}+\frac{(1+m-T)^2}{4}\right] \end{aligned} \tag{3.26}$$

(3.27)

$$\begin{aligned} IC_2 &= \text{interest charged due to instant payment of sold units in time [M,T]} \\ &= \int_M^T \{(1-\alpha)cDN\}dt \\ &= (1-\alpha)cDN(T-M) \end{aligned} \tag{3.28}$$

$$\begin{aligned} IC_3 &= \text{interest charged due to credit payment of sold units in time (0,M]} \\ &= \int_0^M \{(1-\alpha)cD(t+N-M)\}dt \\ &= (1-\alpha)cD\frac{M(2N-M)}{2} \end{aligned} \tag{3.29}$$

$$\begin{aligned} IC_4 &= \text{interest charged due to deteriorated units during (0,M]} \\ &= c(T+N-M)\int_0^M \theta(t)I(t)dt \\ &= cD(T+N-M)\Bigg[(1+m)ln\left(\frac{1+m}{1+m-M}\right) \\ &\quad -M\left\{1-ln\left(\frac{1+m-M}{1+m-T}\right)\right\}\Bigg] \end{aligned} \tag{3.30}$$

$$\begin{aligned} IC_5 &= \text{interest charged due to deteriorated units during (M,T]} \\ &= c\int_M^T \theta(t)I(t)(T+N-t)dt \\ &= cD\Bigg[\frac{ln(1+m-T)}{2}\Big\{(T-M)(3T-M+2N)-(1+m-T)(3+m-T)\Big\} \\ &\quad +ln(1+m-M)\frac{(1+m-M)(3+m-M)}{2}-(T-M)\frac{(6+M-T)}{4}\Bigg] \end{aligned} \tag{3.31}$$

Therefore, the total profit per unit time for the retailer when $N \geq M$ is given by

$$TP_2(s,T) = \begin{cases} TP_{21}(s,T) & \text{if} \quad T \leq M \\ TP_{22}(s,T) & \text{if} \quad M \leq T \end{cases} \tag{3.32}$$

where

$$\begin{aligned} TP_{22}(s,T) &= sD - \frac{cD(1+m)}{T}ln\left(\frac{1+m}{1+m-T}\right) - \frac{A}{T} + \frac{sDI_e\alpha M^2}{2T} \\ &\quad -\frac{hD}{T}\left[\frac{(1+m)^2}{2}ln\left(\frac{1+m}{1+m-T}\right)+\frac{T^2}{4}-\frac{(1+m)T}{2}\right] \\ &\quad -\frac{cDI_c}{T}\Bigg[(1+m)(T+N-M)ln\left(\frac{1+m}{1+m-T}\right)-\frac{N(T+M)}{2} \\ &\quad -\frac{(T-M)(1+m+M)}{2}+(1-\alpha)\left(NT-\frac{M^2}{2}\right)\Bigg] \end{aligned} \tag{3.33}$$

Hence our problem is

$$\text{maximize } TP(s,T) = \begin{cases} TP_1(s,T) & \text{if} \quad N \leq M \\ TP_2(s,T) & \text{if} \quad M \leq N \end{cases} \tag{3.34}$$

where $TP_i(s,T)$, $i = 1,2$ is defined in equations (3.18) and (3.32) respectively.
It is to be noted that, for fixed s, $TP_{11}(s, M-N) = TP_{12}(s, M-N)$, $TP_{12}(s,M) = TP_{13}(s,M)$ and $TP_{21}(s,M) = TP_{22}(s,M)$. Hence for fixed s, $TP_i(s,T)$ is continuous function on $T > 0$, for $i = 1,2$.

3.4 Theoretical Experiment

In this section, how to obtain the optimal ordering cycle length T^*, as well as the optimal selling price s^*, is discussed here for the following two cases.

3.4.1 Optimal solution for the Case of $N \leq M$

For fixed s, the first order partial derivative of $TP_{11}(s,T)$ with respect to T is

$$\frac{\partial TP_{11}(s,T)}{\partial T} = \frac{1}{T^2}\Bigg[A + D(1+m)\Big(c + \frac{h(1+m)}{2}\Big)\Big\{ln\Big(\frac{1+m}{1+m-T}\Big) - \frac{T}{1+m-T}\Big\}\Bigg] - \Big(\frac{hD}{4} + \frac{sDI_e}{2}\Big) \tag{3.35}$$

Motivated by equation (3.35), we assume an auxiliary function, say $F_{11}(T), T \in (0, M-N]$, where

$$F_{11}(T) = \Bigg[A + D(1+m)\Big(c + \frac{h(1+m)}{2}\Big)\Big\{ln\Big(\frac{1+m}{1+m-T}\Big) - \frac{T}{1+m-T}\Big\}\Bigg] - \Big(\frac{hD}{4} + \frac{sDI_e}{2}\Big)T^2 \tag{3.36}$$

Differentiating $F_{11}(T)$ with respect to $T \in (0, M-N]$, we have

$$F'_{11}(T) = \frac{dF_{11}(T)}{dT} = -DT\Bigg[(1+m)\Big(c + \frac{h(1+m)}{2}\Big)\frac{1}{(1+m-T)^2} + \Big(\frac{h}{2} + sI_e\Big)\Bigg] < 0 \tag{3.37}$$

Thus $F_{11}(T)$ is strictly decreasing function with respect to $T \in (0, M-N]$. Moreover $\lim_{T\to\infty} F_{11}(T) = -\infty$, $F_{11}(0) = A > 0$ and

$$F_{11}(M-N) = \Bigg[A + D(1+m)\Big(c + \frac{h(1+m)}{2}\Big)\Big\{ln\Big(\frac{1+m}{1+m-M+N}\Big) - \frac{M-N}{1+m-M+N}\Big\}\Bigg] - \Big(\frac{hD}{4} + \frac{sDI_e}{2}\Big)(M-N)^2 = \triangle_1(say)$$

If $\triangle_1 \leq 0$, then by intermediate value theorem $\exists$ unique value of T [say $T_{11} \in (0, M-N]$] such that $F_{11}(T_{11}) = 0$.
If $\triangle_1 > 0$, we have $F_{11}(T) > 0 \; \forall T \in (0, M-N]$ which implies $TP_{11}(s,T)$ is strictly increasing function of T. Hence $TP_{11}(s,T)$ has a maximum value at the boundary point $T = M-N$.

Lemma 1: *Let T_{11}^* denotes the optimal value of $T \in (0, M-N]$. For fixed s, the profit function $TP_{11}(s,T)$ is concave and reaches its global maximum at point $T = T_{11}^*$*
Proof: From the above discussion, T_{11}^* which maximizes profit function $TP_{11}(s,T)$ for fixed s is given by

$$T_{11}^* = \begin{cases} T_{11} & \text{if} \quad \triangle_1 \leq 0 \\ M-N & \text{if} \quad \triangle_1 \geq 0 \end{cases} \tag{3.38}$$

$$\left[\frac{\partial^2 TP_{11}(s,T)}{\partial T^2}\right]_{T=T_{11}^*} = \frac{F_{11}'(T_{11}^*)}{T_{11}^{*2}} - \frac{2F_{11}(T_{11}^*)}{T_{11}^{*3}} < 0, \text{ since } F_{11}'(T_{11}^*) < 0,\ F_{11}(T_{11}^*) \geq 0$$

Thus T_{11}^* gives global maximum for the profit function $TP_{11}(s,T)$. This completes the proof.□

On the other hand, for fixed T_{11}^*, the first order partial derivative of $TP_{11}(s,T_{11}^*)$ with respect to s is given by

$$\frac{\partial TP_{11}(s,T_{11}^*)}{\partial s} = \frac{b\gamma_1}{s^{b+1}} - \frac{(b-1)\gamma_2}{s^{-b}} \tag{3.39}$$

where

$$\gamma_1 = a\left[\frac{(1+m)}{T_{11}^*}\left(c + \frac{h(1+m)}{2}\right) ln\left(\frac{1+m}{1+m-T_{11}^*}\right) + \frac{hT_{11}^*}{4} - \frac{h(1+m)}{2}\right]$$
$$\gamma_2 = a\left[1 + I_e\left(M - \frac{T_{11}^*}{2} - (1-\alpha)N\right)\right]$$

Equating equation (3.39) with zero and solving for s (denoted by s_{11}^*) we get

$$s_{11}^* = \frac{b\gamma_1}{(b-1)\gamma_2} \tag{3.40}$$

Furthermore, at $s = s_{11}^*$

$$\left[\frac{\partial^2 TP_{11}(s,T_{11}^*)}{\partial s^2}\right]_{s=s_{11}^*} = -\frac{(b-1)\gamma_2}{s_{11}^{*(b+1)}} < 0, \text{ since } \gamma_2 > 0 \tag{3.41}$$

Thus s_{11}^* is the global optimal which maximizes the profit function $TP_{11}(s,T_{11}^*)$ for fixed T_{11}^*. Hence the following lemma.
Lemma 2: *For fixed $T_{11}^* \in (0, M-N]$ the profit per unit time $TP_{11}(s,T_{11}^*)$ has a unique global maximum value at the point $s = s_{11}^*$ which is shown as in equation (3.40).*

Again for fixed s, the first order partial derivative of $TP_{12}(s,T)$ with respect to T is

$$\begin{aligned}\frac{\partial TP_{12}(s,T)}{\partial T} &= \frac{1}{T^2}\Bigg[A + D(1+m)\bigg(c+\frac{h(1+m)}{2}\bigg)\bigg\{ln\bigg(\frac{1+m}{1+m-T}\bigg) \\ &-\frac{T}{1+m-T}\bigg\} - \frac{hDT^2}{4} - \frac{sDI_e}{2}\{\alpha T^2 + (1-\alpha)(M-N)^2\} \\ &-\frac{cDI_c(1-\alpha)}{2}\{T^2-(M-N)^2\} + cDI_cT^2 - cDI_c(1+m)\times \\ &\bigg[\frac{T(T+N-M)}{1+m-T} + (M-N)ln\bigg(\frac{1+m}{1+m-T}\bigg)\bigg]\Bigg] \end{aligned} \tag{3.42}$$

Motivated by equation (3.42), we assume an auxiliary function, say $F_{12}(T)$, $T \in [M-N, M]$, where

$$\begin{aligned}F_{12}(T) &= A + D(1+m)\bigg(c+\frac{h(1+m)}{2}\bigg)\bigg\{ln\bigg(\frac{1+m}{1+m-T}\bigg) - \frac{T}{1+m-T}\bigg\} \\ &-\frac{hDT^2}{4} - \frac{sDI_e}{2}\{\alpha T^2 + (1-\alpha)(M-N)^2\} \\ &-\frac{cDI_c(1-\alpha)}{2}\{T^2-(M-N)^2\} + cDI_cT^2 \\ &-cDI_c(1+m)\bigg[\frac{T(T+N-M)}{1+m-T} + (M-N)ln\bigg(\frac{1+m}{1+m-T}\bigg)\bigg] \end{aligned} \tag{3.43}$$

Differentiating $F_{12}(T)$ with respect to $T \in [M-N, M]$, we have

$$\begin{aligned}F'_{12}(T) = \frac{dF_{12}(T)}{dT} &= -DT\Bigg[(1+m)\bigg(c+\frac{h(1+m)}{2}\bigg)\frac{1}{(1+m-T)^2} \\ &+\bigg(\frac{h}{2}+sI_e\alpha\bigg) + cI_c\bigg\{(1-\alpha) \\ &+\frac{2T}{1+m-T} + \frac{(1+m)(T+N-M)}{(1+m-T)^2}\bigg\}\Bigg] < 0 \end{aligned} \tag{3.44}$$

Thus $F_{12}(T)$ is strictly decreasing function with respect to $T \in [M-N, M]$. Moreover $\lim_{T\to\infty} F_{12}(T) = -\infty$.

$$\begin{aligned}F_{12}(M-N) &= A + D(1+m)\bigg(c+\frac{h(1+m)}{2}\bigg)\bigg\{ln\bigg(\frac{1+m}{1+m-M+N}\bigg) \\ &-\frac{M-N}{1+m-M+N}\bigg\} - \bigg(\frac{hD}{4}+\frac{sDI_e}{2}\bigg)(M-N)^2 \\ &-cDI_c(M-N)\bigg\{(1+m)ln\bigg(\frac{1+m}{1+m-M+N}\bigg) \\ &-(M-N)\bigg\} = \triangle_2(say) \end{aligned} \tag{3.45}$$

$$\begin{aligned} F_{12}(M) &= A + D(1+m)\left(c + \frac{h(1+m)}{2}\right)\left\{ln\left(\frac{1+m}{1+m-M}\right) - \frac{M}{1+m-M}\right\} \\ &\quad -\frac{hDM^2}{4} - \frac{sDI_e}{2}\{\alpha M^2 + (1-\alpha)(M-N)^2\} \\ &\quad -\frac{cDI_c(1-\alpha)}{2}\{M^2 - (M-N)^2\} + cDI_cM^2 \\ &\quad -cDI_c(1+m)\left[\frac{MN}{1+m-M} + (M-N)ln\left(\frac{1+m}{1+m-M}\right)\right] \\ &= \triangle_3(say) \end{aligned} \tag{3.46}$$

If $\triangle_2 < 0$, we have $F_{12}(T) < 0\ \forall T \in [M-N, M]$ which implies that $TP_{12}(s,T)$ is strictly decreasing function of $T \in [M-N, M]$. Hence $TP_{12}(s,T)$ has a maximum value at the boundary point $T = M - N$.
If $\triangle_3 \geq 0$, we have $F_{12}(T) > 0\ \forall T \in [M-N, M]$ which implies that $TP_{12}(s,T)$ is strictly increasing function of $T \in [M-N, M]$. Hence $TP_{12}(s,T)$ has a maximum value at the boundary point $T = M$.
If $\triangle_2 \geq 0$ and $\triangle_3 \leq 0$, then by intermediate value theorem $\exists$ unique value of T [say $T_{12} \in [M-N, M]$] such that $F_{12}(T_{12}) = 0$.
Based on above argument and fact that $\triangle_1 > \triangle_2 > \triangle_3$. Hence we obtain the following lemma.
Lemma 3: *Let T^*_{12} denotes the optimal value of $T \in [M-N, M]$. For fixed s, the profit function $TP_{12}(s,T)$ is concave and reaches its global maximum at point $T = T^*_{12}$.*
Proof: From the above discussion, T^*_{12} which maximizes profit function $TP_{12}(s,T)$ for fixed s is given by

$$T^*_{12} = \begin{cases} M-N & \text{if} \quad \triangle_2 \leq 0 < \triangle_1 \\ T_{12} & \text{if} \quad \triangle_3 \leq 0 \leq \triangle_2 \\ M & \text{if} \quad \triangle_3 \geq 0 \end{cases} \tag{3.47}$$

$$\left[\frac{\partial^2 TP_{12}(s,T)}{\partial T^2}\right]_{T=T^*_{12}} = \frac{F'_{12}(T^*_{12})}{T^{*2}_{12}} - \frac{2F_{12}(T^*_{12})}{T^{*3}_{12}} < 0, \text{ since } F'_{12}(T^*_{12}) < 0,\ F_{12}(T^*_{12}) \geq 0$$

Thus T^*_{12} gives global maximum for the profit function $TP_{12}(s,T)$. This completes the proof.□

On the other hand, for fixed T^*_{12}, the first order partial derivative of $TP_{12}(s, T^*_{12})$ with respect to s is given by

$$\frac{\partial TP_{12}(s,T^*_{12})}{\partial s} = \frac{b\xi_1}{s^{b+1}} - \frac{(b-1)\xi_2}{s^{-b}} \tag{3.48}$$

where

$$
\begin{aligned}
\xi_1 &= a\Bigg[\frac{(1+m)}{T_{12}^*}\Bigg(c+\frac{h(1+m)}{2}\Bigg)ln\Bigg(\frac{1+m}{1+m-T_{12}^*}\Bigg)-\frac{h(1+m)}{2} \\
&\quad +\frac{hT_{12}^*}{4}+\frac{cI_c}{T_{12}^*}\Bigg\{\frac{(1-\alpha)(T_{12}^*+N-M)}{2} \\
&\quad +(1+m)ln\Bigg(\frac{1+m}{1+m-T_{12}^*}\Bigg)-T_{12}^*\Bigg\}(T_{12}^*+N-M)\Bigg] \\
\xi_2 &= a\Bigg[1+\frac{I_e}{2T_{12}^*}[\alpha\{M^2-(M-T_{12}^*)^2\}+(1-\alpha)(M-N)^2]\Bigg]
\end{aligned}
$$

Equating equation (3.48) with zero and solving for s (denoted by s_{12}^*) we get

$$
s_{12}^* = \frac{b\xi_1}{(b-1)\xi_2} \tag{3.49}
$$

Furthermore, at $s = s_{12}^*$

$$
\left[\frac{\partial^2 TP_{12}(s,T_{12}^*)}{\partial s^2}\right]_{s=s_{12}^*} = -\frac{(b-1)\xi_2}{s_{12}^{*(b+1)}} < 0, \quad \text{since } \xi_2 > 0 \tag{3.50}
$$

Thus s_{12}^* is the global optimal which maximizes the profit function $TP_{12}(s,T_{12}^*)$ for fixed T_{12}^*. Hence the following lemma.
Lemma 4: *For fixed $T_{12}^* \in [M-N, M]$ the profit per unit time $TP_{12}(s,T_{12}^*)$ has a unique global maximum value at the point $s = s_{12}^*$ which is shown as in equation (3.49).*
Likewise, for fixed s, the first order partial derivative of $TP_{13}(s,T)$ with respect to T is

$$
\begin{aligned}
\frac{\partial TP_{13}(s,T)}{\partial T} &= \frac{1}{T^2}\Bigg[A+D(1+m)\Bigg(c+\frac{h(1+m)}{2}\Bigg)\Bigg\{ln\Bigg(\frac{1+m}{1+m-T}\Bigg) \\
&\quad -\frac{T}{1+m-T}\Bigg\}-\frac{hDT^2}{4}-sDI_e\{\alpha M^2+(1-\alpha)(M-N)^2\} \\
&\quad -\frac{cDI_c(1+m)T^2}{1+m-T}-cDI_c(1+m)(M-N)\Bigg[ln\Bigg(\frac{1+m}{1+m-T}\Bigg) \\
&\quad -\frac{T}{1+m-T}\Bigg]+\frac{cDI_c}{2}\{M(1+m+M-N) \\
&\quad -N(1-\alpha)(2M-N)\}\Bigg]
\end{aligned} \tag{3.51}
$$

Motivated by equation (3.51), we assume an auxiliary function, say $F_{13}(T)$, $T \in [M, \infty)$, where

$$\begin{aligned} F_{13}(T) &= A + D(1+m)\left(c + \frac{h(1+m)}{2}\right)\left\{ln\left(\frac{1+m}{1+m-T}\right) - \frac{T}{1+m-T}\right\} \\ &\quad -\frac{hDT^2}{4} - sDI_e\{\alpha M^2 + (1-\alpha)(M-N)^2\} - \frac{cDI_c(1+m)T^2}{1+m-T} \\ &\quad -cDI_c(1+m)(M-N)\left[ln\left(\frac{1+m}{1+m-T}\right) - \frac{T}{1+m-T}\right] \\ &\quad +\frac{cDI_c}{2}\{M(1+m+M-N) - N(1-\alpha)(2M-N)\} \end{aligned} \tag{3.52}$$

Differentiating $F_{13}(T)$ with respect to $T \in [M, \infty)$, we have

$$\begin{aligned} F'_{13}(T) &= \frac{dF_{13}(T)}{dT} \\ &= -DT\Bigg[(1+m)\left(c + \frac{h(1+m)}{2}\right)\frac{1}{(1+m-T)^2} + \frac{h}{2} \\ &\quad + \frac{cI_c(1+m)}{(1+m-T)^2}\{(1+m-T) + (1+m-M+N)\}\Bigg] < 0 \end{aligned} \tag{3.53}$$

Thus $F_{13}(T)$ is strictly decreasing function with respect to $T \in [M, \infty)$. Moreover $\lim_{T\to\infty} F_{13}(T) = -\infty$.

$$\begin{aligned} F_{13}(M) &= A + D(1+m)\left(c + \frac{h(1+m)}{2}\right)\left\{ln\left(\frac{1+m}{1+m-M}\right) - \frac{M}{1+m-M}\right\} \\ &\quad -\frac{hDM^2}{4} - sDI_e\{\alpha M^2 + (1-\alpha)(M-N)^2\} \\ &\quad -\frac{cDI_c(1-\alpha)}{2}\{M^2 - (M-N)^2\} + \frac{cDI_cM(1+m+M-N)}{2} \\ &\quad -cDI_c(1+m)\left[\frac{MN}{1+m-M} + (M-N)ln\left(\frac{1+m}{1+m-M}\right)\right] \\ &= \triangle_4(say) \end{aligned} \tag{3.54}$$

If $\triangle_4 < 0$, we have $F_{13}(T) < 0\ \forall T \in [M, \infty)$ which implies that $TP_{13}(s, T)$ is strictly decreasing function of $T \in [M, \infty)$. Hence $TP_{13}(s, T)$ has a maximum value at the boundary point $T = M$.
If $\triangle_4 \geq 0$, then by intermediate value theorem $\exists$ unique value of T [say $T_{13} \in [M, \infty)$] such that $F_{13}(T_{13}) = 0$.
Hence we obtain the following lemma.
Lemma 5: *Let T^*_{13} denotes the optimal value of $T \in [M, \infty)$. For fixed s, the profit function $TP_{13}(s, T)$ is concave and reaches its global maximum at point $T = T^*_{13}$.*

Proof: From the above discussion, T_{13}^* which maximizes profit function $TP_{13}(s,T)$ for fixed s is given by

$$T_{13}^* = \begin{cases} M & \text{if} \quad \triangle_4 \leq 0 < \triangle_3 \\ T_{13} & \text{if} \quad 0 \leq \triangle_4 \end{cases} \tag{3.55}$$

$$\left[\frac{\partial^2 TP_{13}(s,T)}{\partial T^2}\right]_{T=T_{13}^*} = \frac{F_{13}'(T_{13}^*)}{T_{13}^{*2}} - \frac{2F_{13}(T_{13}^*)}{T_{13}^{*3}} < 0, \text{ since } F_{13}'(T_{13}^*) < 0,\ F_{13}(T_{13}^*) \geq 0$$

Thus T_{13}^* gives global maximum for the profit function $TP_{13}(s,T)$. This completes the proof.□

On the other hand, for fixed T_{13}^* the first order partial derivative of $TP_{13}(s,T_{13}^*)$ with respect to s is given by

$$\frac{\partial TP_{13}(s,T_{13}^*)}{\partial s} = \frac{b(\theta_2+\theta_3+\theta_4)}{s^{b+1}} - \frac{(b-1)\theta_1}{s^{-b}} \tag{3.56}$$

where,

$$\theta_1 = a\left[1 + \frac{I_e}{2T_{13}^*}\{\alpha M^2 + (1-\alpha)(M-N)^2\}\right]$$

$$\theta_2 = \frac{ac(1+m)}{T_{13}^*} ln\left(\frac{1+m}{1+m-T_{13}^*}\right)$$

$$\theta_3 = a\left[\frac{h(1+m)^2}{2T_{13}^*} ln\left(\frac{1+m}{1+m-T_{13}^*}\right) + \frac{hT_{13}^*}{4} - \frac{h(1+m)}{2}\right]$$

$$\theta_4 = \frac{acI_c}{T_{13}^*}\left[(1+m)(T_{13}^*+N-M) ln\left(\frac{1+m}{1+m-T_{13}^*}\right) - \frac{N(T_{13}^*+M)}{2} - \frac{(T_{13}^*-M)(1+m+M)}{2} + (1-\alpha)N\left(T_{13}^* - M + \frac{N}{2}\right)\right]$$

Equating equation (3.56) with zero and solving for s (denoted by s_{13}^*) we get

$$s_{13}^* = \frac{b(\theta_2+\theta_3+\theta_4)}{(b-1)\theta_1} \tag{3.57}$$

Furthermore, at $s = s_{13}^*$,

$$\left[\frac{\partial^2 TP_{13}(s,T_{13}^*)}{\partial s^2}\right]_{s=s_{13}^*} = -\frac{(b-1)\theta_1}{s_{13}^{*(b+1)}} < 0, \quad \text{since } \theta_1 > 0 \tag{3.58}$$

Thus s_{13}^* is the global optimal which maximizes the profit function $TP_{13}(s,T_{13}^*)$ for fixed T_{13}^*. Hence the following lemma.

Lemma 6: *For fixed $T_{13}^* \in [M,\infty)$ the profit per unit time $TP_{13}(s,T_{13}^*)$ has a unique global maximum value at the point $s = s_{13}^*$ which is shown as in equation (3.58).*

3.4.2 Optimal solution for the Case of $N \geq M$

For fixed s, the first order partial derivative of $TP_{21}(s,T)$ with respect to T is

$$\begin{aligned}\frac{\partial TP_{21}(s,T)}{\partial T} &= \frac{1}{T^2}\Bigg[A + D(1+m)\Bigg(c + \frac{h(1+m)}{2}\Bigg)\Bigg\{ln\Bigg(\frac{1+m}{1+m-T}\Bigg) \\ &\quad -\frac{T}{1+m-T}\Bigg\} - \frac{hDT^2}{4} - \frac{sDI_e\alpha T^2}{2} \\ &\quad -\frac{cDI_c(T+N-M)T^2}{1+m-T} - \frac{cDI_c(1-\alpha)T^2}{2} \\ &\quad +cDI_c(N-M)\Bigg\{(1+m)ln\Bigg(\frac{1+m}{1+m-T}\Bigg) - T\Bigg\}\Bigg] \qquad (3.59)\end{aligned}$$

Using the similar arguments as in Case 1, there exists unique value of T (say $T_{21} \in (0, M]$) such that $\frac{\partial TP_{21}(s,T)}{\partial T} = 0$. Hence we can easily obtain the following lemma.
For convenience let

$$\begin{aligned}\triangle_5 &= A + D(1+m)\Bigg(c + \frac{h(1+m)}{2}\Bigg)\Bigg\{ln\Bigg(\frac{1+m}{1+m-M}\Bigg) - \frac{M}{1+m-M}\Bigg\} \\ &\quad -\frac{hDM^2}{4} - \frac{sDI_e\alpha M^2}{2} - \frac{cDI_c(1-\alpha)M^2}{2} - \frac{cDI_cM^2N}{1+m-M} \\ &\quad +cDI_c(N-M)\Bigg[(1+m)ln\Bigg(\frac{1+m}{1+m-M}\Bigg) - M\Bigg] \qquad (3.60)\end{aligned}$$

Lemma 7: *Let T^*_{21} denotes the optimal value of $T \in (0, M]$. For fixed s, the profit function $TP_{21}(s,T)$ is concave and reaches its global maximum at point $T = T^*_{21}$.*
Proof: From the above discussion, T^*_{21} which maximizes profit function $TP_{21}(s,T)$ for fixed s is given by

$$T^*_{21} = \begin{cases} T_{21} & \text{if} \quad \triangle_5 \leq 0 \\ M & \text{if} \quad \triangle_5 > 0 \end{cases} \qquad (3.61)$$

$$\Bigg[\frac{\partial^2 TP_{21}(s,T)}{\partial T^2}\Bigg]_{T=T^*_{21}} = \frac{F'_{21}(T^*_{21})}{T^{*2}_{21}} - \frac{2F_{21}(T^*_{21})}{T^{*3}_{21}} < 0, \text{ since } F'_{21}(T^*_{21}) < 0,\ F_{21}(T^*_{21}) \geq 0$$

where

$$\begin{aligned}F_{21}(T) &= A + D(1+m)\Bigg(c + \frac{h(1+m)}{2}\Bigg)\Bigg\{ln\Bigg(\frac{1+m}{1+m-T}\Bigg) - \frac{T}{1+m-T}\Bigg\} \\ &\quad -\frac{hDT^2}{4} - \frac{sDI_e\alpha T^2}{2} - \frac{cDI_c(T+N-M)T^2}{1+m-T} - \frac{cDI_c(1-\alpha)T^2}{2} \\ &\quad +cDI_c(N-M)\Bigg[(1+m)ln\Bigg(\frac{1+m}{1+m-T}\Bigg) - T\Bigg] \qquad (3.62)\end{aligned}$$

$$\begin{aligned} F'_{21}(T) &= \frac{dF_{21}(T)}{dT} \\ &= -DT\Bigg[(1+m)\Big(c+\frac{h(1+m)}{2}\Big)\frac{1}{(1+m-T)^2}+\frac{h}{2} \\ &\quad -sI_e\alpha + cI_c\Big\{(1-\alpha)+\frac{T(T+N-M)}{(1+m-T)^2}+\frac{3T+N-M}{1+m-T}\Big\}\Bigg] \end{aligned} \tag{3.63}$$

Thus T^*_{21} gives global maximum for the profit function $TP_{21}(s,T)$. This completes the proof.□

On the other hand, for fixed T^*_{21} the first order partial derivative of $TP_{21}(s,T^*_{21})$ with respect to s is given by

$$\frac{\partial TP_{21}(s,T^*_{21})}{\partial s} = \frac{b(\beta_2+\beta_3+\beta_4)}{s^{b+1}} - \frac{(b-1)\beta_1}{s^{-b}} \tag{3.64}$$

where

$$\begin{aligned} \beta_1 &= a\Big[1+I_e\alpha(M-\frac{T^*_{21}}{2})\Big] \\ \beta_2 &= \frac{ac(1+m)}{T^*_{21}}ln\Big(\frac{1+m}{1+m-T^*_{21}}\Big) \\ \beta_3 &= a\Big[\frac{h(1+m)^2}{2T^*_{21}}ln\Big(\frac{1+m}{1+m-T^*_{21}}\Big)+\frac{hT^*_{21}}{4}-\frac{h(1+m)}{2}\Big] \\ \beta_4 &= \frac{acI_c}{T^*_{21}}\Bigg[(T^*_{21}+N-M)\Big[(1+m)ln\Big(\frac{1+m}{1+m-T^*_{21}}\Big)-T^*_{21}\Big] \\ &\quad +(1-\alpha)\Big(\frac{T^*_{21}}{2}-M+N\Big)\Bigg] \end{aligned}$$

Equating equation (3.64) with zero and solving for s (denoted by s^*_{21}) we get

$$s^*_{21} = \frac{b(\beta_2+\beta_3+\beta_4)}{(b-1)\beta_1} \tag{3.65}$$

Furthermore, at $s = s^*_{21}$

$$\Big[\frac{\partial^2 TP_{21}(s,T^*_{21})}{\partial s^2}\Big]_{s=s^*_{21}} = -\frac{(b-1)\beta_1}{s^{*(b+1)}_{21}} < 0, \quad \text{since} \quad \beta_1 > 0 \tag{3.66}$$

Thus s^*_{21} is the global optimal which maximizes the profit function $TP_{21}(s,T^*_{21})$ for fixed T^*_{21}. Hence the following lemma.

Lemma 8: *For fixed $T^*_{21} \in (0,M]$ the profit per unit time $TP_{21}(s,T^*_{21})$ has a unique global*

maximum value at the point $s = s^*_{21}$ *which is shown as in equation (3.65).*
Analogously, for fixed s, the first order partial derivative of $TP_{22}(s,T)$ with respect to T is

$$\begin{aligned}\frac{\partial TP_{22}(s,T)}{\partial T} &= \frac{1}{T^2}\Bigg[A + D(1+m)\Big(c+\frac{h(1+m)}{2}\Big)\Big\{ln\Big(\frac{1+m}{1+m-T}\Big) \\ &-\frac{T}{1+m-T}\Big\} - \frac{hDT^2}{4} - \frac{sDI_e\alpha M^2}{2} \\ &-\frac{cDI_c(T+N-M)T(1+m)}{1+m-T} - \frac{cDI_c(1-\alpha)M^2}{2} \\ &+cDI_c(N-M)(1+m)ln\Big(\frac{1+m}{1+m-T}\Big) \\ &+\frac{cDI_cM}{2}(1+m+M-N)\Bigg] \end{aligned} \tag{3.67}$$

Using the similar arguments as in Case 1, there exists unique value of T ($T_{22} \in [M,\infty)$) such that $\frac{\partial TP_{22}(s,T)}{\partial T} = 0$. Hence we can easily obtain the following lemma.
For convenience let

$$\begin{aligned}\triangle_6 &= A + D(1+m)\Big(c+\frac{h(1+m)}{2}\Big)\Big\{ln\Big(\frac{1+m}{1+m-M}\Big) - \frac{M}{1+m-M}\Big\} \\ &-\frac{hDM^2}{4} - \frac{sDI_e\alpha M^2}{2} - \frac{cDI_cMN(1+m)}{1+m-M} - \frac{cDI_c(1-\alpha)M^2}{2} \\ &+cDI_c(N-M)(1+m)ln\Big(\frac{1+m}{1+m-M}\Big) \\ &+\frac{cDI_cM}{2}(1+m+M-N) \end{aligned} \tag{3.68}$$

Lemma 9: *Let* T^*_{22} *denotes the optimal value of* $T \in [M,\infty)$. *For fixed* s, *the profit function* $TP_{22}(s,T)$ *is concave and reaches its global maximum at point* $T = T^*_{22}$.
Proof: From the above discussion, T^*_{22} which maximizes profit function $TP_{22}(s,T)$ for fixed s is given by

$$T^*_{22} = \begin{cases} T_{22} & \text{if} \quad \triangle_6 \geq 0 \\ M & \text{if} \quad \triangle_6 < 0 \end{cases} \tag{3.69}$$

$$\left[\frac{\partial^2 TP_{22}(s,T)}{\partial T^2}\right]_{T=T^*_{22}} = \frac{F'_{22}(T^*_{22})}{T^{*2}_{22}} - \frac{2F_{22}(T^*_{22})}{T^{*3}_{22}} < 0, \text{ since } F'_{22}(T^*_{22}) < 0,\ F_{22}(T^*_{22}) \geq 0$$

where

$$\begin{aligned} F_{22}(T) &= A + D(1+m)\left(c + \frac{h(1+m)}{2}\right)\left\{ln\left(\frac{1+m}{1+m-T}\right) - \frac{T}{1+m-T}\right\} \\ &-\frac{cDI_c(T+N-M)T(1+m)}{1+m-T} - \frac{hDT^2}{4} - \frac{sDI_e\alpha M^2}{2} \\ &-\frac{cDI_c(1-\alpha)M^2}{2} + cDI_c(N-M)(1+m)ln\left(\frac{1+m}{1+m-T}\right) \\ &+\frac{cDI_cM}{2}(1+m+M-N) \end{aligned} \tag{3.70}$$

$$\begin{aligned} F'_{22}(T) &= \frac{dF_{22}(T)}{dT} \\ &= -DT\left[(1+m)\left(c + \frac{h(1+m)}{2}\right)\frac{1}{(1+m-T)^2} + \frac{h}{2}\right. \\ &\left.+\frac{cI_c(1+m)T}{(1+m-T)^2}\{(T+N-M) + 2(1+m-T)\}\right] \end{aligned} \tag{3.71}$$

Thus T^*_{22} gives global maximum for the profit function $TP_{22}(s,T)$. This completes the proof.□

On the other hand, for fixed T^*_{22} the first order partial derivative of $TP_{22}(s,T^*_{22})$ with respect to s is given by

$$\frac{\partial TP_{22}(s,T^*_{22})}{\partial s} = \frac{b(\eta_2+\eta_3+\eta_4)}{s^{b+1}} - \frac{(b-1)\eta_1}{s^{-b}} \tag{3.72}$$

where

$$\begin{aligned} \eta_1 &= a\left[1 + \frac{I_e\alpha M^2}{2T^*_{22}}\right] \\ \eta_2 &= \frac{ac(1+m)}{T^*_{22}}ln\left(\frac{1+m}{1+m-T^*_{22}}\right) \\ \eta_3 &= a\left[\frac{h(1+m)^2}{2T^*_{22}}ln\left(\frac{1+m}{1+m-T^*_{22}}\right) + \frac{hT^*_{22}}{4} - \frac{h(1+m)}{2}\right] \\ \eta_4 &= \frac{acI_c}{T^*_{22}}\left[(1+m)(T^*_{22}+N-M)ln\left(\frac{1+m}{1+m-T^*_{22}}\right) - \frac{(T^*_{22}-M)(1+m+M)}{2}\right. \\ &\left.-\frac{N(T^*_{22}+M)}{2} + (1-\alpha)\left(NT^*_{22} - \frac{M^2}{2}\right)\right] \end{aligned}$$

Equating equation (3.72) with zero and solving for s (denoted by s^*_{22}) we get

$$s^*_{22} = \frac{b(\eta_2+\eta_3+\eta_4)}{(b-1)\eta_1} \tag{3.73}$$

Furthermore, at $s = s_{22}^*$

$$\left[\frac{\partial^2 TP_{22}(s, T_{22}^*)}{\partial s^2}\right]_{s=s_{22}^*} = -\frac{(b-1)\eta_1}{s_{22}^{*(b+1)}} < 0, \text{ since } \eta_1 > 0 \tag{3.74}$$

Thus s_{22}^* is the global optimal which maximizes the profit function $TP_{22}(s, T_{22}^*)$ for fixed T_{22}^*. Hence the following lemma.
Lemma 10: *For fixed $T_{22}^* \in [M, \infty)$ the profit per unit time $TP_{22}(s, T_{22}^*)$ has a unique global maximum value at the point $s = s_{22}^*$ which is shown as in equation (3.73).*

3.5 Solution methodology: A Routine Framework for GA

The numerical experiment of the proposed model has been done using the heuristic search method say Genetic Algorithm (GA). The discussion about the Genetic Algorithm process and algorithms are given in the section-2.2.4.
At the beginning of the GA module, the different parameters of GA i.e. generation number (MAXGEN), population size (POPSIZE), probability of cross-over (PXOVER), probability of mutation (PMUT), random seed (RSEED), distribution index for SBX (DISBX) and for mutation (DIMUT) and the others input data have to be supplied. As there is no clear indication as to how large a population should be, here with POPSIZE = no of variables$\times 12$, the expected result is obtained. Here the chromosomes are structured by some real numbers, where a chromosome is a string of genes which are specified by the decision variables of the problem namely- the unit selling price (s) and the length of the business period (T). The variable boundaries may be fixed or flexible. The fitness function is the profit function ($TP(s, T)$) given by equation (3.34).

3.6 Numerical Experiment

The optimum results are obtained and some sensitivity analyses for the model have been performed using the solution methodology given by sec. -3.5. To illustrate the problem numerically, the following input data are used.

3.6.1 Input Data

An inventory system is considered with the following parametric values.

$D = as^{-b}$ units/year where, $a = 5 \times 10^6$, $b = 2.3$, $A = \$130/\text{order}$, $h = \$7/\text{unit/year}$, $c = \$15/\text{unit}$, $I_c = 15\%/\text{year}$, $I_e = 12\%/\text{year}$, $m = 1$ year, $\alpha = 0.5$.

GA parameters: To find the numeric solution of the model, the above mentioned Gentic algorithm is used with corresponding parameters as follows:

POPSIZE=50 MAXGEN=200, PXOVER=0.8, PMUT=0.2, RSEED=1.2, DISBX=2, DIMUT=100, T-(0.0001 to 2), s-(10 to 40), M-($10/365$ to $60/365$), and N-(15 to 60).

3.6.2 Optimum Result

The optimum result of the model for the given parametric values are

(i) Optimum value of M = $60/365$ year and N = $15/365$ year.

(ii) Total business length T^* = 0.103 year.

(iii) Optimum quantity Q^* = 256.75 unit.

(iv) Optimum value of selling price s^* = $ 27.59/unit.

(v) Average interest earned (AIE) = $ 7206.27.

(vi) Average interest paid (AIP) = $ 0.0.

(vii) Average stock holding cost (AHC) = $ 890.93

(viii) Optimum average profit TP^* = $ 30710.16.

A sensitivity analysis has been performed for different values of $M, \; N$ and given in Table-3.1.

3.7 Discussion

Based on the above computational results, the following managerial insights can be obtained.
• It is observed that for fixed α and M, when customer credit period N increases, the optimal retail price (s^*) and optimal cycle time (T^*) increase, whereas the optimal total profit per unit time ($TP^*(s^*, T^*)$) decreases. These results imply that a longer delay in payment period provided by the retailer leads to lower demand rate and higher retail price. It may be interesting to observe that when $\alpha = 0.5$, the optimal replenishment cycle time increases as long $M \leq N$ and then decreases subsequently.

Table 3.1: *Sensitivity analysis for* $\alpha = 0.5$

M	N	T^*	s^*	Q^*	AIE	AIP	TP^*
60	10	0.0978	27.4836	245.52	8408.18	0.0	30950.28
	15	0.0979	27.5118	245.35	7819.8	0.0	30891.37
	30	0.0987	27.5904	245.75	559.88	6.072	30696.52
	45	0.1002	27.695	247.42	490.78	55.5	30493.02
	60	0.1009	27.789	247.26	453.67	149.7	30341.16
45	10	0.0983	27.6335	243.85	484.96	0.414	30601.11
	15	0.099	27.6663	244.96	432.15	6.21	30520.94
	30	0.1005	27.7678	246.68	324.61	55.57	30318.45
	45	0.1012	27.8659	246.44	288.4	149.23	30167.46
	60	0.1015	27.9536	245.41	286.63	263.29	30042.03
30	10	0.1010	29.1874	221.08	180.46	1694.05	28433.88
	15	0.1015	29.2253	221.54	115.13	1735.6	28347.73
	30	0.1022	29.3659	220	122.49	1823.5	28188.42
	45	0.0847	27.9778	203	157.4	1715.35	30210.67
	60	0.0847	28.0678	201.99	156.75	2404.48	30101.82
15	10	0.1036	32.5214	176.9	29.39	4368.56	24776.3
	15	0.1038	32.5479	176.96	26.38	4370.5	24758.24
	30	0.1035	32.5743	176.11	26.42	4633.36	24753.29
	45	0.1039	32.5969	176.53	26.3	4826.42	24713.27
	60	0.1043	32.63	176.81	26.16	5013.47	24673.71
10	10	0.1041	33.6233	164.7	11.21	4981.87	23775.08
	15	0.1040	33.6258	164.5	11.21	5052.35	23777.01
	30	0.1050	33.666	165.68	11.09	5163.1	23732.9
	45	0.1048	33.666	165.36	11.11	5271.47	23734.81
	60	0.1052	33.6746	165.91	11.07	5383.32	23714.07

• For fixed value of N and α, it can be noted that the optimal retail price and the optimal length of replenishment cycle decrease, whereas per unit time retailer's profit ($TP^*(s^*, T^*)$) increases with an increase in retailers credit period M. These results imply that a longer credit period provided by the supplier may ultimately cause the retailers to shorten the replenishment cycle length to take advantage of trade credit frequently.
• With an increase in the value of the of the parameter α, the optimal retail price and the optimal length of replenishment cycle decrease, whereas retailer's profit per unit time increases. These results indicate that if the amount of the part payment of purchase cost is more, then the demand can be stimulated by reducing the retail price and thereby the retailer can raise the profit.

3.8 Conclusion

Here a supplier-retailer inventory system for deteriorating items with expiration date, price-dependent demand and two-level trade credit has been developed and the corresponding mathematical problem has been solved. A sensitivity analysis with different trade credit values is performed.

The presented model can be extended to formulate a SCM with supplier-wholesaler-retailer and to consider three level trade-credit offered by supplier, wholesaler and retailer. Moreover, a new policy for the payment of dues can be applied. In this policy, retailer clears all his / her dues as and when he / she has the required amount.

Part III

Inventory Models in Random, Fuzzy and Fuzzy-Random Environment

Chapter 4

Two layers supply chain imperfect production inventory model with two level credit period and production rate dependent defective rate

4.1 Introduction

Business organizations all over the world are striving hard to evolve strategies to survive in the area of competition ushered in by globalization. Supply chain management (SCM) is one such strategy. It is an effective methodology and presents an integrated approach to resolve issues in sourcing customer service, demand flow and distribution. The focus is on the customer. The results are in the form of reduced operational costs, improved flow of supplies, reduction in delays of production and increased customer satisfaction. While the goal of supply chain management is to reduce cost of producing and reaching the finished products to the customers, inventory control is the means to achieve the goal. Researchers as well as practitioners in manufacturing industries have given importance to develop inventory control problems in supply chain management. All steps from supply of raw materials to finished products can be included into a supply chain, connecting raw materials supplier, manufacturer, retailer and finally customers. Recent reviews on supply chain management are provided by Weng [252], Munson and Rosenblatt [187], Yang and Wee [264], Khouja [133], Yao et al. [271], Chaharsooghi et al. [37], Wang et al. [249] and others.

Now-a-days, with the advent of multi-nationals in the developing countries, there is a stiff competition amongst the multi- nationals to capture the market. Thus in the recent competitive market, the inventory/stock is decoratively exhibited and colorably displayed through electronic media to attract the customers and thus to boost the sale. For this reason, Datta and Pal [73], Mandal and Phaujder [174], Mondal and Maiti [175] and others considered linear form of stock dependent demand. Salameh and Jaber [215] developed an inventory model with such demand for imperfect quality items using the EPQ/EOQ formula and assumed that inferior quality items are sold as a single batch at the end of the total screening process. Thereafter, Goyal and Cardenas-Barron [93] extended the idea of Salameh and Jaber's [215] model and proposed a practical approach to determine EPQ for items with imperfect quality. Yu et al. [272] generalized the models of Salameh and Jaber [215], incorporating deterioration and partial back ordering. Liu and Yang [152] investigated a single stage production system with imperfect process delivering two types of defects: reworkable and non-reworkable items. The reworkable items are sent for reworking, whereas non-reworkable items are immediately discarded from the system. They determined the optimal lot size that maximized the expected total profit over the expected time length of the production cycle. Ma et al. [157] considered the effects of imperfect production processes and the decision on whether and when to implement a screening process for defective items generated during a production run. Sana [219] developed an economic production lot size model in an imperfect production system.

Naturally, inventory models are based on the assumption that the retailer pays for the item immediately after the units are received. However, it may not be true for today's competitive business transactions. Now-a-days, it is normally found that the supplier allows a certain fixed time period (termed as credit period) to his/her retailers for settling the account that the retailer owes to the supplier for the item supplied. Goyal [93] developed an EOQ model under the conditions of permissible delay in payments. Chung and Liao [64] studied a lot-sizing problem under a supplier's trade credit depending on the retailer's order quantity. Manna [177] considered a three-layer supply chain in an imperfect production inventory model with two storage facilities under fuzzy rough environment. In recent business environment, trade credit is used as a tool by a supplier to encourage the retailer to procure a greater volume of goods, to earn a reasonable profit. On the other hand, trade credit offers a lower unit purchasing cost as well as representing an important source of short-term external finance for retailers.

This study differs from the others in the following view.

1. The production system, considered here undergoes an out-of-control (starting point of imperfect production) state from an in-control state. A mixture of perfect and imperfect quality items are produced by the manufacturer where the defective rate is random which follows an uniform distribution.

2. This is a supply chain model in which the manufacturer deals with more than one retailer with unknown production rate.

3. Partial trade credit sharing is considered in this model.

4. The rework process is one of common criterion to control the out-of-control state. In

this study rework process is applied and some non-repairable items are sold with reduced price.

5. The demand of the customer is considered as a function of retailer's stock.

In this chapter, a single period SCM model is considered with a manufacturer and n retailers. A mixture of perfect and imperfect quality items are produced by the manufacturer and supplies those to the retailers. Manufacturer also offers a period of delay in payment to the retailers. The defective rate is random in nature which follows uniform distribution and certain percentage of defective items is reworked as a perfect quality items. Each retailer shares his/her delay period with his/her customers partially. Set-up cost of the manufacturer is linearly dependent on the production rate. Also the open end customer demand at each of the retailers depends on the displayed stock of the retailer. The whole model is formulated as profit maximization problem to maximize the SCM profit. The model is solved numerically using GRG method using Lingo-12.0 software.

The rest of this chapter is organized as follows. Section-4.2 contains all the notations and assumptions which are used to formulate the model. The formulation of the model is discussed in Section-4.3. Section-4.4 contains the solution methodology which is used to solve the model numerically. In section-4.5 a numerical experiment has been made with some practical data and the optimal result is given in tabular form. Section-4.6 and 4.7 contains some discussions and conclusions on the model respectively.

4.2 Notations and Assumptions

To develop the model, the following notations and assumptions have been used to develop the proposed model.

4.2.1 Notations:

For convenience, the following notations are used to formulate the model.

$q_m(t)$	inventory level at any time t of perfect quality items for the manufacturer.
$q_r(t)$	inventory level at any time t of perfect quality items for the retailer.
P	production rate in units.
θ	defective rate.
δ	percentage of rework of defective units per unit time.
d_r	selling rate of perfect quality items of the manufacturer, where, $d_r = \sum_{i=1}^{n} d_r^i$.
d_r^i	demand rate of perfect quality items of the $i-th$ retailer.

$q_r^i(t)$	inventory position of $i-th$ retailer.
$d_c^i(t) = d_0^i + d_1^i q_r^i(t)$	demand rate of perfect quality items of the customers of $i-th$ retailer.
t_1	production run-time in one period.
t_2	time of zero sock situation of the retailer.
M	time length of credit period of retailers, offered by the manufacturer.
N^i	time length of the customers offered by $i-th$ retailer.
T^i	time at which the selling season ends for $i-th$ retailer.
n	number of retailers where the item is sold.
I_{em}	rate of interest per year earned manufacturer from retailer.
I_{er}	rate of interest per year earned retailer from customer.
s_c	screening cost per unit item.
$A_m = A_{m0} + A_{m1}P$	set up cost of manufacturer.
h_m	holding cost per unit for per unit time for perfect item in manufacturer.
r_{cm}	reworking cost per unit for manufacturer.
c_p	production cost per unit.
s_m	selling price per unit of perfect quality items for manufacturer.
A_r^i	set up cost of the $i-th$ retailer.
h_r^i	holding cost per unit for per unit time of perfect quality items of the $i-th$ retailer.
s_r^i	selling price per unit of perfect quality items of the $i-th$ retailer.

4.2.2 Assumptions:

The problem is constructed under the following considerations.

i) A manufacturer produces a mixture of perfect and imperfect quality items in single period business. Some portion of imperfect items are reworked as a perfect quality items.

ii) The defective rate θ is a random variable which follows uniform distribution.

iii) The whole model is formulated for a finite time horizon.

iv) The period length is different for different retailers.

v) Production rate (P) is a decision variable.

vi) Set up cost of manufacturer has been considered as a linear function of production rate.

vii) It is assumed that the length of credit period (M) offered by manufacturer must be less than the cycle length T^i of the $i-th$ retailer i.e. $M < T^i$ for $i = 1, 2, \cdots, n$.

viii) The length of the credit period N^i offered by $i-th$ retailer to the end customers must be less than M.

ix) The customer demand d_c^i, faced by $i-th$ retailer is a linear function of stock level $q_r^i(t)$ of himself/herself.

4.3 Model Formulation

4.3.1 Mathematical formulation for manufacturer

The rate of change of inventory level of manufacturer [c.f. 4.1] for perfect quality items can be represented by the following differential equations:

$$\frac{dq_m}{dt} = \begin{cases} P - d_r - (1-\delta)\theta P, & 0 \le t \le t_1 \\ -d_r, & t_1 \le t \le t_2 \end{cases} \tag{4.1}$$

with boundary conditions $q_m(0) = 0, \ q_m(t_2) = 0$.
The solution of the differential equations (4.1) are given by

$$q_m(t) = \begin{cases} \{P - d_r - (1-\delta)\theta P\}t, & 0 \le t \le t_1 \\ -d_r(t - t_2), & t_1 \le t \le t_2 \end{cases}$$

From the continuity conditions of $q_m(t)$ at $t = t_1$, the following is obtained,

$$t_2 = \frac{1}{d_r}\{P - (1-\delta)\theta P\}t_1$$

Inventory holding cost for perfect items is:

$$\begin{aligned} HCM &= h_m\Big[\int_0^{t_1} q_m(t)\,dt + \int_{t_1}^{t_2} q_m(t)\,dt\Big] \\ &= h_m\Big[\int_0^{t_1} \{P - d_r - (1-\delta)\theta P\}t\ dt + \int_{t_1}^{t_2} \{-d_r(t-t_2)\}\ dt \\ &= \frac{h_m}{2}\Big[\{P - d_r - (1-\delta)\theta P\}t_1^2 + d_r(t_1 - t_2)^2\Big] \end{aligned} \tag{4.2}$$

Production cost for the manufacturer= c_pPt_1.
Inspection cost = s_cPt_1.
Reworking cost for manufacture = $r_{cm}\int_0^{t_1}\delta\theta P\,dt = r_{cm}\delta\theta Pt_1$
Set up cost of the manufacturer=A_m.
Revenue of perfect quality items for the manufacturer = $s_md_rt_2$.
Disposal cost during $(0, t_2)$ = $d_c(1-\delta)\theta Pt_1$

Total expected profit TEP_m of manufacturer during the period $(0, t_2)$ is given by

$$\begin{aligned} TEP_m &= s_m d_r t_2 - (c_p + s_c)Pt_1 - r_{cm}\delta\theta Pt_1 - d_c(1-\delta)\theta Pt_1 \\ &\quad -\frac{h_m}{2}\Big[\{P - d_r - (1-\delta)\theta P\}t_1^2 + d_r(t_1 - t_2)^2\Big] - A_m \end{aligned} \tag{4.3}$$

4.3.2 Formulation for $i - th$ retailer

The $i - th$ retailer receives his/her required quantity per unit time from the manufacturer and fulfill the customers' demand rate d_c^i. Those retailers start their business on or before the production run time t_1, pay r portion of the price amount payable initially and the remaining $(1 - r)$ portion pay at the end of his/her business period. But those retailers arrive after the production run time t_1, pay the total amount at their business starting time. They pay the initial amount by getting loan from a bank at the rate of interest of I_p per year. Every retailer's earns interest at the rate of I_e by depositing sales revenue continuously. The inventory level $q_r^i(t)$ [c.f. 4.1] for the $i - th$ retailer's is governed by the following differential equation.

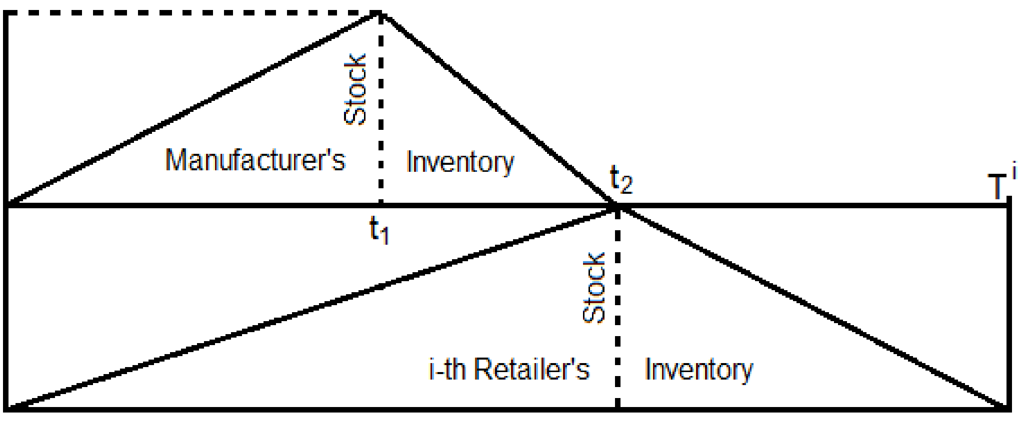


Figure 4.1: Inventory Policy for perfect quality items

$$\frac{dq_r^i(t)}{dt} = \begin{cases} (d_r^i - d_c^i), & 0 \le t \le t_2 \\ -d_c^i, & t_2 \le t \le T^i \end{cases}$$

with boundary conditions $q_r^i(0) = 0$ and $q_r^i(T^i) = 0$.
The customer demand is $d_c^i(t) = d_0^i + d_1^i q_r^i(t)$, where $d_0^i > 0$ and $0 < d_1^i < 1$.
Therefore the solutions of above differential equations are given by

$$q_r^i(t) = \begin{cases} \frac{(d_r^i - d_0^i)}{d_1^i}(1 - e^{-d_1^i t}), & 0 \le t \le t_2 \\ -\frac{d_0^i}{d_1^i}[1 - e^{-d_1^i(t - T^i)}], & t_2 \le t \le T^i \end{cases}$$

From the continuity conditions of $q_r^i(t)$ at $t = t_2$, we have
$T^i = t_2 + \frac{1}{d_1^i} log\{1 + \frac{(d_r^i - d_0^i)}{d_0^i}(1 - e^{-d_1^i t_2})\}$

Holding cost of the $i - th$ retailer is given by

$$\begin{aligned} HCR^i &= h_r^i \Big[\int_0^{t_2} q_r^i(t)\, dt + \int_{t_2}^{T^i} q_r^i(t)\, dt \Big] \\ &= h_r^i \Big[\frac{(d_r^i - d_0^i)}{d_1^i} \Big\{ t_2 + \frac{1}{d_1^i}(e^{-d_1^i t_2} - 1) \Big\} - \frac{d_0^i}{d_1^i} \Big\{ (T^i - t_2) + \frac{1}{d_1^i}\{1 - e^{-d_1^i(t_2 - T^i)}\} \Big\} \Big] \end{aligned}$$

Holding cost (HCR) for all retailers' is given by

$$\begin{aligned} HCR &= \sum_{i=1}^{n} h_r^i \Big[\frac{(d_r^i - d_0^i)}{d_1^i} \Big\{ t_2 + \frac{1}{d_1^i}(e^{-d_1^i t_2} - 1) \Big\} - \frac{d_0^i}{d_1^i} \Big\{ (T^i - t_2) \\ &\quad + \frac{1}{d_1^i}\{1 - e^{-d_1^i(t_2 - T^i)}\} \Big\} \Big] \end{aligned} \tag{4.4}$$

Sales revenue from perfect quality items of the $i - th$ retailer is given by

$$\begin{aligned} SRR^i &= s_r^i \Big[\int_0^{t_2} (d_0^i + d_1^i q_r^i(t))\, dt + \int_{t_2}^{T} (d_0^i + d_1^i q_r^i(t))\, dt \Big] \\ &= s_r^i \Big[d_0^i T^i + (d_r^i - d_0^i) \Big\{ t_2 + \frac{1}{d_1^i}(e^{-d_1^i t_2} - 1) \Big\} - d_0^i \Big\{ (T^i - t_2) \\ &\qquad + \frac{1}{d_1^i}\{1 - e^{-d_1^i(t_2 - T^i)}\} \Big\} \Big] \\ &= s_r^i \Big[d_r^i t_2 + \frac{(d_r^i - d_0^i)}{d_1^i}(e^{-d_1^i t_2} - 1) - \frac{d_0^i}{d_1^i}\{1 - e^{-d_1^i(t_2 - T^i)}\} \Big] \end{aligned}$$

All retailers' total sales revenue (SRR) is given by

$$SRR = \sum_{i=1}^{n} s_r^i \Big[d_r^i t_2 + \frac{(d_r^i - d_0^i)}{d_1^i}(e^{-d_1^i t_2} - 1) - \frac{d_0^i}{d_1^i}\{1 - e^{-d_1^i(t_2 - T^i)}\} \Big] \tag{4.5}$$

All retailers' total purchase cost (PCR) is given by

$$PCR = \sum_{i=1}^{n} s_m d_r^i t_2 \tag{4.6}$$

Here it is assumed that the retailers trade credit period offered by the manufacturer is M and that of customers offered by the retailer is N^i (where $N^i < M$). The retailer is charged by the manufacturer, an interest at the rate of I_p per year per unit for the unpaid amount after the delay period and can earn an interest at the rate of I_e ($I_e > I_p$) per year per unit for the amount sold during the period (N^i, M) respectively. Depending on the cycle times of the retailer and offering as well as receiving credit periods, three different cases may arise, which have been discussed separately.

Case-1 $(N^i < M < t_2 < T^i)$: Interest paid by the $i-th$ retailer (IP_1^i) is given by

$$\begin{aligned}
IP_1^i &= s_m I_p \int_M^{T^i} q_r^i(t)\,dt \\
&= s_m I_p\Big[\int_M^{t_2} q_r^i(t)\,dt + \int_{t_2}^{T^i} q_r^i(t)\,dt\Big] \\
&= s_m I_p\Big[\frac{(d_r^i - d_0^i)}{d_1^i}\Big\{(t_2 - M) + \frac{1}{d_1^i}(e^{-d_1^i t_2} - e^{-d_1^i M})\Big\} - \frac{d_0^i}{d_1^i}\Big\{(T^i - t_2) \\
&\qquad + \frac{1}{d_1^i}\{1 - e^{-d_1^i(t_2 - T^i)}\}\Big\}\Big]
\end{aligned}$$

Interest earned by the $i-th$ retailer (IE_1^i),

$$\begin{aligned}
IE_1^i &= s_r^i I_e\Big[(T^i - N^i)\int_0^{N^i} d_c^i(t)\,dt + (T^i - M)\int_{N^i}^{M} (M - t) d_c^i(t)\,dt \\
&\qquad + (T^i - t_2)\int_M^{t_2} (t_2 - t) d_c^i(t)\,dt + \int_{t_2}^{T^i} (T^i - t) d_c^i(t)\,dt\Big] \\
&= s_r^i I_e\Big[(T^i - N^i)\Big\{d_0^i N^i + (d_r^i - d_0)\Big\{N + \frac{1}{d_1^i}(e^{-d_1^i N^i} - 1)\Big\}\Big\} \\
&\quad + (T^i - M)\Big\{\frac{d_r^i}{2}(M - N^i)^2 - (M - N^i)\frac{e^{-d_1^i N^i}}{d_1^i} + \frac{1}{(d_1^i)^2}(e^{-d_1^i N^i} - e^{-d_1^i M})\Big\} \\
&\quad + (T^i - t_2)\Big\{\frac{d_r^i}{2}(M - t_2)^2 - (t_2 - M)\frac{e^{-d_1^i M}}{d_1^i} + \frac{1}{(d_1^i)^2}(e^{-d_1^i M} - e^{-d_1^i t_2})\Big\} \\
&\quad + \frac{d_0^i}{d_1^i}(T^i - t_2)e^{-d_1^i(t_2 - T^i)} + \frac{d_0^i}{(d_1^i)^2}\{1 - e^{-d_1^i(t_2 - T^i)}\}\Big]
\end{aligned}$$

All retailers' total interest payable (IP_1) is expressed as

$$\begin{aligned}
IP_1 &= \sum_{i=1}^{n} s_m I_p\Big[\frac{(d_r^i - d_0^i)}{d_1^i}\Big\{(t_2 - M) + \frac{1}{d_1^i}(e^{-d_1^i t_2} - e^{-d_1^i M})\Big\} - \frac{d_0^i}{d_1^i}\Big\{(T^i - t_2) \\
&\qquad + \frac{1}{d_1^i}\{1 - e^{-d_1^i(t_2 - T^i)}\}\Big\}\Big] \qquad (4.7)
\end{aligned}$$

All retailers' total interest earned (IE_1) is obtained as

$$\begin{aligned} IE_1 &= \sum_{i=1}^{n} s_r^i I_e \Big[(T^i - N^i)\Big\{d_0^i N^i + (d_r^i - d_0)\Big\{N + \frac{1}{d_1^i}(e^{-d_1^i N^i} - 1)\Big\}\Big\} \\ &+(T^i - M)\Big\{\frac{d_r^i}{2}(M - N^i)^2 - (M - N^i)\frac{e^{-d_1^i N^i}}{d_1^i} + \frac{1}{(d_1^i)^2}(e^{-d_1^i N^i} - e^{-d_1^i M})\Big\} \\ &+(T^i - t_2)\Big\{\frac{d_r^i}{2}(M - t_2)^2 - (t_2 - M)\frac{e^{-d_1^i M}}{d_1^i} + \frac{1}{(d_1^i)^2}(e^{-d_1^i M} - e^{-d_1^i t_2})\Big\} \\ &+\frac{d_0^i}{d_1^i}(T^i - t_2)e^{-d_1^i(t_2 - T^i)} + \frac{d_0^i}{(d_1^i)^2}\{1 - e^{-d_1^i(t_2 - T^i)}\}\Big] \end{aligned} \tag{4.8}$$

Therefore, all retailers' total profit is given by

$$TEP_r^1(P, t_1) = SRR - PCR - HCR - IP_1 + IE_1 - \sum_{i=1}^{n} A_r^i \tag{4.9}$$

where, SRR, PCR, HCR, IP_1 and IE_1 are given equations (4.5), (4.6), (4.4), (4.7) and (4.8) respectively. So, the total profit (ITP) for this case of the integrated system is written as

$$E[ITP_1(P, t_1)] = E[TEP_m(P, t_1)] + E[TEP_r^{(1)}(P, t_1)] \tag{4.10}$$

where, $TEP_m(P, t_1)$, $TEP_r^{(1)}(P, t_1)$ are given by equation (4.3) and (4.9) respectively.

Case-2 $N^i < t_2 < M < T^i$: Interest paid by the retailer (IP_2^i),

$$\begin{aligned} IP_2^i &= s_m I_p \int_M^{T^i} q_r^i(t)\, dt \\ &= s_m I_p \int_M^{T^i} -\frac{d_0^i}{d_1^i}[1 - e^{-d_1^i(t - T^i)}]\, dt \\ &= \frac{d_0^i s_m I_p}{d_1^i}\Big[\frac{1}{d_1^i}\{e^{-d_1^i(M - T^i)} - 1\} - (T^i - M)\Big] \end{aligned}$$

Interest earned by the retailer (IE_2^i),

$$\begin{aligned}
IE_2^i &= s_r^i I_e\Big[(T^i - N^i)\int_0^{N^i} d_c^i(t)\,dt + (T^i - t_2)\int_{N^i}^{t_2}(t_2 - t)d^i{}_c(t)\,dt \\
&\quad + (T^i - M)\int_{t_2}^{M}(M - t)d_c^i(t)\,dt + \int_M^{T^i}(T^i - t)d_c^i(t)\,dt\Big] \\
&= s_r^i I_e\Big[(T^i - N^i)\Big\{d_0^i N^i + (d_r^i - d_0^i)\Big\{N^i + \frac{1}{d_1^i}(e^{-d_1^i N^i} - 1)\Big\}\Big\} \\
&\quad + (T^i - t_2^i)\Big\{\frac{d_r^i}{2}(t_2^i - N^i)^2 - (t_2 - N^i)\frac{e^{-d_1^i N^i}}{d_1^i} + \frac{1}{(d_1^i)^2}(e^{-d_1^i N^i} - e^{-d_1^i t_2})\Big\} \\
&\quad + (T^i - M)\Big\{\frac{d_0^i}{d_1^i}(M - t_2)e^{-d_1^i(t_2 - M)} + \frac{d_0^i}{(d^i)_1^2}\{1 - e^{-d_1^i(t_2 - M)}\}\Big\} \\
&\quad + \frac{d_0^i}{d_1^i}(T^i - M)e^{-d_1^i(M - T^i)} + \frac{d_0}{(d_1^i)^2}\{1 - e^{-d_1^i(M - T^i)}\}\Big]
\end{aligned}$$

All retailers' total interest payable (IP_2) is expressed as

$$IP_2 = \sum_{i=1}^{n}\frac{d_0^i s_m I_p}{d_1^i}\Big[\frac{1}{d_1^i}\{e^{-d_1^i(M - T^i)} - 1\} - (T^i - M)\Big] \tag{4.11}$$

All retailers' total interest earned (IE_2) is obtained as

$$\begin{aligned}
IE_2 &= \sum_{i=1}^{n} s_r^i I_e\Big[(T^i - N^i)\Big\{d_0^i N^i + (d_r^i - d_0^i)\Big\{N^i + \frac{1}{d_1^i}(e^{-d_1^i N^i} - 1)\Big\}\Big\} \\
&\quad + (T^i - t_2^i)\Big\{\frac{d_r^i}{2}(t_2^i - N^i)^2 - (t_2 - N^i)\frac{e^{-d_1^i N^i}}{d_1^i} + \frac{1}{(d_1^i)^2}(e^{-d_1^i N^i} - e^{-d_1^i t_2})\Big\} \\
&\quad + (T^i - M)\Big\{\frac{d_0^i}{d_1^i}(M - t_2)e^{-d_1^i(t_2 - M)} + \frac{d_0^i}{(d^i)_1^2}\{1 - e^{-d_1^i(t_2 - M)}\}\Big\} \\
&\quad + \frac{d_0^i}{d_1^i}(T^i - M)e^{-d_1^i(M - T^i)} + \frac{d_0}{(d_1^i)^2}\{1 - e^{-d_1^i(M - T^i)}\}\Big]
\end{aligned} \tag{4.12}$$

Therefore, all retailers' total profit is given by

$$TEP_r^{(2)}(P, t_1) = SRR - PCR - HCR - IP_2 + IE_2 - \sum_{i=1}^{n} A_r^i \tag{4.13}$$

where, SRR, PCR, HCR, IP_2 and IE_2 are given equations (4.5), (4.6), (4.4), (4.11) and (4.12) respectively.

So, the total profit (ITP) for this case of the integrated system is written as

$$E[ITP_2(P, t_1)] = E[TEP_m(P, t_1)] + E[TEP_r^{(2)}(P, t_1)] \tag{4.14}$$

where, $TEP_m(P,t_1)$, $TEP_r^{(2)}(P,t_1)$ are given by equation (4.3) and (4.13) respectively.

Case 3: $t_2 < N^i < M < T^i$: Interest paid by the retailer (IP_3^i),

$$\begin{aligned} IP_3^i &= s_m I_p \int_M^{T^i} q_r^i(t)\,dt \\ &= s_m I_p \int_M^{T^i} -\frac{d_0^i}{d_1^i}[1-e^{-d_1^i(t-T^i)}]\,dt \\ &= \frac{d_0 s_m I_p}{d_1^i}\Big[\frac{1}{d_1^i}\{e^{-d_1^i(M-T^i)}-1\}-(T^i-M)\Big] \end{aligned}$$

Interest earned by the retailer (IE_3^i),

$$\begin{aligned} IE_3^i &= s_r^i I_e\Big[(T^i-N^i)\Big\{\int_0^{t_2} d_c^i(t)\,dt+\int_{t_2}^{N^i} d_c^i(t)\,dt\Big\}+\int_{N^i}^{T^i}(T^i-t)d_c^i(t)\,dt\Big] \\ &= s_r^i I_e\Big[(T^i-N^i)\Big\{\int_0^{t_2}\{d_0^i+d_1^i q_r^i(t)\}\,dt+\int_{t_2}^{N^i}\{d_0^i+d_1^i q_r^i(t)\}\,dt\Big\} \\ &\qquad +\int_{N^i}^{T}(T^i-t)\{d_0^i+d_1^i q_r^i(t)\}\,dt\Big] \\ &= s_r^i I_e\Big[(T^i-N^i)\Big\{d_0^i N^i+(d_r^i-d_0^i)\{t_2+\frac{1}{d_1^i}(e^{-d_1^i t_2}-1)\}-d_0^i\Big\{(N^i-t_2) \\ &\quad +\frac{1}{d_1^i}\{1-e^{-d_1^i(t_2-N^i)}\}\Big\}\Big\}+\frac{d_0^i}{d_1^i}(T^i-N^i)e^{-d_1^i(N^i-T^i)}+\frac{d_0^i}{(d_1^i)^2}\{1-e^{-d_1^i(N^i-T^i)}\}\Big] \end{aligned}$$

All retailers' total interest payable (IP_3) is expressed as

$$IP_3 = \sum_{i=1}^{n}\frac{d_0 s_m I_p}{d_1^i}\Big[\frac{1}{d_1^i}\{e^{-d_1^i(M-T^i)}-1\}-(T^i-M)\Big] \tag{4.15}$$

All retailers' total interest earned (IE_3) is obtained as

$$\begin{aligned} IE_3 &= \sum_{i=1}^{n} s_r^i I_e\Big[(T^i-N^i)\Big\{d_0^i N^i+(d_r^i-d_0^i)\{t_2+\frac{1}{d_1^i}(e^{-d_1^i t_2}-1)\}-d_0^i\Big\{(N^i-t_2) \\ &\quad +\frac{1}{d_1^i}\{1-e^{-d_1^i(t_2-N^i)}\}\Big\}\Big\}+\frac{d_0^i}{d_1^i}(T^i-N^i)e^{-d_1^i(N^i-T^i)} \\ &\quad +\frac{d_0^i}{(d_1^i)^2}\{1-e^{-d_1^i(N^i-T^i)}\}\Big] \end{aligned} \tag{4.16}$$

Therefore, all retailers' total profit is given by

$$TEP_r^{(3)}(P,t_1) = SRR-PCR-HCR-IP_3+IE_3-\sum_{i=1}^{n}A_r^i \tag{4.17}$$

where, SRR, PCR, HCR, IP_3 and IE_3 are given equations (4.5), (4.6), (4.4), (4.15) and (4.16) respectively.

So, the total profit (ITP) for this case of the integrated system is written as

$$E[ITP_3(P,t_1)] = E[TEP_m(P,t_1)] + E[TEP_r^{(3)}(P,t_1)] \tag{4.18}$$

where, $TEP_m(P,t_1)$, $TEP_r^{(3)}(P,t_1)$ are given by equation (4.3) and (4.17) respectively.

When manufacturer and retailers' have decided to share resources to undertake mutually beneficial cooperation, the joint total profit which is a function of P and t_1 can be obtained by maximized $E[ITP(P,t_1)]$ is given by

$$\text{Maximize } E[ITP(P,t_1)] = \begin{cases} E[ITP_1(P,t_1)], & \text{if } N^i < M < t_2 < T^i \\ E[ITP_2(P,t_1)], & \text{if } N^i < t_2 < M < T^i \\ E[ITP_3(P,t_1)], & \text{if } t_2 < N^i < M < T^i \end{cases} \tag{4.19}$$

where, $ITP_1(P,t_1)$, $ITP_2(P,t_1)$ and $ITP_3(P,t_1)$ are given by equation (4.10), (4.14) and (4.18) respectively.

4.4 Solution Methodology

The proposed non-linear problem is solved by a gradient based non-linear optimization method- Generalized Reduced Gradient (GRG) method [c.f. sec.-2.2.2] using LINGO Solver 12.0 for a given set of particular input data.

4.5 Numerical Experiment

4.5.1 Input Data

To verify the model numerically some realistic data are collected from rice mill with two of its retailer which are given by the following.

For Manufacturer: $A_{m0} = \$48$, $A_{m1} = \$0.05$, $c_p = \$24$ / unit, $d_c = \$0.6$ / unit, $h_m = \$0.90$, $s_c = \$0.80$ / unit, $s_m = \$89$ / unit, $r_{cm} = \$2$ / unit, $\delta = 0.70$, $I_{em} = 12\%$ / year.

The defective rate θ is a random variable which follows uniform distribution with corresponding probability density function as

$$\begin{aligned} f(x) &= \frac{1}{b-a}, \quad \text{for } a < x < b \\ &= 0, \quad \text{elsewhere. Here } a = 2,\ b = 4.5 \end{aligned}$$

For 1st Retailer: $d_r^1 = 30$ $d_0^1 = 4$, $d_1^1 = 0.3$, $s_r^1 = \$122$ / unit, $A_r^1 = \$85$, $h_r^1 = \$1.5$, $I_{er}^1 = 10\%$ / year.

For 2nd Retailer: $d_r^2 = 32$, $d_0^2 = 5$ $d_1^2 = 0.3$, $s_r^2 = \$85$ / unit, $A_r^2 = \$40$, $h_r^2 = \$0.55$, $I_{er}^2 = 10\%$ / year.

4.5.2 Optimal Results

After solving the equation (4.19) for the above input data, the obtained optimum results are given in Table-4.1 for different cases.

Table 4.1: *Optimum results for all of the different cases*

Cases	(N^1, N^2)	M	(P)	(t_1)	(t_2)	(T^1, T^2)	Ex. Profit $E[ITP(P, t_1)]$
Case-1 $(N^i < M < t_2 < T^i)$	(0.11, 0.11)	0.24	93.32	0.13	0.42	(0.46, 0.48)	2381.09
Case-2 $(N^i < t_2 < M < T^i)$	(0.09, 0.10)	0.58	45.82	0.37	0.56	(0.59, 0.58)	2956.81
Case-3 $(t_2 < N^i < M < T^i)$	(0.44, 0.42)	0.58	97.61	0.12	0.41	(0.54, 0.49)	2316.64

4.6 Discussion

The optimum results in the Table-4.1 are as per expectation. Case-2 of the proposed model gives the maximum profit than the other two cases. In this case (Case-2), the credit periods to customers offered by the retailers are lowest than the other two cases. Due to this retailers' losses due to the offer of trade credit to customers is minimum. The time periods for the retailers are also maximum than the other two cases. This fetches more revenues to the retailers. But the production rate of Case-2 is minimum than those of Case-1 and Case-3. This consideration is supposed to produce less amount which fetches less revenue. But, combination of these three considerations gives maximum profit as first two assumptions overpower the third assumption. Among the two cases- Case-1 and Case-2, though the customers' trade credit are almost same the time periods (T^1, T^2), trade credit (M), the production period (t_1) for Case-2 are more than those of Case-1 and due to this, the total profit for Case-2 is more than the Case-1. Among Case-2 and Case-3, customers' trade credits (N^1, N^2) and the production period (t_1) dominate the scenario and the total profit for Case-3.It is interesting to note that the total profits for Case-1 and Case-3 are not much different. Except (N^1, N^2) and M, the other parameters for these cases are almost same. Here, both (N^1, N^2) and M for Case-3 are higher than Case-1. The retailers get more time (M) to replay the dues and in return, give more trade credits to customers. These parameters balances the total profit.

4.7 Conclusion

In this chapter a production inventory model has been developed for a supply chain involving manufacturer, retailers and the end customers in a target to find the perfect production rate and exact maintenance policy for which the total supply chain profit will be maximum.

• The manufacturer produces an imperfect item with random defective rate. Researcher may develop the model for different type of defective rates (such as, fuzzy, fuzzy-random, rough etc.)

• A retailer shares a certain percentage of his/her delay in payment facility with the customers to push the sale. There are many scope to extend this study by considering different type of credit length. Also the effect of new method of payment [given in sec.-9.4.1] may be considered here.

• The model can also be developed for different types of customer demand.

• Finally, the numerical results are obtained using the data given by a merchant. So the results have greater importance from the managerial point of view.

Chapter 5

Two plant optimal production problem in random time horizon

5.1 Introduction

In any manufacturing system, taking the perfect production decision for demand satisfaction is one of common problem to the manufacturer. Higher production rate increases the stock holding cost and lower production rate creates shortage situation. Also there are some other factors which also affect the production rate. Different type production models are continuously studied by several researchers to find the optimal production policy for different business situations. Levin *et al.* [145] gave a brief discussion about contemporary policy for managing operating systems. GrubbstroK and Winkr [94] used control theory to develop an inventory trigger control policy. Khouja and Mehrez [131] studied an EPQ model with imperfect quality items where the production rate is variable. He *et al.* [107] studied a production inventory model for deteriorating items for different market demands. Cárdenas-Barrón and Sana [36] formulated a production inventory model for two echelon supply chain where demand depends on the sale. It is always seen the requirement of larger space to store items increases according to the growth of business. But, the situation may not support to enlarge the go-down. In this case the situation can be tackle by making another warehouse at a distance from the business place. Jaggi *et al.* [118] studied an inventory model of deteriorating items with two warehouses. Liang and Zhou [147] have also studied a two warehouse inventory model for deteriorating items under conditionally permissible delay in payment. A two storage inventory model is formulated by Guchhait *et al.* [96] with variable demand under permissible delay in payment. Kar *et al.* [129] presented an inventory model of muti-deteriorating items where a business manager sells business materials from two different shops due to congested space.

The normal phenomenon of a real life oriented inventory problem is that the time horizon is uncertain. Moon *et al.* [185] have presented an economic order quantity model with random planning horizon. A two storage inventory model is constructed by Roy *et al.* [211] with fuzzy deterioration for a random planning horizon. Datta and Pal [74] studied the effects of inflation and time value of money on an inventory model with linear time dependent demand rate where shortage is allowed. A fuzzy inventory model with two warehouses is studied by Maiti [166] and solved using the possibility measure dependent fuzzy goal technique.

Every production firm starts its business with a budget which is to be maintain over the whole business horizon and it is very difficult in reality. Chen and Wang [49] have studied a supply chain with trade-credit contract under the consideration of budget restriction.

In-spite of all these studies, we present here a production inventory model which is newer from the previous studies in the following context.

1. In this EPQ model, two production houses are considered to control the stock out situation under a random environment which makes this investigation different from the others. Here market demand is satisfied from both of th production house.

2. The randomness of the different parameters of the model are removed using chance constraint method and taking expected value.

3. The production rates are unknown functions of time and the demand rates are known functions of time.

4. None has investigated a two plant production-inventory models with dynamic production rate, trended demand, random shortage (occurs only at one production house) under a random planning horizon till now.

In this model, a production house is considered with two production plants situated at different places. A single item is produced at both of the plants under a single management with different production rates for a random time horizon. All the inventory parameters are also random. Production rate and Demand at both of the plants are linear function of time. At plant-1, the demand rate is not lower than the production rate, so shortage may arise at this plant. At plant-2, the demand rate is not greater than the production rate, so there may be a over stock situation. In this process, if there is an excess demand at a particular production center then excess produced amount is shifted to another production center having shortage of items. The model is formulated as a cost minimization problem for the time dependent production-inventory model in the form of an integral. The objective is to find the optimum production rates for two production centers. The problem is solved using gradient based non-linear optimization technique (GRG) using Lingo 12.0 software. Some particular cases re considered, derived and solved. The problems are illustrated through some numerical examples and presented in tabular and graphical forms.

The next of this chapter is organized as follows. Section-5.2 contains the notations and assumptions that we have made for the development of the model. The formulation and the theoretical development of the models are given in section-5.3. A solution methodology presented in the section-5.4 and the numerical experiment done through that solution methodology is presented in section-5.5 .The corresponding discussions and conclusions are given in section-5.6 and -5.7 respectively.

5.2 Notations and Assumptions

In the proposed model, the following notations and assumptions are used for the i-th plant [i=1,2].

5.2.1 Notations

$\hat{T}$	length of each cycle which random in nature.
Z	budget cost.
m	number of times stock transported from plant-1 to plant-2 over the whole business period (number of replenishment cycle).
$\hat{C}_t$	random transportation cost.
$S_2(t)$	shortage level at plant-2 at time t in each cycle.
$\hat{\pi}_{21}$	per item per unit time shortage cost for plant-2.
$\mu_{\hat{T}}$	mean value of $\hat{T}$.
$\hat{h}_{i1}$	per unit item per unit time the holding cost for the i-th plant.
$\hat{C}_{iu}$	per unit item production cost for the i-th plant.
$D_i(t) = d_{i0} + d_{i1}t$	per unit time demand rate for the i-th plant.
$U_i(t) = u_{i0} + u_{i1}t$	per unit time production rate.
$X_i(t)$ and $I_i(t)$	inventory level at time t for $i-th$ plant where, [$X_i(t) = I_i(t)$ for $X_i(t) > 0$].
β_j	a real number which is used to fixed the surplus stock of plant-1 in the interval $[t_{j-1}, t_j]$ that can be transported to plant-2 in the interval $(t_j, t_j + 0)$ where, $0 \leq \beta_j < 1$ and $j = 1, 2, \cdots, m$.

5.2.2 Assumptions

i) This is a single period inventory model with random time horizon.

ii) Shortages are allowed only plant-2 and over stock situation occurs only at plant-1.

iii) Both the plants are in a single management system.

iv) The production rate is a unknown function of time and considered as a control variable.

v) The demand rate is a known function of time.

vi) After the demand satisfaction, the excess amount of stock at plant-1 is transported to the plant-2 if there be unwanted stock out situation occurs.

vii) There is only one cycle over a finite business period and the transfer of stock is done with in a zero lead time.

5.3 Model Formulation

5.3.1 Model-I: Random Time horizon for both Plant

In this investigation a production-inventory control problem is considered with two production center namely, plant-1 and plant-2 for a finite but random business planning horizon $\hat{T}$. Here, the randomness of the time horizon is removed using chance constraint method. Let the deterministic value of $\hat{T}$ is T_r. Then the corresponding chance-constraint is

$$Prob(|\hat{T} - T_r| < \epsilon_1) \geq \eta_1 \tag{5.1}$$

where, $\hat{T}$ is normally distributed with corresponding mean and variance $m_{\hat{T}}\ and\ \sigma_{\hat{T}}$ respectively. If k_1 represents the value of the standard normal variable for which

$$\begin{aligned} \eta_1 &= F(k_1) \\ &= \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{k_1} e^{-\frac{t^2}{2}} dt \end{aligned} \tag{5.2}$$

The cumulative probability $P[t \leq k_1]$ is available in standard statistical table for different values of k_1. Then according to chance constraint technique [c.f. 2.1.5], the constraint (5.1) becomes

$$\mu_{\hat{T}} - \epsilon_1 - \sigma_{\hat{T}} k_1 \leq T_r \leq \mu_{\hat{T}} + \epsilon_1 - \sigma_{\hat{T}} k_1 \tag{5.3}$$

Now the whole business plane is subdivided into m equal sub-intervals. The $j-th$ sub-interval is $[t_{j-1}, t_j]$ where, $t_j = \frac{jT_r}{m}$ for, $j = 0, 1, \cdots, m.$

The position of plant-2 is at the heart of market with unavailability of space and raw materials and some non-supportive circumstances for production. Therefore, the production rate $[U_2(t)]$ is poor but according to its position the demand rate $[D_2(t)]$ may be higher than $[U_2(t)]$ at plant-2 which may create a shortage situation.

For plant-1 it is considered that due to the availability of raw material, man power support, space availability and some of other advantages production rate $[U_1(t)]$ is higher than its demand rate $[D_1(t)]$. So, the stock continuously increases upto the time of end of production process.

Moreover, lower production rate and higher demand rate create shortage at plant-2 during business running period and then it is fully back-logged within a small duration using the excess amount of item produced at plant-1. The transportation of excess item of plant-1 to plant-2 is made in bulk release pattern.

Since the business horizon is finite so, the inventory level at both of the plants are continuous function of time over the business period except for a finite number of times.

In the first interval $[0, t_1]$, $\beta_1 I_1(t_1)$ amount is transported to plant-2 from plant-1 within a small time gap $(t_1, t_1 + 0)$. Similarly, for the second interval $(t_1 + 0, t_2]$ the amount transported from plant-1 to the 2nd one is $\beta_2 I_1(t_2)$ in small time gap $(t_2, t_2 + 0)$. This process continues in each of the remaining intervals $(t_{j-1} + 0, t_j]$, for $j = 3, \cdots m$ till the end of the business period.

Thus the inventory position at the different time are given by

$$I_1(t+0) = \begin{cases} (1-\beta_j)I_1(t), & for \;\; t = t_j \\ 0, & for \; t = T_r \end{cases} \tag{5.4}$$

$$I_2(t) = \begin{cases} 0, & for \;\; t = t_j \\ \beta_j I_1(t), & for \;\; t = t_j + 0 \\ 0, & for \;\; t = T_r \end{cases} \tag{5.5}$$

The differential equation expressing the inventory level $X_i(t)$ in production rate $U_i(t)$ and demand rate $D_i(t)$ for i-th (i=1,2) plant in the time horizon is

$$\frac{dX_i(t)}{dt} = U_i(t) - D_i(t) \tag{5.6}$$

$$\text{where, } X_2(t) = \begin{cases} 0, & for \;\; t = t_j \\ \beta_j I_1(t), & for \;\; t = t_j + 0 \\ 0, & for \;\; t = T_r \end{cases} \tag{5.7}$$

$$\text{with, } I_i(t) = max[X_i(t), 0], \; S_2(t) = max[-X_2(t), 0]. \tag{5.8}$$

Thus the objective function of the problem is given by

$$Minimize \; J = \int_0^{T_r} \left[\sum_{i=1}^{2} \{\hat{h}_{i1} I_i(t) + \hat{C}_{iu} U_i(t)\} + \hat{\pi}_{21} S_2(t) \right] dt + \sum_{j=1}^{m-1} \hat{C}_t \beta_j I_1(t_j) \tag{5.9}$$

where T_r is given by the equation (5.3) and the other constraints are given by the following-

$$\sum_{i=1}^{2}\int_0^{T_r} \hat{C}_{iu}U_i(t)dt \leq Z \tag{5.10}$$
$$X_1(t) \geq 0 \tag{5.11}$$
$$0 \leq D_1(t) \leq U_1(t) \tag{5.12}$$
$$0 \leq U_2(t) \leq D_2(t) \tag{5.13}$$

Equivalent deterministic form of the model

The whole problem given by (5.9) with the constraints (5.10), (5.11), (5.12) and (5.13) is converted into its deterministic form using two method. Method-1 is the chance constraint method and Method-2 is taking the expected value.

Method-1(Using Chance Constraint Method):

Let $h_{ri1}, C_{riu}, \pi_{r21}, C_{rt}$ are the deterministic value of the random numbers $\hat{h}_{i1}, \hat{C}_{iu}, \hat{\pi}_{21}, \hat{C}_t$ respectively. Then the objective function (5.9) becomes

$$\text{Minimize } J = \int_0^{T_r}[\sum_{i=1}^{2}\{h_{ri1}I_i(t) + C_{riu}U_i(t)\} + \pi_{r21}S_2(t)]dt + \sum_{j=1}^{m-1} C_{rt}\beta_j I_1(t_j) \tag{5.14}$$

and the budget constraint (5.10) becomes

$$\sum_{i=1}^{2}\int_0^{T_r} C_{riu}U_i(t)dt \leq Z \tag{5.15}$$

and the other constraints are

$$Prob(|h_{r1} - h_{ri1}| < \epsilon_2) \geq \eta_2$$
$$Prob(|\hat{C}_{iu} - C_{riu}| < \epsilon_3) \geq \eta_3$$
$$Prob(|\hat{\pi}_{21} - \pi_{r21}| < \epsilon_4) \geq \eta_4$$
$$Prob(|\hat{C}_t - C_{rt}| < \epsilon_5) \geq \eta_5$$

If k_i represents the standard normal value for which $F(k_i) = \eta_i$ for, $i = 2, 3, 4, 5$, where $F(x)$ is the distribution function of standard normal variable given by equation (**??**), then following the chance constraint method [c.f. 2.1.3] the above inequalities are reduces to

$$\mu_{\hat{h}_{i1}} - \epsilon_2 - \sigma_{\hat{h}_{i1}}k_2 \leq \hat{h}_{i1} \leq \mu_{\hat{h}_{i1}} + \epsilon_2 - \sigma_{\hat{h}_{i1}}k_2 \tag{5.16}$$
$$\mu_{\hat{C}_{iu}} - \epsilon_3 - \sigma_{\hat{C}_{iu}}k_3 \leq \hat{h}_{i1} \leq \mu_{\hat{C}_{iu}} + \epsilon_3 - \sigma_{\hat{C}_{iu}}k_3 \tag{5.17}$$
$$\mu_{\pi\hat{2}1} - \epsilon_4 - \sigma_{\pi\hat{2}1}k_4 \leq \mu_{\pi\hat{2}1} + \epsilon_4 - \sigma_{\pi\hat{2}1}k_4 \tag{5.18}$$
$$\mu_{\hat{C}_t} - \epsilon_5 - \sigma_{\hat{C}_t}k_5 \leq \mu_{\hat{C}_t} + \epsilon_5 - \sigma_{\hat{C}_t}k_5 \tag{5.19}$$

Where, $\mu_{\hat{h}_{i1}}$, $\mu_{\hat{C}_{iu}}$, $\mu_{\hat{\pi}_{21}}$, $\mu_{\hat{C}_t}$ and $\sigma_{\hat{h}_{i1}}$, $\sigma_{\hat{C}_{iu}}$, $\sigma_{\hat{\pi}_{21}}$, $\sigma_{\hat{C}_t}$ are the mean and standard deviation (S.D.) of the normal variables $\hat{h}_{i1}$, $\hat{C}_{iu}$, $\hat{\pi}_{21}$, and $\hat{C}_t$ respectively.

Method-2(Considering the expected value) :

Here, the expected value of the random parameters $\hat{h}_{i1}$, $\hat{C}_{iu}$, $\hat{\pi}_{21}$ and $\hat{C}_t$ are considered, where $\hat{h}_{i1}$, $\hat{C}_{iu}$, $\hat{\pi}_{21}$ and $\hat{C}_t$ all are normal variate with parameters ($\mu_{\hat{h}_{i1}}$, $\sigma_{\hat{h}_{i1}}$), ($\mu_{\hat{C}_{iu}}$, $\sigma_{\hat{C}_{iu}}$), ($\mu_{\hat{\pi}_{21}}$, $\sigma_{\hat{\pi}_{21}}$), ($\mu_{\hat{C}_t}$, $\sigma_{\hat{C}_t}$) respectively. If expected value of a random parameter $\hat{h}_{i1}$ is represented as E[$\hat{h}_{i1}$] (and, similar for other parameters), then the equation (5.9) becomes

$$\begin{aligned} Minimize J &= \int_0^{T_r} [\sum_{i=1}^{2}\{E[\hat{h}_{i1}]I_i(t) + E[\hat{C}_{iu}]U_i(t)\} + E[\hat{\pi}_{21}]S_2(t)]dt \\ &\quad + \sum_{j=1}^{m-1} E[\hat{C}_t]\beta_j I_1(t_j) \end{aligned} \quad (5.20)$$

and the constraints are

$$\sum_{i=1}^{2}\int_0^{T_r} E[\hat{C}_{iu}]U_i(t)dt \leq Z \quad (5.21)$$

$$Prob(|\hat{T} - T_r| < \epsilon_1) \geq \eta_1 \quad (5.22)$$

There are some particular cases which can derived from the presented model. Such as

5.3.2 Model-II: Time horizon is crisp for both plant

Here, the time horizon for both of the plants are considered as crisp in nature. If T is the time horizon in this case, then the problem takes the form

$$Minimize\ J = \int_0^{T} \left[\sum_{i=1}^{2}\{\hat{h}_{i1}I_i(t) + \hat{C}_{iu}U_i(t)\} + \hat{\pi}_{21}S_2(t)\right]dt + \sum_{j=1}^{m-1} \hat{C}_t\beta_j I_1(t_j) \quad (5.23)$$

where the budget constraints are given by

$$\sum_{i=1}^{2}\int_0^{T} \hat{C}_{iu}U_i(t)dt \leq Z \quad (5.24)$$

and the other constraints are given by (5.11), (5.12) and (5.13).

5.3.3 Theoretical Experiment

The problem is evaluated using the Optimal Control Theory (c.f. sec.-2.2.3). According to this method the problem given by equations (5.14), (5.15), the corresponding Hamiltonian function is

$$H = \sum_{i=1}^{2}[-\{h_{ri1}I_i(t) + C_{riu}U_i(t)\} + q_i(t)\{U_i(t) - d_{i0} - d_{i1}t\}] - \pi_{r21}S_2(t). \tag{5.25}$$

and the Lagrangian function for the constraints is

$$L = H + \lambda_1 I_1(t) + \lambda_2 I_2(t) + \lambda_3\left[\sum_{i=1}^{2} C_{riu}U_i(t) - \frac{Z}{T_r}\right] \tag{5.26}$$

where, $\lambda_i \geq 0$, for $i = 1, 2, 3$ are the Lagranges multipliers. The Kuhn-Tucker conditions are

$$\lambda_1 I_1(t) = 0,\ \lambda_2 I_2(t) = 0,\ \lambda_3\left[\sum_{i=1}^{2} C_{riu}U_i(t) - \frac{Z}{T_r}\right] = 0. \tag{5.27}$$

where, the corresponding ad-joint functions $q_i(t)$, $i = 1, 2$ are given by the first order differential equations,

$$\frac{dq_i(t)}{dI_i(t)} = -\frac{\delta L}{\delta I_i(t)},\ for,\ i = 1, 2.$$

which can be reduces to

$$\frac{dq_1(t)}{dI_1(t)} = h_{r11} - \lambda_1,\ for\ I_1(t) \geq 0,\ and \tag{5.28}$$

$$\frac{dq_2(t)}{dI_2(t)} = h_{r21} - \lambda_2,\ for\ I_2(t) \geq 0 \tag{5.29}$$

$$= -\hat{\pi}_{21},\ for\ X_2(t) < 0 \tag{5.30}$$

where, $q_i(T_r) = 0,\ for\ i = 1, 2$
Also following the maximum principle [c.f. sec.-2.2.3], the Lagrangian function is maximized at every point of time with respect to the control function $U_i(t)$. This leads to the following relations.

$$\frac{\delta L}{\delta U_i(t)} = q_i(t) - (1 - \lambda_3)E[\hat{C}_{iu}] \tag{5.31}$$

Here, $U_i(t)$ is the control variable and bounded with lower and upper bounds. Thus the following three cases may arise.

case-1: $\frac{\delta L}{\delta U_i(t)} > 0$, **case-2:** $\frac{\delta L}{\delta U_i(t)} = 0$, **case-3:** $\frac{\delta L}{\delta U_i(t)} < 0$.

For case-1, the Lagrangian function is an increasing function of production function $U_i(t)$.
For case-2, the Lagrangian function is independent of production function $U_i(t)$.
For case-3, the Lagrangian function is a decreasing function of production function $U_i(t)$.
Using the boundary conditions given by (5.12), (5.13) the optimal production function can be reduced for case-1 and case-3 as the following.

$$\begin{aligned} U_1(t) &= u_{10} + u_{11}t,\ for\ q_1(t) > (1-\lambda_3)C_{11u} && (5.32)\\ &= 0,\ for\ q_1(t) < (1-\lambda_3)C_{11u} && (5.33)\\ U_2(t) &= u_{20} + u_{21}t,\ for\ q_2(t) > (1-\lambda_3)C_{12u} && (5.34)\\ &= 0,\ for\ q_2(t) < (1-\lambda_3)C_{12u} && (5.35) \end{aligned}$$

Let equation (5.33) and(5.35) are satisfied for $0 \le t \le t_{11}$ and $0 \le t \le t_{21}$ respectively.
Then using the conditions (5.28) to (5.31) in equations (5.33) to (5.35), the optimum production function is reduced as

$$\begin{aligned} U_1(t) &= u_{10} + u_{11}t,\ for\ 0 \le t \le t_{11} && (5.36)\\ &= 0,\ for\ t_{11} \le t \le T_r && (5.37)\\ U_2(t) &= u_{20} + u_{21}t,\ for\ 0 \le t \le t_{12} && (5.38)\\ &= 0,\ for\ t_{12} \le t \le T_r && (5.39) \end{aligned}$$

Using equations (5.37), (5.38) and $I_i(t) = max[X_i(t), 0]$, $S_2(t) = max[-X_2(t), 0]$, the optimum stock function in [0,T_r] for plant-1 is obtained as

$$\begin{aligned} I_1(t) &= I_1(t_j+0) + (u_{10}-d_{10})(t-t_j) - (u_{11}-d_{11})\frac{(t^2-t_j^2)}{2} && (5.40)\\ &\qquad for,\ t_j < t \le t_{11} \\ &= I_1(t_{11}) - d_{10}(t-t_{11}) + d_{11}\frac{(t^2-t_{11}^2)}{2} \qquad for,\ t_{11} \le t \le t_{j+1} && (5.41)\\ &= I_1(t_{j+1}+0) - d_{10}(t-t_{j+1}) - d_{11}\frac{(t^2-t_{j+1}^2)}{2}\ for,\ t > t_{j+1} && (5.42) \end{aligned}$$

Similarly, using the equations (5.39), (5.39) and $I_i(t) = max[X_i(t), 0]$, $S_2(t) = max[-X_2(t), 0]$, the optimum stock and shortage functions in [0,T_r] for plant-2 is reduced as

$$\begin{aligned} X_2(t) &= I_2(t_j+0) + (u_{20}-d_{20})(t-t_j) - (u_{21}-d_{21})\frac{(t^2-t_j^2)}{2} && (5.43)\\ &\qquad for,\ t_j < t \le t_{21} \\ &= I_2(t_{12}) - d_{20}(t-t_21) + d_{21}\frac{(t^2-t_{21}^2)}{2}\ for,\ t_{21} \le t \le t_{j+1} && (5.44)\\ &= I_2(t_{j+1}+0) - d_{20}(t-t_{j+1}) - d_{21}\frac{(t^2-t_{j+1}^2)}{2}\ for,\ t > t_{j+1} && (5.45) \end{aligned}$$

Thus the problem reduces to

$$\text{Minimize } J = \sum_{i=1}^{2}(H_i + P_i) + S_2 + \sum_{j=1}^{m-1} C_{rt}\beta_j I_1(t_j) \tag{5.46}$$

subject to the constraints given by (5.3) and (5.10) to (5.13)., where $H_i,\ P_i\ and\ S_2$ are given by

$$H_i = \int_0^{T_r} h_{ri1} I_i(t)dt. \tag{5.47}$$

$$P_i = \int_0^{T_r} C_{riu} U_i(t)dt. \tag{5.48}$$

$$= C_{riu}(u_{i0}t_{i1} + \frac{u_{i1}}{2}t_{i1}^2). \tag{5.49}$$

$$and\ \ S_2 = \int_0^{T_r} \pi_{r21} S_2(t)dt. \tag{5.50}$$

$$= \pi_{r21}\left[(d_{20} - u_{20})\frac{t_2}{2} + (d_{21} - u_{21})\frac{{t_2}^3}{6}\right] \tag{5.51}$$

5.4 Solution methodology

The objective function given by equation (5.46), with its corresponding constraints is evaluated numerically, using gradient based optimization technique- Generalized Reduced Gradient (GRG) Method [c.f. sec.-2.2.2] through Lingo software of 12.0 version.

5.5 Numerical Experiments

To illustrate the models numerically, the following data are considered according to some manufacturer suggestion.

5.5.1 Input Data

$d_{10} = 11$, $d_{11} = 0.2$, $n = 5$, $d_{20} = 17$, $d_{21} = 0.25$ (demands are in units).
The mean and standard deviation (s.d.) of the random parameters are given by the Table-5.1.

Table 5.1: *Mean and s.d. of random parameters*

		$\hat{T}$	$\hat{C}_{iu}$	$\hat{h}_{i1}$	$\hat{\pi}_{21}$	$\hat{C}_t$
plant	mean	10	6.10	0.31	-	0.35
-1	s.d.	3	0.20	0.15	-	0.2
	Lt.of int.		(5.48,6.52)	(0.21,0.51)		
plant	mean	10	6.43	0.15	0.25	
-2	s.d.	3	0.25	0.15	0.15	
	Lt.of int.		(5.7,6.81)	(0.22,0.58)	(0.25,0.57)	(0.26,0.56)
ϵ		0.02	0.021	0.025	0.025	0.024
η		0.95	0.95	0.90	0.93	0.94

5.5.2 Optimal Result

Using the methodology mentioned in sec.5.4 the optimal results using the above inputted data are obtained for different models and presented in Table-5.2. For the above mentioned data, the stock positions at two plants at the ends of different replenishment cycles are given.

Table 5.2: *Optimum result for different models*

Model	Method	Plant	u_{i0}	u_{i1}	T_r	Total Cost
-I	-1	-1	12.02	0.24	9.275	1729.3
		-2	16.57	0.36	9.372	
	-2	-1	11.98	0.27	9.267	1734.65
		-2	16.04	0.41	9.357	
-II	-1	-1	12.11	0.23	9.684	1781.21
		-2	16.53	0.21	9.89	
	-2	-1	12.09	0.25	9.776	1801.02
		-2	16.23	0.24	9.866	

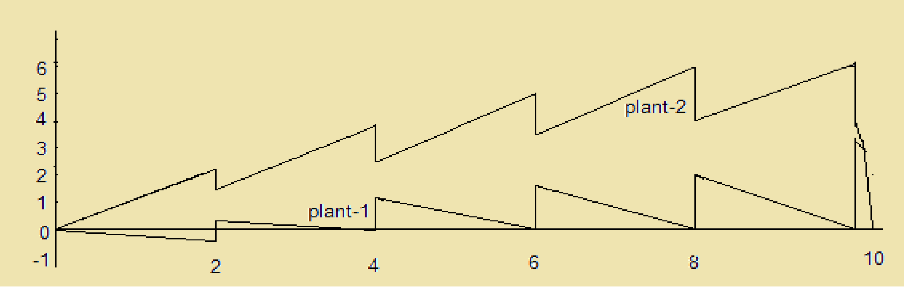


Figure 5.1: Stock level at different time at plant-1 and plant-2

Table 5.3: *Sensitivity analysis for different models*

Model	Method	Plant	t_j	0	1.897	3.794	5.691	7.588	9.275	9.374	9.485
		-1	$I_1(t_j)$	0	1.999	3.245	4.07	4.66	4.833	2.839	0
	-1	Plant	t_j	0	1.897	3.794	5.691	7.588	9.275	9.374	9.485
		-2	$I_2(t_j)$	0	-0.62	0	0	0	1.92	2.989	0
			$I_2(t_j+0)$	0	0.262	1.43	1.792	2.056	3.13	2.989	0
	Method	Plant	t_j	0	1.903	3.806	5.709	7.612	9.567	9.461	9.515
		-1	$I_1(t_j)$	0	2.112	3.4136	4.433	5.123	5.262	3.212	0
-I	-2	Plant	t_j	0	1.903	3.806	5.709	7.612	9.067	9.461	9.515
		-2	$I_2(t_j)$	0	-0.68	0	0	0	1.435	3.513	0
			$I_2(t_j+0)$	0	0.313	1.604	2.084	2.408	3.912	3.513	0
Model	Method	Plant	t_j	0	2	4	6	8	9.684	9.89	10
		-1	$I_1(t_j)$	0	2.28	3.896	5.077	5.972	6.235	2.951	0
	-1	Plant	t_j	0	2	4	6	8	9.684	9.89	10
		-2	$I_2(t_j)$	0	-0.57	0	0	0	1.52	3.625	0
			$I_2(t_j+0)$	0	0.21	1.340	1.746	2.054	3.712	3.625	0
	Method	Plant	t_j	0	2	4	6	8	9.776	9.857	10
		-1	$I_1(t_j)$	0	2.28	3.8816	5.231	6.212	6.842	3.11	0
-II	-2	Plant	t_j	0	2	4	6	8	9.36	9.82	10
		-2	$I_2(t_j)$	0	-0.64	0	0	0	1.61	4.112	0
			$I_2(t_j+0)$	0	0.297	1.595	2.150	2.553	4.432	4.112	0

5.6 Discussion

The following discussions can be made from the above results.

• The model presented in this chapter highlights on a production inventory model with unknown production rate which is a function of time and gives a procedure to get the optimum production function.

• From the optimum results given in Table-5.2, it can be seen that the optimal production rate functions for plant-1, plant-2 are $U_1(t) = 16.57 + 0.36t \; and \; U_2(t) = 12.02 + 0.24t$ respectively. From these functions, its clear that the production rate at plant-1 is greater than plant-2 as per expectation. Moreover, the demand function for the respective plant-1 and plant-2 are $D_1(t) = 11 + 0.2t \; and \; D_2(t) = 17 + 0.25t$ respectively. Hence it is obvious that there is a excess stock at the plant-1 which is supplied to plant-2 in each replenishment cycles. This indicates that the process presented here has worked positively as per expectation from the model formulation.

• Moreover the stock situation is presented in Table-5.3 also supports the above discussion. From Table-5.3 it can be seen that the stock situation at time $t = t_1$ there is a shortage occurred at the plant-2 and in a infinitesimal small duration of time (at $t = t_1 + 0$) the shortage is covered by the excess stock occurred at the plant-1.

• As the time increases, the amount of stock increases at both of the plants. This happen as the production rate is a linear function of time and also the demand for both the plants.

• Ti is seen that during the nd of the business period, last business time for deterministic model (Model-II) is 10 units where this parameter for the random model (Model-I) is slightly different from 10 units. This is as per expectation.

5.7 Conclusion

In comparing with the existing literature on production-inventory models, the followings are the main contributions in the proposed model:
• For the first time, a two plant production-inventory model is considered with time dependent production rate which is an unknown function of time in a random planning horizon with shortage at one of the two plants.
• Manufacturer continuously produces items at both plants and the shortage situation is covered instantly as it occurs at one of the two plants using the excess stock occurs at the other plant.
• There are many scopes for the future research in this context. The models can be formulate in other deterministic and non-deterministic environments such as fuzzy, fuzzy-random, fuzzy-rough etc.
• The model also can be reformulated using the lead time, trade credit to get some other realistic pictures of productions and costs.

Chapter 6

Optimum Production Policy for a Production Inventory Model in Random Time Horizon

6.1 Introduction

According to Levin *et al.* [145] " At times, the presence of inventory has a motivational effect on the people around it. It is common belief that large piles of goods displayed in super market will lead the customers to buy more. So inventory can be used to increase the demand for an item in the market". For this reason, in the recent competitive market within the developing countries like India, Brazil etc. the inventory / stock is decoratively exhibited and colorably displayed through electronic media to attract the customers and to boost the sale. Hence several authors such as Datta and pal [75], Mandal and Phaujder [174], Maiti and Maiti [170] and others considered stock dependent demand in their models.

Normally, most of the classical production-inventory models are developed without considering the effect of inflation and time value of money. But according to the recent economical situation of most of the countries, the effect of inflation and time value of money cannot be ignored. Also for various factors, the time value of money in the world economy changes in very small duration. The first inventory model with the inflationary effect was developed by Buzacott [27]. Jolai *et al.* [125] examined the effect of inflation on an EOQ production lot size model of deteriorating items with partial backlogging. Effect of inflation was also considered by Jaggi [119] in an inventory model with two warehouse and partial back logging. Neetu and Tomer [192] considered inflationary effect in some inventory models for deteriorating items with infinite planning horizon. Moon and Lee [184] presented an EOQ model under inflation and discount with a random product life cycle.

In production-inventory models, production rate is an important controlling factor. A manufacturer normally tries to slowdown i.e. slash-down the production rate to avoid the unnecessary idle stock in the go-down. So the production rate decreases as the inventory level goes up. This contradicts the modern marketing policy where demand proportional to the displayed stock level. Roy *et al.* [212, 213] solved some inventory problems with constant production rate and without shortages under a random planning horizon.

The present investigation differs from existing literature in the following pointof view.

1. The present production-inventory model involves stock dependent production rate and stock dependent demand rate over a random planning horizon.

2. The present model is developed with and without shortages and the unit production rate is also considered as crisp, random, fuzzy and fuzzy-random in nature.

3. Till now, none has investigated inventory models under the above assumption.

In this study, a production inventory model is developed with displayed stock dependent demand over a random planning horizon under inflation taking time value of money into account. The random business period follows exponential distribution with known mean. The unit production rate is partly stock dependent and the unit production cost is considered as crisp, random, fuzzy and fuzzy-random in nature and different models are presented for each of these considerations. All models are formulated as profit maximization problems which are developed with and without shortages. Models are solved using the non-linear optimization technique- Generalized Reduced Gradient (GRG) technique and illustrated through some numerical examples. Some sensitivity analysis also made for different production rates.

The rest of the chapter is organized as follows. Some notations and assumptions which are considered in this model are given in Section-6.2. Section-6.3 contains the formulation of the models for different considerations. A short note given about the solution procedure in Section-6.4 and section-6.5 presents some numerical experiment with optimal result which are obtained using our proposed solution method. A brief discussion and conclusion is given in Section-6.6 and 6.7 respectively.

6.2 Notations and Assumptions

The corresponding production-inventory model is developed on the basis of the following assumptions and notations.

6.2.1 Notations

n	Number of full cycles.
t_u	time from the beginning of each cycle, when shortages are fully backlogged .
u_t	$u_0 - u_1 x(t)$, production function where u_0 and u_1 are unknown constants.
$d(t)$	$d_0 + d_1 x(t)$, demand rate where d_0 and d_1 are unknown constants.

t_1	time when stock is maximum and production is stopped.
t_2	time when stock is zero and shortages begins.
t_3	time when shortages are maximum and production is started.
c_1	holding cost per unit item per unit time.
s_1	shortage cost per unit item per unit time.
s_2	per unit item reduced selling price.
$x(t)$	inventory position in each cycle at time t, where, $(j-1)t_u \le t \le jt_u$.
H	random time horizon, where h is the real time horizon.
Q_1	maximum stock level in each cycle.
Q_2	maximum shortage amount in each cycle.
e^{-Rt}	difference between discount and inflation at any time t, where R is a known constant.
$p_0 e^{-(j-1)\gamma}$	per unit item production cost where, p_0 and γ are known constants.
m_0	mark-up imposed upon the unit production cost to fix the unit selling price.
$c_3^1 + c_3^2 e^{-\delta j}$	the set-up cost for the j-th cycle.

6.2.2 Assumptions

i) Demand rate is a known linear function of displayed stock amount.

ii) Shortages are allowed and fully backlogged whenever it is possible.

iii) Unit production rate is unknown and partly depends on displayed stock and decreases according to time increment.

iv) Lead time is negligible.

v) Business period is random and follows an exponential distribution with known mean.

6.3 Formulation of the Models

6.3.1 Model-A: Model with deterministic production cost and shortages

Formulation for first n full cycles

In the $j-th$ cycle $(j = 1, 2, \cdots, n)$, production starts with maximum production rate at the beginning of the cycle and stopped at $t = (j-1)t_u + t_1$, when the stock is maximum. The shortage begins at the time $t = (j-1)t_u + t_2$ and at the time $t = (j-1)t_u + t_3$, when shortage is Q, the production starts again and the shortage is fully backlogged at the time end of cycle i.e. at $t = jt_u$. So following Fig.-6.1 the leading differential equation for the $j-th$ cycle is given by

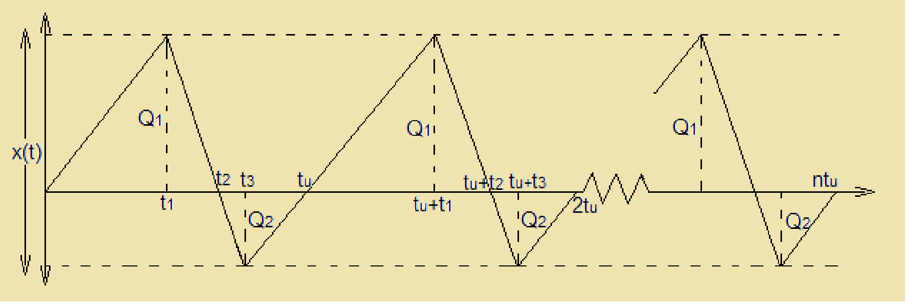


Figure 6.1: Model of the problem for n full cycle

$$\frac{dx(t)}{dt} = \begin{cases} u_0 - d_0 - (u_1 + d_1)x(t) & for,\ (j-1)t_u \le t \le (j-1)t_u + t_1 \\ -d_0 - d_1 x(t) & for,\ (j-1)t_u + t_1 \le t \le (j-1)t_u + t_3 \\ u_0 - d_0 - (u_1 + d_1)x(t) & for,\ (j-1)t_u + t_3 \le t \le jt_u \end{cases} \quad (6.1)$$

with the boundary conditions

$$x(t) = \begin{cases} 0 & for,\ t = (j-1)t_u \\ Q_1 & for,\ t = (j-1)t_u + t_1 \\ 0 & for,\ t = (j-1)t_u + t_2 \\ Q_2 & for,\ t = (j-1)t_u + t_3 \\ 0 & for,\ t = jt_u, \quad where, \quad j = 1, 2, \cdots, n. \end{cases} \quad (6.2)$$

Using the boundary conditions given by (6.2), the solution of the differential equation (6.1) is given by

$$x(t) \begin{cases} for,\ (j-1)t_u \le t \le (j-1)t_u + t_1, \\ \quad = \dfrac{u_0 - d_0}{u_1 + d_1}\left[1 - e^{-(u_1+d_1)\{t-(j-1)t_u\}}\right] \\ for,\ (j-1)t_u + t_1 \le t \le (j-1)t_u + t_3, \\ \quad = \left(Q_1 + \dfrac{d_0}{d_1}\right)e^{-d_1\{t-(j-1)t_u-t_1\}} - \dfrac{d_0}{d_1} \\ for,\ (j-1)t_u + t_3 \le t \le jt_u, \\ \quad = \left(Q_2 - \dfrac{u_0 - d_0}{u_1 + d_1}\right)e^{-d_1\{t-(j-1)t_u-t_3\}} - \dfrac{u_0 - d_0}{u_1 + d_1} \end{cases} \quad (6.3)$$

where,

$$Q_1 = \frac{u_0 - d_0}{u_1 + d_1}\Big[1 - e^{-(u_1+d_1)t_1}\Big] \tag{6.4}$$

$$t_2 = t_1 + \frac{1}{d_1} log\Big[\frac{d_1 Q_1 + d_0}{d_0}\Big] \tag{6.5}$$

$$Q_2 = -\frac{d_0}{d_1}[1 - e^{-d_1(t_3-t_2)}] \tag{6.6}$$

$$t_u = t_3 + \frac{1}{d_1} log\Big(1 - Q_2 \frac{u_1 + d_1}{u_0 - d_0}\Big) \tag{6.7}$$

Expected value of holding cost for n full cycles

Present value of holding cost of the inventory for $j - th\,(1 \leq j \leq n)$ cycle, HC_j is given by

$$\begin{aligned} HC_j &= c_1\Big[\int_{(j-1)t_u}^{(j-1)t_u+t_1} + \int_{(j-1)t_u+t_1}^{(j-1)t_u+t_2}\Big] x(t)e^{-Rt}dt \\ &= c_1 \int_{(j-1)t_u}^{(j-1)t_u+t_1} \frac{u_0 - d_0}{u_1 + d_1}\Big[1 - e^{-(u_1+d_1)\{t-(j-1)t_u\}}\Big] e^{-Rt}dt \\ &\quad + c_1 \int_{(j-1)t_u+t_1}^{(j-1)t_u+t_2} \Big[\Big\{Q_1 + \frac{d_0}{d_1}\Big\} e^{-d_1\{t-(j-1)t_u-t_1\}} - \frac{d_0}{d_1}\Big] e^{-Rt}dt \\ &= c_1[I_{1j} + I_{2j}] \;\; (say) \end{aligned} \tag{6.8}$$

where,

$$\begin{aligned} I_{1j} &= \int_{(j-1)t_u}^{(j-1)t_u+t_1} \frac{u_0 - d_0}{u_1 + d_1}\Big[1 - e^{-(u_1+d_1)\{t-(j-1)t_u\}}\Big] e^{-Rt}dt \\ &= \frac{u_0 - d_0}{u_1 + d_1}\Big[\frac{1 - e^{Rt_1}}{R} - \frac{1 - e^{-(u_1+d_1+R)t_1}}{u_1 + d_1 + R}\Big] e^{-R(j-1)t_u} \end{aligned}$$

$$\begin{aligned} and,\;\; I_{2j} &= \int_{(j-1)t_u+t_1}^{(j-1)t_u+t_2} \Big[\Big\{Q_1 + \frac{d_0}{d_1}\Big\} e^{-d_1\{t-(j-1)t_u-t_1\}} - \frac{d_0}{d_1}\Big] e^{-Rt}dt \\ &= \Big[\Big(Q_1 + \frac{d_0}{d_1}\Big) \frac{e^{d_1 t_1}\{e^{-(R+d_1)t_1} - e^{-(R+d_1)t_2}\}}{d_1 + R} \\ &\qquad - \frac{d_0\{e^{-Rt_1} - e^{-Rt_2}\}}{d_1 R}\Big] e^{-R(j-1)t_u} \end{aligned}$$

Therefore, the present value of total holding cost (HC) for n full cycle is

$$HC = \sum_{j=1}^{n} HC_j = c_1 \sum_{j=1}^{n} (I_{1j} + I_{2j})$$

Since, the business planning horizon H is a random variable which follows exponential distribution with corresponding probability density function as

$$f(h) = \begin{cases} \lambda e^{-\lambda h}, & h \geq 0 \\ 0, & otherwise \end{cases}$$

Therefore, the expected value of holding cost for n full business cycle is

$$\begin{aligned} E\{HC\} &= \sum_{n=0}^{\infty} \int_{nt_u}^{(n+1)t_u} HC\lambda e^{-\lambda h} dh \\ &= c_1 \sum_{n=0}^{\infty} \int_{nt_u}^{(n+1)t_u} \sum_{j=1}^{n} (I_{1j} + I_{2j}) \lambda e^{-\lambda h} dh \\ &= c_1 \Bigg[\frac{u_0 - d_0}{u_1 + d_1} \Bigg\{ \frac{1 - e^{-Rt_1}}{R} - \frac{1 - e^{-(u_1+d_1+R)t_1}}{u_1 + d_1 + R} \Bigg\} \Bigg\{ \frac{e^{-\lambda t_u}}{1 - e^{-(R+\lambda)t_u}} \Bigg\} \\ &\quad + \Bigg\{ \frac{e^{d_1 t_1}(Q_1 + \frac{d_0}{d_1})}{d_1 + R} \{e^{-(R+d_1)t_1} - e^{-(R+d_1)t_2}\} - \frac{d_0(e^{Rt_1} - e^{Rt_2})}{d_1 R} \Bigg\} \times \\ &\quad \Bigg\{ \frac{1 - e^{-\lambda t_u}}{1 - e^{-(R+\lambda)t_u}} \Bigg\} \Bigg] \end{aligned} \tag{6.9}$$

Expected value of production cost for n full cycles

The present value of production cost for j-th full cycle ($j = 1, 2, \cdots, n$), PC_j, is given by

$$\begin{aligned} PC_j &= p_0 e^{-(j-1)\gamma} \Bigg[\int_{(j-1)t_u}^{(j-1)t_u + t_1} + \int_{(j-1)t_u + t_3}^{jt_u} \Bigg] \Bigg\{ u_0 - u_1 x(t) \Bigg\} e^{-Rt} dt \\ &= [PC_{1j} + PC_{2j}] \ (say) \end{aligned} \tag{6.10}$$

Where,

$$\begin{aligned} PC_{1j} &= p_0 e^{-(j-1)\gamma} \int_{(j-1)t_u}^{(j-1)t_u + t_1} \Bigg\{ u_0 - u_1 x(t) \Bigg\} e^{-Rt} dt \\ &= p_0 e^{-(j-1)(\gamma + Rt_u)} \Bigg[\frac{(u_0 d_1 + u_1 d_0)(1 - e^{-Rt_1})}{R(u_1 + d_1)} + \frac{u_1(u_0 - d_0)\{1 - e^{-(u_1+d_1+R)t_1}\}}{(u_1 + d_1)(u - 1 + d_1 + R)} \Bigg] \end{aligned}$$

$$\begin{aligned} and, \ PC_{2j} &= p_0 e^{-(j-1)\gamma} \int_{(j-1)t_u + t_3}^{jt_u} \Bigg\{ u_0 - u_1 x(t) \Bigg\} e^{-Rt} dt \\ &= p_0 e^{-(j-1)(\gamma + Rt_u)} \Bigg[\frac{(u_0 d_1 + u_1 d_0)(e^{-Rt_3} - e^{Rt_u})}{R(u_1 + d_1)} \\ &\quad - \Bigg\{ Q_2 - \frac{u_0 - d_0}{u_1 + d_1} \Bigg\} \frac{u_1 e^{(u_1+d_1)t_3} \{e^{-(u_1+d_1+R)t_3} - e^{-(u_1+d_1+R)t_u}\}}{(u_1 + d_1 + R)} \Bigg] \end{aligned}$$

Then the expected value of total production cost (PC) over the n full cycle is

$$\begin{aligned}
E\{PC\} &= \sum_{n=0}^{\infty}\int_{nt_u}^{(n+1)t_u}\sum_{j=1}^{n}(PC_{1j}+PC_{2j})\lambda e^{-\lambda h}dh \\
&= \frac{p_0 e^{-\lambda t_u}}{1-e^{-(R+\lambda)t_u-\gamma}}\left[\frac{(u_0 d_1+u_1 d_0)(1-e^{-Rt_1})}{R(u_1+d_1)}\right. \\
&\quad +\frac{u_1(u_0-d_0)\{1-e^{-(u-1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)}+\frac{(u_0 d_1+u_1 d_0)(e^{-Rt_3}-e^{-Rt_u})}{R(u_1+d_1)} \\
&\quad \left.-u_1\left(Q_2-\frac{u_0-d_0}{u_1+d_1}\right)\frac{e^{-Rt_3}-e^{-(u_1+d_1)(t_u-t_3)-Rt_u}}{u_1+d_1+R}\right] \qquad (6.11)
\end{aligned}$$

Expected value of shortage cost for n full cycles

The present value of shortage cost for the j-th full cycle (SC_j) where $j=1,2,\cdots,n$ is given by

$$\begin{aligned}
SC_j &= s_1\left[\int_{(j-1)t_u+t_2}^{(j-1)t_u+t_3}+\int_{(j-1)t_u+t_3}^{jt_u}\right]\{d_0+d_1x(t)\}e^{-Rt}dt \\
&= s_1\{SC_{1j}+SC_{2j}\}\quad (say) \\
\text{where, } SC_{1j} &= \int_{(j-1)t_u+t_2}^{(j-1)t_u+t_3}\{d_0+d_1x(t)\}e^{-Rt}dt \\
&= \frac{d_0\{e^{-(d_1+R)t_2}-e^{-(d_1+R)t_3}\}}{R+d_1}e^{-R(j-1)t_u}
\end{aligned}$$

$$\begin{aligned}
and,\quad SC_{2j} &= \int_{(j-1)t_u+t_3}^{jt_u}\{d_0+d_1x(t)\}e^{-Rt}dt \\
&= \left[\left(Q_2+\frac{u_0-d_0}{u_1+d_1}\right)\frac{e^{-(u_1+d_1)t_3}\{e^{-(u_1+d_1+R)t_3}-e^{(u_1+d_1+R)t_u}\}}{u_1+d_1+R}\right. \\
&\quad \left.+\frac{(u_0-d_0)(e^{-Rt_3}-e^{Rt_u})}{R(u_1+d_1)}\right]e^{-R(j-1)t_u}
\end{aligned}$$

Thus the expected value of total shortage cost (SC) over n full cycles is given by

$$\begin{aligned}
E\{SC\} &= s_1\sum_{n=0}^{\infty}\int_{nt_u}^{(n+1)t_u}\sum_{j=1}^{n}(SC_{1j}+SC_{2j})\lambda e^{-\lambda h}dh \\
&= \frac{s_1 e^{-\lambda t_u}}{1-e^{-(R+\lambda)t_u}}\left[\frac{d_0\{e^{-(d_1+R)t_2}-e^{-(d_1+R)t_3}\}}{R+d_1}\right. \\
&\quad +\left(Q_2+\frac{u_0-d_0}{u_1+d_1}\right)\frac{\{e^{-2(u_1+d_1)t_3-Rt_3}-e^{(u_1+d_1)(t_u-t_3)+Rt_u}\}}{u_1+d_1+R} \\
&\quad \left.+\frac{(u_0-d_0)(e^{-Rt_3}-e^{Rt_u})}{R(u_1+d_1)}\right] \qquad (6.12)
\end{aligned}$$

Expected value of set-up cost for n full cycles

Present value of set-up cost in j-th full cycle where, $j = 1, 2, \cdots, n$ is

$$[c_3^1 + c_3^2 e^{-\delta j}]e^{-R(j-1)t_u}.$$

Therefore the expected value of total set-up cost (SUC) for n full cycle is given by

$$\begin{aligned} E\{SUC\} &= \sum_{n=0}^{\infty} \int_{nt_u}^{(n+1)t_u} \sum_{j=1}^{n} [c_3^1 + c_3^2 e^{-\delta j}] e^{-R(j-1)t_u} \lambda e^{-\lambda h} dh \\ &= \frac{c_3^1 e^{-\lambda t_u}}{1 - e^{-(R+\lambda)t_u}} + \frac{c_3^2 e^{-\delta j} e^{-\lambda t_u}}{1 - e^{-(Rt_u + \lambda t_u + \delta)}} \end{aligned} \tag{6.13}$$

Expected value of sell revenue for n full cycles

The present value of sell revenue (SR_j) in j-th full cycle where, $j = 1, 2, \cdots, n$ is given by

$$\begin{aligned} SR_j &= m_0 p_0 e^{-(j-1)\gamma} \left[\int_{(j-1)t_u}^{(j-1)t_u + t_1} + \int_{(j-1)t_u + t_1}^{(j-1)t_u + t_2} + \int_{(j-1)t_u + t_3}^{jt_u} \right] \{d_0 + d_1 x(t)\} e^{-Rt} dt \\ &= m_0 p_0 e^{-(j-1)\gamma} [SR_{1j} + SR_{2j} + SR_{3j}] \end{aligned}$$

where,

$$\begin{aligned} SR_{1j} &= \int_{(j-1)t_u}^{(j-1)t_u + t_1} \{d_0 + d_1 x(t)\} e^{-Rt} dt \\ &= \left[\frac{(u_0 d_1 + u_1 d_0)(1 - e^{-Rt_1})}{R(u_1 + d_1)} - \frac{d_1(u_0 - d_0)\{1 - e^{-(u_1 + d_1 + R)t_1}\}}{(u_1 + d_1)(u_1 + d_1 + R)} \right] e^{-R(j-1)t_u} \\ SR_{2j} &= \int_{(j-1)t_u + t_1}^{(j-1)t_u + t_2} \{d_0 + d_1 x(t)\} e^{-Rt} dt \\ &= \left[(d_0 + d_1 Q_1) \frac{e^{-Rt_1} - e^{-d_1(t_2 - t_1) - Rt_1}}{d_1 + R} \right] e^{-R(j-1)t_u} \\ & and \\ SR_{3j} &= \int_{(j-1)t_u + t_3}^{jt_u} \{d_0 + d_1 x(t)\} e^{-Rt} dt \\ &= e^{-(j-1)Rt_u} \left[\frac{(u_0 d_1 + u_1 d_0)(e^{-Rt_3} - e^{-Rt_u})}{R(u_1 + d_1)} + \left(Q_2 - \frac{u_0 - d_0}{u_1 + d_1} \right) \times \right. \\ & \left. \qquad \frac{d_1\{e^{-Rt_3} - e^{-(u_1 + d_1)(t_u - t_3) - Rt_u}\}}{u_1 + d_1 + R} \right] \end{aligned}$$

Therefore the expected value of total sale revenue (SR) for n full cycle is given by

$$\begin{aligned}
E\{SR\} &= \sum_{n=0}^{\infty}\int_{nt_u}^{(n+1)t_u} m_0p_0\sum_{j=1}^{n} e^{-(j-1)\gamma}[SR_{1j}+SR_{2j}+SR_{3j}]\lambda e^{-\lambda h}dh \\
&= \frac{m_0p_0e^{-\lambda t_u}}{1-e^{-(Rt_u+\lambda t_u+\gamma)}}\Bigg[\frac{(u_0d_1+u_1d_0)(1-e^{-Rt_1})}{R(u_1+d_1)} \\
&\quad+\frac{d_1(u_0-d_0)\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)}+\frac{d_0+d_1Q_1}{d_1+R}\{e^{-Rt_1}-e^{-d_1(t_2-t_1)-Rt_2}\} \\
&\quad+\frac{(u_0d_1+u_1d_0)(e^{-Rt_3}-e^{-Rt_u})}{R(u_1+d_1)}+\left(Q_2-\frac{u_0-d_0}{u_1+d_1}\right)\times \\
&\qquad\frac{d_1\{e^{-Rt_3}-e^{-(u_1+d_1)(t_u-t_3)-Rt_u}\}}{u_1+d_1+R}\Bigg] \qquad (6.14)
\end{aligned}$$

Expected value of total profit for n full cycles

Thus the expected value of total profit (TP_1) in n full cycles can be obtained using the following expression.

$$E\{TP_1\} = E\{SR\}-E\{HC\}-E\{PC\}-E\{SC\}-E\{SUC\} \qquad (6.15)$$

Where, the expected value of $HC,\ PC,\ SC,\ SUC,\ and\ SR$ are given by equations (6.9), (6.11), (6.12), (6.13) and (6.14) respectively.

Formulation for last cycle

The leading differential equation describing the inventory level $x(t)$ in the last cycle ($nt_u \leq t \leq (n+1)t_u$), are given by

$$\frac{dx(t)}{dt} = \begin{cases} u_0-d_0-(u_1+d_1)x(t) & for,\ nt_u \leq t \leq nt_u+t_1 \\ -d_0-d_1x(t) & for,\ nt_u+t_1 \leq t \leq nt_u+t_3 \\ u_0-d_0-(u_1+d_1)x(t) & for,\ nt_u+t_3 \leq t \leq (n+1)t_u \end{cases} \qquad (6.16)$$

with the boundary conditions

$$x(t) = \begin{cases} 0 & for,\ t=nt_u \\ Q_1 & for,\ t=nt_u+t_1 \\ 0 & for,\ t=nt_u+t_2 \\ Q_2 & for,\ t=nt_u+t_3 \\ 0 & for,\ t=(n+1)t_u \end{cases} \qquad (6.17)$$

Using the boundary conditions (6.17) the solution of the differential equation (6.16) is given by

$$x(t)\begin{cases} for,\ nt_u \le t \le nt_u+t_1 \\ \quad = \dfrac{u_0-d_0}{u_1+d_1}[1-e^{-(u_1+d_1)(t-nt_u)}] \\ for,\ nt_u+t_1 \le t \le nt_u+t_3 \\ \quad = \left[Q_1+\dfrac{d_0}{d_1}\right]e^{-d_1(t-nt_u-t_1)}-\dfrac{d_0}{d_1} \\ for,\ nt_u+t_3 \le t \le (n+1)t_u \\ \quad = \left[Q_2-\dfrac{u_0-d_0}{u_1+d_1}\right]e^{-d_1(t-nt_u-t_3)}-\dfrac{u_0-d_0}{u_1+d_1} \end{cases} \tag{6.18}$$

Therefore the expected value of total set-up cost (SUCL) for last cycle is given by

$$\begin{aligned} E\{SUCL\} &= \sum_{n=0}^{\infty}\int_{nt_u}^{(n+1)t_u}[c_3^1+c_3^2e^{-\delta(n+1)}]e^{-Rnt_u}\lambda e^{-\lambda h}dh \\ &= \frac{c_3^1(1-e^{-\lambda t_u})}{1-e^{-(R+\lambda)t_u}}+\frac{c_3^2e^{-\delta}(1-1e^{-\lambda t_u})}{1-e^{-(Rt_u+\lambda t_u+\delta)}} \end{aligned} \tag{6.19}$$

Now the following cases may arise:

Case-1 ($nt_u \le h \le nt_u+t_1$)

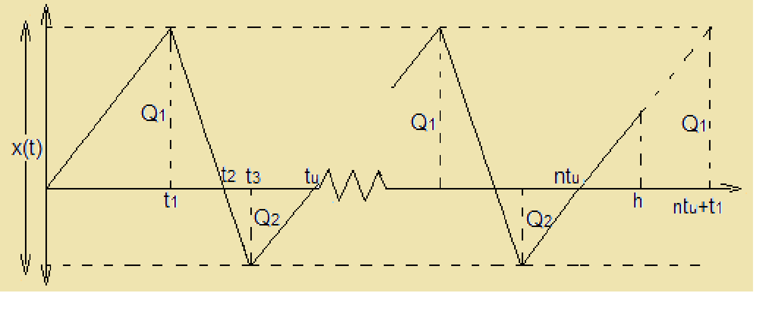


Figure 6.2: Model structure for last cycle case-1

According to Fig.-6.2 Present value of holding cost of the inventory for the last cycle, HCL_1 is given by

$$\begin{aligned} HCL_1 &= c_1\int_{nt_u}^{h}x(t)e^{-Rt}dt \\ &= c_1\frac{u_0-d_0}{u_1+d_1}\left[\frac{e^{-Rnt_u}-e^{Rh}}{R}-\frac{e^{-2(u_1+d_1)nt_u-Rnt_u}-e^{-(u_1+d_1)(nt_u+h)-Rh}}{u_1+d_1+R}\right] \end{aligned}$$

Therefore, the expected value of holding cost for the last cycle ($nt_u \leq t \leq nt_u + t_1$) is

$$\begin{aligned} E\{HCL_1\} &= \sum_{n=0}^{\infty} \int_{nt_u}^{nt_u+t_1} HCL_1 \lambda e^{-\lambda h} dh \\ &= \frac{c_1(u_0-d_0)}{(u_1+d_1)\{1-e^{-(R+\lambda)t_u}\}} \left[\frac{1-e^{-\lambda t_1}}{R} - \frac{\lambda\{1-e^{-(R+\lambda)t_1}\}}{R(R+\lambda)} \right. \\ &\quad \left. - \frac{1-e^{-\lambda t_1}}{u_1+d_1+R} + \frac{\lambda\{1-e^{-(u_1+d_1+R+\lambda)t_1}\}}{(u_1+d_1+R)(u_1+d_1+R+\lambda)} \right] \end{aligned} \tag{6.20}$$

Present value of production cost (PCL_1) for this case of the last cycle is

$$\begin{aligned} PCL_1 &= p_0 e^{-n\gamma} \int_{nt_u}^{h} \{u_0 - u_1 x(t)\} e^{-Rt} dt \\ &= p_0 e^{-n\gamma} \left[\frac{(u_0 d_1 + u_1 d_0)(e^{-Rnt_u} - e^{-Rh})}{R(u_1+d_1)} \right. \\ &\quad \left. - \frac{u_1(u_0-d_0)\{e^{-Rnt_u} - e^{-(u_1+d_1)(h-nt_u)-Rh}\}}{(u_1+d_1)(u_1+d_1+R)} \right] \end{aligned}$$

The expected value of production cost is given by

$$\begin{aligned} E\{PCL_1\} &= \sum_{n=0}^{\infty} \int_{nt_u}^{nt_u+t_1} PCL_1 \lambda e^{-\lambda h} dh \\ &= \frac{p_0}{1-e^{-(R+\lambda)t_u-\gamma}} \left[\frac{(u_0 d_1 + u_1 d_0)}{R(u_1+d_1)} \left\{ (1-e^{-\lambda t_1}) - \frac{\lambda(1-e^{-Rt_1-\lambda t_1})}{R+\lambda} \right\} \right. \\ &\quad - \frac{u_1(u_0-d_0)}{(u_1+d_1)(u_1+d_1+R)} \left\{ (1-e^{-\lambda t_1}) \right. \\ &\quad \left. \left. - \frac{\lambda\{1-e^{-(u_1+d_1+R+\lambda)t_1}\}}{u_1+d_1+R+\lambda} \right\} \right] \end{aligned} \tag{6.21}$$

The present value of sale revenue (SRL_1^1) for the last cycle is given by

$$\begin{aligned} SRL_1^1 &= m_0 p_0 e^{-\gamma n} \int_{nt_u}^{h} \{d_0 + d_1 x(t)\} e^{-Rt} dt \\ &= m_0 p_0 e^{-\gamma n} \left[\frac{u_0 d_1 + u_1 d_0}{R(u_1+d_1)} (e^{-Rnt_u} - e^{-Rh}) \right. \\ &\quad \left. - \frac{u_1(u_0-d_0)\{e^{-Rnt_u} - e^{-(u_1+d_1)(h-nt_u)-Rh}\}}{(u_1+d_1)(u_1+d_1+R)} \right] \end{aligned}$$

and the expected value of sale revenue in this case is given by

$$\begin{aligned} E\{SRL_1^1\} &= \sum_{n=0}^{\infty}\int_{nt_u}^{nt_u+t_1} SRL_1^1\lambda e^{-\lambda h}dh \\ &= \frac{m_0p_0}{1-e^{-(R+\lambda)t_u-\gamma}}\left[\frac{u_0d_1+u_1d_0}{R(u_1+d_1)}\left\{(1-e^{-\lambda t_1})-\frac{\lambda\{1-e^{-(R+\lambda)t_1}\}}{R+\lambda}\right\}\right. \\ &\quad -\frac{d_1(u_0-d_0)}{(u_1+d_1)(u_1+d_1+R)}\left\{(1-e^{\lambda t_1})\right. \\ &\quad \left.\left.-\frac{\lambda\{1-e^{-(u_1+d_1+R+\lambda)t_1}\}}{u_1+d_1+R+\lambda}\right\}\right] \end{aligned} \tag{6.22}$$

Also in this case the business cycle ends before the stock is finished. So, the remaining amount of stock is sold in reduced price. Thus the expected value of revenue earned from reduce sale (SRL_1^2) is given by

$$\begin{aligned} E\{SRL_1^2\} &= s_2\sum_{n=0}^{\infty}\int_{nt_u}^{nt_u+t_1} x(h)\lambda e^{-\lambda h}dh \\ &= \frac{s_2\lambda(u_0-d_0)}{(u_1+d_1)\{1-e^{-(R+\lambda)t_u}\}}\left[\frac{1-e^{-(R+\lambda)t_1}}{R+\lambda}\right. \\ &\quad \left.-\frac{1-e^{-(u_1+d_1+R+\lambda)t_1}}{u_1+d_1+R+\lambda}\right] \end{aligned} \tag{6.23}$$

Case-2 ($nt_u + t_1 \leq h \leq nt_u + t_2$)

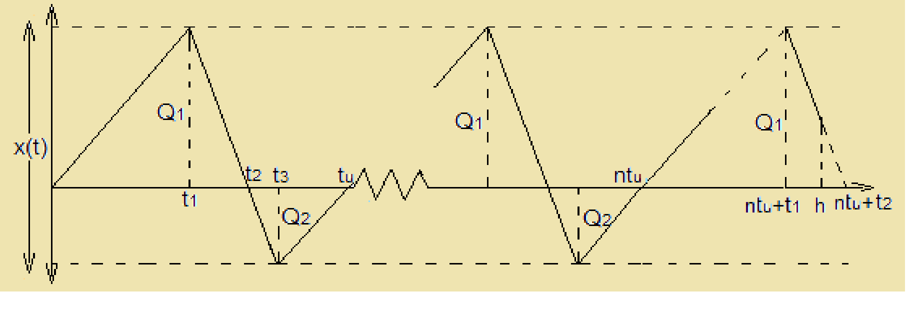


Figure 6.3: Model structure for last cycle case-2

In this case, a time gap ($nt_u + t_1$, t_2) is added (see Fig.-6.3) the present value of holding cost (HCL_2) is given by

$$\begin{aligned}
HCL_2 &= c_1\left[\int_{nt_u}^{nt_u+t_1} + \int_{nt_u+t_1}^{h}\right] x(t)e^{-Rt}dt \\
&= \frac{c_1(u_0-d_0)}{u_1+d_1}\left[\frac{1-e^{Rt_u}}{R} - \frac{1-e^{-(u_1+d_1+R)t_1}}{u_1+d_1+R}\right]e^{-Rnt_u} \\
&\quad + c_1\left[(Q_2+\frac{d_0}{d_1})\frac{e^{-R(nt_u+t_1)}-e^{-d_1(h-nt_u+t_1)-Rh}}{d_1+R} - \frac{d_0\{e^{-R(nt_u+t_1)}-e^{-Rh}\}}{d_1R}\right]
\end{aligned}$$

and the expected value of holding cost in this case is given by

$$\begin{aligned}
E\{HCL_2\} &= \sum_{n=0}^{\infty}\int_{nt_u+t_1}^{nt_u+t_2} HCL_2\lambda e^{-\lambda h}dh \\
&= \frac{c_1}{1-e^{-(R+\lambda)t_u}}\left[\frac{c_1(u_0-d_0)}{u_1+d_1}\left\{\frac{1-e^{Rt_1}}{R} - (e^{-\lambda t_1}-e^{-\lambda t_2})\times\right.\right. \\
&\quad \left.\frac{1-e^{-(u_1+d_1+R)t_1}}{u_1+d_1+R}\right\} + \frac{d_1Q_1+d_0}{d_1(d_1+R)}\left\{e^{-Rt_1}(e^{-\lambda t_1}-e^{-\lambda t_2})\right. \\
&\quad \left.\left. - \frac{\lambda\{e^{-(u_1+R)t_1}-e^{-d_1(t_2-t_1)-(u_1+R)t_2}\}}{d_1+R+\lambda}\right\}\right] \qquad (6.24)
\end{aligned}$$

Present value of sale revenue in this case (SRL_2^1) for the last cycle is given by

$$\begin{aligned}
SRL_2^1 &= m_0p_0e^{-\gamma n}\left[\int_{nt_u}^{nt_u+t_1} + \int_{nt_u+t_1}^{h}\right]\{d_0+d_1x(t)\}e^{-Rt}dt \\
&= m_0p_0e^{-\gamma n}\left[\frac{u_0d_1+u_1d_0}{R(u_1+d_1)}\{e^{-Rnt_u}-e^{-R(nt_u+t_1)}\}\right. \\
&\quad - \frac{d_1(u_0-d_0)e^{-Rnt_u}\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)} \\
&\quad \left. + \frac{d_0+d_1Q_1}{d_1+R}\{e^{-R(nt_u+t_1)}-e^{-d_1(h-nt_u-t_1)-Rh}\}\right]
\end{aligned}$$

The expected value of sale revenue in this case is

$$\begin{aligned}
E\{SRL_2^1\} &= \sum_{n=0}^{\infty}\int_{nt_u+t_1}^{nt_u+t_2} SRL_2^1\lambda e^{-\lambda h}dh \\
&= \frac{m_0p_0}{1-e^{-(Rt_u+\lambda t_u+\gamma)}}\left[\left\{\frac{u_1d_0+d_1u_0}{R(u_1+d_1)}(1-e^{-Rt_1})\right.\right. \\
&\quad \left. - \frac{d_1(u_0-d_0)\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)} + \frac{(d_0+d_1Q_1)}{d_1+R}e^{-Rt_1}\right\}(e^{-\lambda t_1}-e^{-\lambda t_2}) \\
&\quad \left. + \frac{\lambda(d_0+d_1Q_1)}{(d_1+R)(d_1+R+\lambda)}\left\{e^{-(u_1+R)t_1}-e^{-(t_2-t_1)d_1-(u_1+R)t_2}\right\}\right] \qquad (6.25)
\end{aligned}$$

According to the time horizon, a situation of reduced sale is also occurs in this case. So the expected value of earn from reduced sale (SRL_2^2) is given by

$$\begin{aligned} E\{SRL_2^2\} &= s_2 \sum_{n=0}^{\infty} \int_{nt_u+t_1}^{nt_u+t_2} \{Q_1 - x(h)\}\lambda e^{-\lambda h} dh \\ &= \frac{s_2(d_1 Q_1 + d_0)}{d_1\{1 - e^{-(R+\lambda)t_u}\}} \left[\frac{\lambda\{e^{-(R+\lambda)t_1} - e^{-(R+\lambda)t_u}\}}{R+\lambda} \right. \\ &\quad \left. - \frac{\lambda\{e^{-(u_1+d_1+R+\lambda)t_1} - e^{-(u_1+d_1+R+\lambda)t_u}\}}{d_1 + R + \lambda} \right] \end{aligned} \tag{6.26}$$

Case-3 ($nt_u + t_2 \leq h \leq nt_u + t_3$)

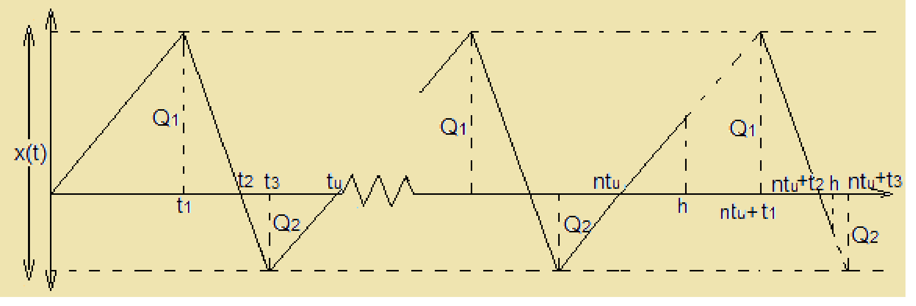


Figure 6.4: Model structure for last cycle case-3

In this case as the stock is finished so a situation of shortage arises (see Fig.-6.4). So, the shortage cost in this case is given by

$$\begin{aligned} SCL_1 &= s_1 \int_{nt_u+t_2}^{nt_u+t_3} \{d_0 + d_1 x(t)\} e^{-Rt} dt \\ &= \frac{s_1 d_0}{R + d_1} \left[e^{-R(nt_u+t_2)} - e^{-(h-nt_u-t_2)d_1 - Rh} \right] \end{aligned}$$

The expected value of shortage cost in this case is given by

$$\begin{aligned} E\{SCL_1\} &= \sum_{n=0}^{\infty} \int_{nt_u+t_2}^{nt_u+t_3} SCL_1 \lambda e^{-\lambda h} dh \\ &= \frac{s_1 d_0}{(d_1 + R)\{1 - e^{-(R+\lambda)t_u}\}} \left[(e^{-\lambda t_2} - e^{-\lambda t_3}) \right. \\ &\quad \left. - \frac{\lambda\{e^{-(R+\lambda)t_2} - e^{-(t_3-t_2)d_1 - (R+\lambda)t_3}\}}{d_1 + R + \lambda} \right] \end{aligned} \tag{6.27}$$

Case-4 ($nt_u + t_3 \leq h \leq (n+1)t_u$)

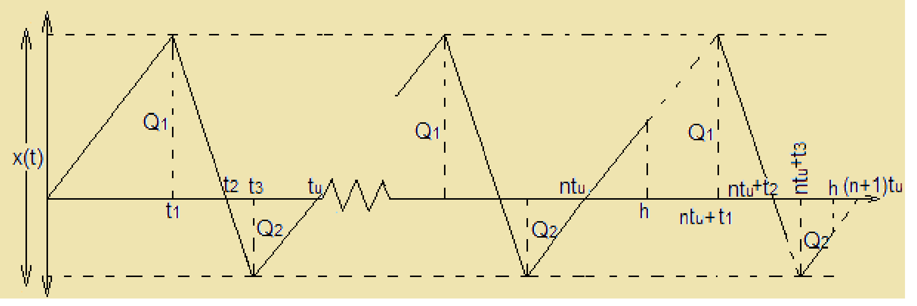


Figure 6.5: Model structure for last cycle case-4

In this case there is a need of production to backlog the shortage (see Fig.-6.6), so the production cost (PCL_4) is given by

$$\begin{aligned} PCL_4 &= p_0 e^{-\gamma n}\left[\int_{nt_u}^{nt_u+t_1} + \int_{nt_u+t_3}^{h}\right]\{u_0 - u_1 x(t)\}e^{-Rt}dt \\ &= p_0 e^{-\gamma n}\left[\left\{\frac{(u_0 d_1 + u_1 d_0)(1-e^{-Rt_1})}{R(u_1+d_1)} + \frac{u_1(u_0-d_0)\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)}\right\}\times\right. \\ & e^{-Rnt_u} + \frac{(u_0 d_1 + u_1 d_0)\{e^{-R(nt_u+t_3)} - e^{-Rh}\}}{R(u_1+d_1)} - \left(Q_2 - \frac{u_0-d_0}{u_1+d_1}\right)\times \\ & \left.\frac{u_1\{e^{-R(nt_u+t_3)} - e^{-(u_1+d_1)(h-nt_u-t_3)-Rh}\}}{u_1+d_1+R}\right] \end{aligned}$$

The expected value of PCL_4 is given by

$$\begin{aligned} E\{PCL_4\} &= \sum_{n=0}^{\infty}\int_{nt_u+t_3}^{(n+1)t_u} PCL_4 \lambda e^{-\lambda h} dh \\ &= \left[\left\{\frac{u_0 d_1 + u_1 d_0}{R(u_1+d_1)}(1-e^{-Rt_1}) + \frac{u_1(u_0-d_0)\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)}\right\}\times\right. \\ & (e^{-\lambda t_3} - e^{-\lambda t_u}) + \frac{u_0 d_1 + u_1 d_0}{R(u_1+d_1)}\left\{e^{-Rt_3}(e^{-\lambda t_3} - e^{-\lambda t_u})\right. \\ & \left. -\frac{\lambda\{e^{-(R+\lambda)t_3} - e^{-(R+\lambda)t_u}\}}{R+\lambda}\right\} - \left(Q_2 - \frac{u_0-d_0}{u_1+d_1}\right)\frac{u_1}{u_1+d_1+R}\times \\ & \left.\left\{e^{-Rt_3}(e^{-\lambda t_3} - e^{-\lambda t_u}) - \frac{\lambda\{e^{-(R+\lambda)t_3} - e^{-(u_1+d_1)(t_u-t_3)-(R+\lambda)t_u}\}}{u_1+d_1+R+\lambda}\right\}\right] \\ & \times \frac{p_0}{1-e^{-(R+\lambda)t_u-\gamma}} \end{aligned} \tag{6.28}$$

Now the earned sale revenue (SRL_4^1) in this case for the last cycle

$$\begin{aligned}
SRL_4^1 &= m_0 p_0 e^{-\gamma n}\left[\int_{nt_u}^{nt_u+t_1} + \int_{nt_u+t_1}^{nt_u+t_2} + \int_{nt_u+t_3}^{h}\right]\{d_0 + d_1 x(t)\}e^{-Rt}dt \\
&= m_0 p_0 e^{-\gamma n}\left[\frac{u_0 d_1 + u_1 d_0}{R(u_1+d_1)}\{e^{-Rnt_u} - e^{-R(nt_u+t_1)}\}\right. \\
&\qquad - \frac{d_1(u_0-d_0)e^{-Rnt_u}\{1 - e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)} \\
&\qquad + \frac{(d_0 + d_1 Q_1)e^{-Rnt_u}\{e^{-Rt_1} - e^{-d_1(t_2-t_1)-Rt_2}\}}{d_1+R} \\
&\qquad + \frac{(u_0 d_1 + u_1 d_0)}{R(u_1+d_1)}\{e^{-R(nt_u+t_3)} - e^{-Rh}\} \\
&\qquad \left. + \left(Q_2 - \frac{u_0-d_0}{u_1+d_1}\right)\frac{d_1\{e^{-R(nt_u+t_3)} - e^{-(u_1+d_1)(h-nt_u-t_3)-Rh}\}}{u_1+d_1+R}\right]
\end{aligned}$$

So the expected value of sale revenue (SRL_4^1) is

$$\begin{aligned}
E\{SRL_4^1\} &= \sum_{n=0}^{\infty}\int_{nt_u+t_3}^{(n+1)t_u} SRL_4^1 \lambda e^{-\lambda h}dh \\
&= \frac{m_0 p_0}{1 - e^{-(R+\lambda)t_u-\gamma}}\left[\frac{u_0 d_1 + u_1 d_0}{R(u_1+d_1)}(1 - e^{-Rt_1})\right. \\
&\qquad - \frac{d_1(u_0-d_0)\{1 - e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)} \\
&\qquad + \frac{(d_0 + d_1 Q_1)\{e^{-Rt_1} - e^{-d_1(t_2-t_1)-Rt_2}\}}{d_1+R} \\
&\qquad + \frac{u_0 d_1 + u_1 d_0}{R(u_1+d_1)}\left\{e^{-Rt_3}(e^{-\lambda t_3} - e^{-\lambda t_u}) - \frac{\lambda\{e^{-(R+\lambda)t_3} - e^{-(R+\lambda)t_u}\}}{R+\lambda}\right\} \\
&\qquad + \left(Q_2 - \frac{u_0-d_0}{u_1+d_1}\right)\frac{d_1}{u_1+d_1+R}\left\{e^{-Rt_3}(e^{-\lambda t_3}\right. \\
&\qquad \left.\left. -e^{-\lambda t_u}) - \frac{\lambda\{e^{-(R+\lambda)t_3} - e^{-(u_1+d_1)(t_u-t_3)-(R+\lambda)t_u}\}}{u_1+d_1+R+\lambda}\right\}\right] \quad (6.29)
\end{aligned}$$

Shortage is also arises in this case. So, the present value of shortage cost (SCL_2) is given by

$$\begin{aligned} SCL_2 &= s_1\left[\int_{nt_u+t_2}^{nt_u+t_3} + \int_{nt_u+t_3}^{h}\right]\{d_0 - d_1x(t)\}e^{-Rt}dt \\ &= s_1\left[\frac{d_0e^{-Rnt_u}}{R+d_1}\left\{e^{-(d_1+R)t_2} - e^{-(d_1+R)t_3}\right\}\right. \\ &\quad +\left(Q_2 + \frac{u_0-d_0}{u_1+d_1}\right)\left\{\frac{\{e^{-R(nt_u+t_3)} - e^{-(u_1+d_1)(h-nt_u-t_3)-Rh}\}}{u_1+d_1+R}\right\} \\ &\quad \left.+\frac{u_0-d_0}{R(u_1+d_1)}\left\{e^{-R(nt_u+t_3)} - e^{-Rh}\right\}\right] \end{aligned}$$

Thus the expected value of shortage cost (SCL_2) in this case for the last cycle is given by

$$\begin{aligned} E\{SCL_2\} &= \sum_{n=0}^{\infty}\int_{nt_u+t_3}^{h} SCL_2\lambda e^{-\lambda h}dh \\ &= \frac{s_1}{1-e^{-(R+\lambda)t_u}}\left[\frac{d_0(e^{-\lambda t_3} - e^{-\lambda t_u})}{R+d_1}\left\{e^{-(d_1+R)t_2} - e^{-(d_1+R)t_3}\right\}\right. \\ &\quad +\left(Q_2 + \frac{u_0-d_0}{u_1+d_1}\right)\left\{\frac{e^{-Rt_3}(e^{-\lambda t_3} - e^{-\lambda t_u})}{u_1+d_1+R}\right. \\ &\quad \left.-\frac{\lambda\{e^{-(R+\lambda)t_3} - e^{-(u_1+d_1)(t_u-t_3)-(R+\lambda)t_u}\}}{(u_1+d_1+R)(u_1+d_1+R+\lambda)}\right\} + \frac{u_0-d_0}{R(u_1+d_1)} \\ &\quad \left.\times\left\{e^{-Rt_3}(e^{-\lambda t_3} - e^{-\lambda t_u}) - \frac{\lambda\{e^{-(R+\lambda)t_3} - e^{-(R+\lambda)t_u}\}}{R+\lambda}\right\}\right] \end{aligned} \tag{6.30}$$

Expected value of total profit for last cycle

Thus the expected value of the profit (TP_2) for the last cycle is

$$\begin{aligned} E\{TP_2\} &= E\{SRL_1^1\} + E\{SRL_1^2\} + E\{SRL_2^1\} + E\{SRL_2^2\} + E\{SRL_4^1\} \\ &\quad -E\{SUCL\} - E\{HCL_1\} - E\{HCL_2\} - E\{PCL_1\} - E\{PCL_4\} \\ &\quad -E\{SCL_1\} - E\{SCL_2\} \end{aligned} \tag{6.31}$$

where the expected value of the parameters SRL_1^1, SRL_1^2, SRL_2^1, SRL_2^2, SRL_4^1, $SUCL$, HCL_1, HCL_2, PCL_1, PCL_4, SCL_1, and SCL_2 for different cases are given by the expressions (6.22), (6.23), (6.25), (6.26), (6.29), (6.19), (6.20), (6.24), (6.21), (6.28), (6.27) and (6.30) respectively.

6.3.1.1 Expected value of total profit for Model-A

Therefore the expected value of total profit (TP) over the whole business period is given by

$$E\{TP\} = E\{TP_1\} + E\{TP_2\} \tag{6.32}$$

where the expected value of TP_1 and TP_2 are given by 6.15 and 6.31 respectively.

6.3.2 Model-B: Model with deterministic production cost and without shortages

Formulation for first n full cycles

In this model, production starts with maximum rate at the beginning of each cycle and stopped at $t = (j-1)t_u + t_1$ where,$(j = 1, 2, \cdots, n)$ when the stock reaches the maximum level. The stock level becomes zero at the end of the cycle i.e. at $t = jt_u$. So, the leading differential equation describing different inventory situation, is given by

$$\frac{dx(t)}{dt} = \begin{cases} u_0 - d_0 - (u_1 + d_1)x(t) & for,\ (j-1)t_u \le t \le (j-1)t_u + t_1 \\ -d_0 - d_1 x(t) & for,\ (j-1)t_u + t_1 \le t \le jt_u \end{cases} \tag{6.33}$$

with the boundary conditions

$$x(t) = \begin{cases} 0 & for,\ t = (j-1)t_u \\ Q_1 & for,\ t = (j-1)t_u + t_1 \\ 0 & for,\ t = jt_u, \quad where, \quad j = 1, 2, \cdots, n. \end{cases} \tag{6.34}$$

Using the boundary conditions given by (6.2), the solution of the differential equation (6.1) is given by

$$x(t) \begin{cases} for,\ (j-1)t_u \le t \le (j-1)t_u + t_1 \\ \qquad \dfrac{u_0 - d_0}{u_1 + d_1}\left[1 - e^{-(u_1+d_1)\{t-(j-1)t_u\}}\right] \\ for,\ (j-1)t_u + t_1 \le t \le jt_u \\ \qquad \left(Q_1 + \dfrac{d_0}{d_1}\right)e^{-d_1\{t-(j-1)t_u-t_1\}} - \dfrac{d_0}{d_1} \end{cases} \tag{6.35}$$

where,

$$Q_1 = \frac{u_0 - d_0}{u_1 + d_1}\left[1 - e^{-(u_1+d_1)t_1}\right]$$
$$t_u = t_1 + \frac{1}{d_1}log\left[\frac{d_1 Q_1 + d_0}{d_0}\right]$$

Holding Cost

According to the model stock level becomes zero at the end of each cycle [i.e. at $t = t_u$]. Therefore, the expression for the holding cost in j-th cycle (HC_j) can be obtained from the equation (6.8) replacing t_2 by t_u. Thus the expected value of holding cost in n full cycle is given by

$$\begin{aligned}
E\{HC\} &= \sum_{n=0}^{\infty}\int_{nt_u}^{(n+1)t_u}\left(\sum_{j=1}^{n} HC_j\right)\lambda e^{-\lambda h}dh \\
&= \frac{c_1(1-e^{-\lambda t_u})}{1-e^{-(R+\lambda)t_u}}\left[\frac{u_0-d_0}{u_1+d_1}\left\{\frac{1-e^{-Rt_1}}{R}-\frac{1-e^{-(u_1+d_1+R)t_1}}{u_1+d_1+R}\right\}+\right. \\
&\left.\left\{\frac{e^{d_1t_1}(Q_1+\frac{d_0}{d_1})}{d_1+R}\{e^{-(R+d_1)t_1}-e^{-(R+d_1)t_u}\}-\frac{d_0(e^{Rt_1}-e^{Rt_u})}{d_1R}\right\}\right] \quad (6.36)
\end{aligned}$$

Production Cost

The production runs for same duration $[(j-1)t_u \le t \le (j-1)t_u + t_1]$ as in the model-A and the expression for inventory level is also same for this time gap. So present value of production cost for j-th cycle, PC_j for each $j = 1, 2, \cdots, n$ and the expected value of total production cost ($E\{PC\}$) for n full cycle are given by the equations (6.10), (6.11) respectively.

Sale Revenue

The present value of sell revenue (SR_j) in j-th full cycle where, $j = 1, 2, \cdots, n$ is given by

$$\begin{aligned}
SR_j &= m_0p_0e^{-(j-1)\gamma}\left[\int_{(j-1)t_u}^{(j-1)t_u+t_1}+\int_{(j-1)t_u+t_1}^{jt_u}\right]\{d_0+d_1x(t)\}e^{-Rt}dt \\
&= m_0p_0e^{-(j-1)\gamma}[SR_{1j}+SR_{2j}]
\end{aligned}$$

where,

$$\begin{aligned}
SR_{1j} &= \int_{(j-1)t_u}^{(j-1)t_u+t_1}\{d_0+d_1x(t)\}e^{-Rt}dt \\
&= \left[\frac{(u_0d_1+u_1d_0)(1-e^{-Rt_1})}{R(u_1+d_1)}-\frac{d_1(u_0-d_0)\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)}\right]e^{-R(j-1)t_u} \\
SR_{2j} &= \int_{(j-1)t_u+t_1}^{jt_u}\{d_0+d_1x(t)\}e^{-Rt}dt \\
&= \left[(d_0+d_1Q_1)\frac{e^{-Rt_1}-e^{-d_1(t_u-t_1)-Rt_1}}{d_1+R}\right]e^{-R(j-1)t_u}
\end{aligned}$$

Therefore the expected value of total sale revenue (SR) for n full cycle is given by

$$\begin{aligned}
E\{SR\} &= \sum_{n=0}^{\infty}\int_{nt_u}^{(n+1)t_u} m_0p_0\sum_{j=1}^{n}e^{-(j-1)\gamma}[SR_{1j}+SR_{2j}]\lambda e^{-\lambda h}dh \\
&= \left[\frac{(u_0d_1+u_1d_0)(1-e^{-Rt_1})}{R(u_1+d_1)}+\frac{d_1(u_0-d_0)\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)}\right. \\
&\left.+\frac{d_0+d_1Q_1}{d_1+R}\{e^{-Rt_1}-e^{-d_1(t_u-t_1)-Rt_u}\}\right]\frac{m_0p_0e^{-\lambda t_u}}{1-e^{-(Rt_u+\lambda t_u+\gamma)}} \quad (6.37)
\end{aligned}$$

Set-up cost

The set-up cost and its expected value $E\{SUC\}$ is also same as the model-A. So $E\{SUC\}$ is given by equation (6.13).

Total Profit

Thus the expected value of total profit (TP_1) for n full cycles is given by

$$E\{TP_1\} = E\{SR\} - E\{HC\} - E\{PC\} - E\{SUC\} \tag{6.38}$$

where the expected value of the parameters SR, HC, PC and SUC are given by the equations (6.37), (6.36), (6.11) and (6.13) respectively.

Formulation for last cycle

The leading differential equation describing the inventory level $x(t)$ in the last cycle ($nt_u \leq t \leq (n+1)t_u$), are given by

$$\frac{dx(t)}{dt} = \begin{cases} u_0 - d_0 - (u_1 + d_1)x(t) & for,\ nt_u \leq t \leq nt_u + t_1 \\ -d_0 - d_1 x(t) & for,\ nt_u + t_1 \leq t \leq (n+1)t_u \end{cases} \tag{6.39}$$

with the boundary conditions

$$x(t) = \begin{cases} 0 & for,\ t = nt_u \\ Q_1 & for,\ t = nt_u + t_1 \\ 0 & for,\ t = (n+1)t_u \end{cases} \tag{6.40}$$

Using the boundary conditions (6.40) the solution of the differential equation (6.39) is given by

$$x(t) = \begin{cases} \dfrac{u_0 - d_0}{u_1 + d_1}[1 - e^{-(u_1+d_1)(t-nt_u)}] & for,\ nt_u \leq t \leq nt_u + t_1 \\ \left[Q_1 + \dfrac{d_0}{d_1}\right] e^{-d_1(t-nt_u-t_1)} - \dfrac{d_0}{d_1} & for,\ nt_u + t_1 \leq t \leq (n+1)t_u \end{cases} \tag{6.41}$$

Also,

$$Q_1 = \frac{u_0 - d_0}{u_1 + d_1}\left[1 - e^{-(u_1+d_1)t_1}\right]$$

Now the following two cases arises-

Case-1: $(nt_u \leq t \leq nt_u + t_1)$

This case is same as the Case-1 of model-A. So the expressions for the expected values of sale revenue for normal sale and reduce sale (SRL_1^1 and SRL_1^2), holding cost (HCL_1), set-up cost ($SUCL$) and purchasing cost (PCL_1) are given by equations (6.22), (6.23), (6.20), (6.19) and (6.21) respectively.

Case-2: $(nt_u + t_1 \le t \le (n+1)t_u)$

In this case also the calculation and expressions for different parameters and their respective expected values becomes same as case-2 of model-A if t_2 is replaced by t_u in the expressions of the parameters of case-2 of model-A. Thus the expected values of sale revenue earned from normal and reduced sale (SRL_2^1 and SRL_2^2) and holding cost (HCL_2) are given by

$$\begin{aligned} E\{SRL_2^1\} &= \frac{m_0 p_0}{1-e^{-(Rt_u+\lambda t_u+\gamma)}}\Bigg[\Bigg\{\frac{u_1 d_0 + d_1 u_0}{R(u_1+d_1)}(1-e^{-Rt_1}) + \frac{(d_0+d_1Q_1)}{d_1+R}e^{-Rt_1} \\ &\quad -\frac{d_1(u_0-d_0)\{1-e^{-(u_1+d_1+R)t_1}\}}{(u_1+d_1)(u_1+d_1+R)}\Bigg\}(e^{-\lambda t_1}-e^{-\lambda t_u}) \\ &\quad +\frac{\lambda(d_0+d_1Q_1)}{(d_1+R)(d_1+R+\lambda)}\Bigg\{e^{-(u_1+R)t_1} - e^{-(t_u-t_1)d_1-(u_1+R)t_u}\Bigg\}\Bigg] \end{aligned} \tag{6.42}$$

$$\begin{aligned} E\{SRL_2^2\} &= \frac{s_2(d_1Q_1+d_0)}{d_1\{1-e^{-(R+\lambda)t_u}\}}\Bigg[\frac{\lambda\{e^{-(R+\lambda)t_1}-e^{-(R+\lambda)t_u}\}}{R+\lambda} \\ &\quad -\frac{\lambda\{e^{-(u_1+d_1+R+\lambda)t_1}-e^{-(u_1+d_1+R+\lambda)t_u}\}}{d_1+R+\lambda}\Bigg] \end{aligned} \tag{6.43}$$

$$\begin{aligned} E\{HCL_2\} &= \frac{c_1}{1-e^{-(R+\lambda)t_u}}\Bigg[\frac{c_1(u_0-d_0)}{u_1+d_1}\Bigg\{\frac{1-e^{Rt_1}}{R} - (e^{-\lambda t_1}-e^{-\lambda t_u})\times \\ &\quad \frac{1-e^{-(u_1+d_1+R)t_1}}{u_1+d_1+R}\Bigg\} + \frac{d_1Q_1+d_0}{d_1(d_1+R)}\Bigg\{e^{-Rt_1}(e^{-\lambda t_1}-e^{-\lambda t_u}) \\ &\quad -\frac{\lambda\{e^{-(u_1+R)t_1}-e^{-d_1(t_u-t_1)-(u_1+R)t_u}\}}{d_1+R+\lambda}\Bigg\}\Bigg] \end{aligned} \tag{6.44}$$

Therefore, the expected value of profit (TP_2) for the last cycle is given by

$$\begin{aligned} E\{TP_2\} &= E\{SRL_1^1\} + E\{SRL_1^2\} + E\{SRL_2^1\} + E\{SRL_2^2\} \\ &\quad -E\{HCL_1\} - E\{PCL_1\} - E\{HCL_2\} - E\{SURL\} \end{aligned} \tag{6.45}$$

where, the expected values of sale revenue earned from normal sale (SRL_1^1, SRL_2^1) and reduce sale (SRL_1^2, SRL_2^2), holding costs (HCL_1, HCL_2), purchasing cost (PCL_1) and set-up cost ($SUCL$) are given by equations (6.22), (6.42), (6.23), (6.43), (6.20), (6.44), (6.21) and (6.19) respectively.

Total expected profit for Model-B

Thus the expected value of total profit (TP) over the whole time horizon is given by

$$E\{TP\} = E\{TP_1\} + E\{TP_2\} \tag{6.46}$$

where, $E\{TP_1\}$ *and* $E\{TP_2\}$ are given by(6.38) and (6.45) respectively.

6.3.3 Model-C1 (or, Model-C2): Model with shortages (or, without shortages) in which the unit production cost is random.

The only difference of Models-C1, -C2 from Model-A is that the production cost is random for the Models-C1, -C2 where it is crisp for Model-A. The unit production cost p_0 is random which follows uniform distribution with known mean m_{p_0} and corresponding probability density function

$$\phi(t) \;=\; \begin{cases} \dfrac{1}{p_2 - p_1} & for,\, p_1 \;\le\; t \;\le\; p_2 \\ 0 & otherwise \end{cases} \tag{6.47}$$

where, $m_{p_0} \;=\; \frac{p_1+p_2}{2}$.
The expressions for the Models -C1, -C2 can be obtained from the expressions of Model-A and Model-B respectively by replacing p_0 by m_{p_0}.

6.3.4 Model-D1 (or, Model-D2): Model with (or, without) shortages where unit production cost is fuzzy in nature

In these models, the planning horizon is stochastic and the unit production cost p_0 is fuzzy in nature. The fuzzyness of the models are removed and reduced to its equivalent crisp model using the method proposed by Maiti and Maiti [170]. According to this method, one can maximize the crisp variable z such that the possibility measure of the event $E\{TP\} \;\ge\; z$ exceeds some predefined level (α) according to the decision maker choice in an optimistic sense and in this view the models are reduces to

Maximize z
subject to $Pos\{E\{TP\} \ge\; z\} \ge\; \alpha$.
where, TP is the total profit of the models and $Pos\{E\{TP\} \;\ge\; z\}$ is calculated using the following formula.

$$Pos(\breve{a} * \breve{b}) \;\;=\;\; [sup\{min(\mu_{\breve{a}}(x),\, \mu_{\breve{b}}(y))\},\, x, y\epsilon R,\, x * y]$$

where $\breve{a}\; and\; \breve{b}$ are two fuzzy quantities having membership functions $\mu_{\breve{a}}(x)\; and\; \mu_{\breve{b}}(x)$ respectively, the abbreviation Pos refers to possibility and $*$ represents any of the relations $<, >, \le, \ge$.

6.3.5 Model-E1 (or, Model-E2): Model with (or, without) shortages in which unit production cost is fuzzy-random

In Model-E1 and -E2, the unit production cost p_0 is considered as fuzzy-random, i.e. p_0 takes fuzzy values following some discrete probability distribution. Here it is considered that the fuzzy values of p_0 are give by the triangular fuzzy numbers $p_{01},\, p_{02}, \cdots ,\, p_{0m}$ with corresponding probabilities $Prob.\{p_0 = p_{0j}\} = pr_j,\ j = 1, 2, \cdots, m$. If the j-th triangular fuzzy numbers p_{0j} is constructed as $p_{0j} = (p_{0j}^1, p_{0j}^2, p_{0j}^3)$ where $j = 1, 2, \cdots, m$ then according to Liu & Liu [154], the expected value [c.f. 2.8] of the fuzzy-random variable p_0 is given by

$$E\{p_0\} = \sum_{j=1}^{m} pr_j \frac{(1-\rho)p_{0j}^1 + p_{0j}^2 + \rho p_{0j}^3}{2} \tag{6.48}$$

The required mathematical expressions for the different model parameters and total expected profit ($E\{TP\}$) of model-E1 and model-E2 can be obtained replacing p_0 by $E\{p_0\}$ in the corresponding expressions of model-A and model-B respectively. Note that, here $\rho = \frac{1}{2}$.

6.4 Solution Methodology

The main aim of the models are to maximize the expected value of total profit. A gradient based optimization technique- Generalised Reduced Gradient method (GRG) [c.f.2.2.2] is used to obtain the optimum results for all of the models through Lingo 12.0 software using the inputs given below.

6.5 Numerical Experiment

A numerical result can establish the logical view of a model. To illustrate the above production-inventory models numerically the following inputs are considered.

6.5.1 Input data

$d_0 = 20,\ d_1 = 0.2,\ Q_2 = 10,\ R = 0.09,\ \lambda = 0.01,\ c_1 = 0.2,\ \gamma = 0.05,\ m_0 = 2.3,\ c_3^1 = 20,\ c_3^2 = 5,\ \delta = 0.1,\ s_1 = 0.5,\ s_2 = 5.5,$
Different type of unit production cost(p_0) is considered as follows:

Table 6.1: *Values of different type of unit production cost (p_0)*

Type	Constant	Random	Fuzzy(TNF^*)	Fuzzy-random
Value	$p_0 = 4$	$p_1 = 4,\ p_2 = 4.4,$ $E[p_0] = 4.2$	$(3.5, 4.0, 4.6)$ $\alpha = 0.7$	$p_{01} = (0, 0.5, 1),\ pr_1 = 0.5$ $p_{02} = (1, 2, 2.5),\ pr_2 = 0.25$ $p_{03} = (2, 2.7, 3.3),\ pr_3 = 0.25$

*The term 'TNF' refers to triangular fuzzy number.

$u_0,\ u_1$ are the decision variables for all of the models.

6.5.2 Optimum results

Using the above mentioned methodology, obtained optimum results for different models are presented in Table-6.2.

Table 6.2: *Optimum results for all of the different models*

Type of p_0	Model	u_0	u_1	t_1	t_2	t_3	t_u	Profit
crisp	model-A	49.43	0.87	0.80	1.53	2.05	3.61	695.75
	model-B	59.11	0.19	0.35	-	-	0.95	722.15
Random	model-C1	65.22	0.31	1.09	2.70	3.22	3.76	748.87
	model-C2	69.73	0.23	0.35	-	-	0.97	765.25
Fuzzy	model-D1	44.33	0.79	1.02	1.75	2.77	3.98	814.15
	model-D2	47.23	0.89	0.63	-	-	1.20	862.55
Fuzzy-random	model-E1	46.31	0.72	0.91	1.66	2.19	3.69	716.83
	model-E2	52.21	0.14	0.42	-	-	1.01	760.89

6.6 Discussion

In Table-6.2, the maximized profit for each of the models are presented. Models-A, -C1, -D1,-E1 are developed with shortages and the models-B, -C2, -D2, -E2 are developed without shortages. In any business plan allowing shortages, penalty for shortages always lower down the profit. So, the models without shortages are more profitable than the models with shortages. Also the values of u_0 for the models without shortages are greater than the models with shortages. This is also a cause of less profit for the models with shortages than the models without shortages.

Table 6.3: *Percentage of change of objective value for percentage of change of p_0 for Model-A*

change of $p_0(\%)$	-7.5	-5	-2.5	0	2.5	3.75	5
change of $E\{TP\}(\%)$	-8.46	-5.64	-2.8	0	2.82	4.23	5.65

Table-6.3 presents a sensitivity analysis of changes in objective value with respect to the change in unit production cost for model-A. According to Table-6.3, the objective value is proportional to the unit production cost and the increment in unit production cost forces objective value to change more rapidly.

Figure-6.6 is drawn with different values of profit due to different values of possibility measure for Models-D1, -D2. From this figure, it can be point out that there is a linear

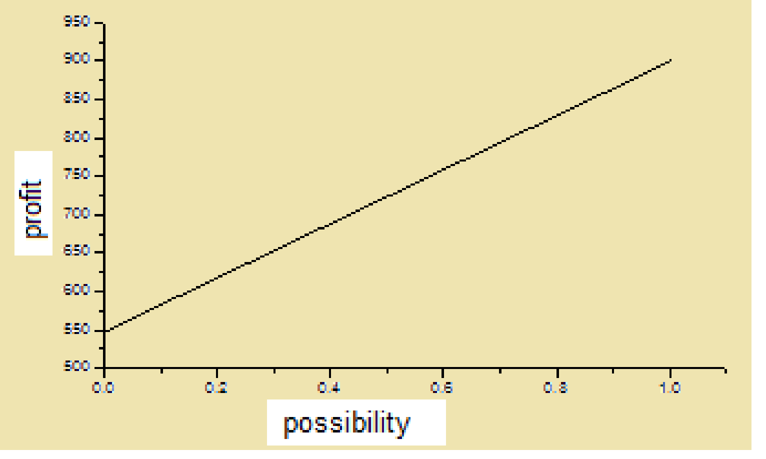


Figure 6.6: Possibility (α) / expected total profit

relation between profit and possibility measure. So, increment in possibility degree increases the profit.

6.7 conclusion

In each production-inventory firm, control in production is desirable as each concern does not wants undesirable shortage as it invites loss of goodwill. Therefore, the production in a firm should be controlled. But, to the best of our knowledge, there is no suitable production policy to control production in an uncertain environment in the exiting literature. Moreover, normally production rate is slashed down as the inventory level goes up but according to the modern marketing policy, demand goes up along with the larger displayed stock. For the first time, this investigation outlines a production-inventory policy in the above mentioned contradicting situation in deterministic, random, fuzzy and fuzzy-random environments with production as a decision variable. This study can also be extended under the consideration of trade credit, price discount, rework etc. Hence there is great research scope for the future researchers.

Part IV

Inventory Models with Fuzzy Logic

Chapter 7

A Supply Chain Model with fuzzy logic under Random planning Horizon via Genetic Algorithm

7.1 Introduction

Supply chain modeling is a process which optimizes the whole supply chain profit or cost by co-operative management of materials and information between supply chain members, which cannot be reached by the supply chain members individually. In decentralized and uncoordinated SCM, chain members are inclined to make decisions by solely considering their individual interests. Such decisions may conflict to each other, as viewed from the standpoint of the whole supply chain. Co-coordinating supply chain members through well-designed mechanisms, such as cost discount, quantity discount [253], etc. get some operational advantages in the study of supply chain management [28, 88]. Matching supply with demand [135, 255], limitation on the capacity of the suppliers and uncertainty [201, 277] are the most common features of a real life oriented SCMs [270]. Jeuland and Shugan [123] for instance, applied quantity discounts for marketing channel coordination, assuming that demand is a decreasing function of the buyer's sales price. On the other hand, Hsieh and Wu [111] examined capacity allocation, ordering, and pricing decisions in a supply chain with uncertain demand and supply. Also various SCMs have been proposed to improve supply chain performances [7, 252, 254, 256]. SCMs in fuzzy environment [263] becomes more difficult to control and improve due to vague and subjective information and the qualitative measures are often needed to be translated into quantitative measures. Therefore modeling with fuzzy set theory is a useful performance monitoring tool to be incorporated.

Fuzzy inference technique has been introduced in SCMs to monitor and control supply chain variables, optimizing the supply chain process and meeting customers' requirements [4]. Kannan *et al.* [128] presented an integrated approach, of fuzzy multi attribute utility theory and multi-objective programming, for rating and selecting the best green suppliers according to economic and environmental criteria and the allocating the optimum order quantities among them. Kumar *et al.* [141] proposed a supply chain model based on fuzzy logic using four input single output Mamdani fuzzy inference system to handle the various attributes associated with supplier evaluation process.

Classical inventory models and SCMs are usually developed over infinite planning horizon [149, 159]. This is an unrealistic assumption on the real world situations. According to Gurnani [100] and Chung and Kim [60], the assumption of an infinite planning horizon is not realistic due to several reasons such as variation of inventory costs, changes in product specifications and designs, technological changes, etc. Moreover, for seasonal products like fruits, vegetables, warm garments, etc., business period is not infinite. Actually, every year a seasonal product does not end at a particular time. Over the years, a seasonal product does have different time periods. There is an inherent uncertainty in these time horizons. This uncertainty can be represented randomly with a distribution. Recently, Moon and Lee [184] presented an EOQ model under inflation and discounting with a random product life cycle.

As a problem solving tool, GAs are now-a-days more useful than the traditional direct and gradient-based optimization techniques, mixed-integer linear programming method [**?**] etc. The primary reasons of their success are their broad applicability, easy to use and global perspective . GAs [58, 90] are adaptive computational procedures modeled on the mechanics of natural genetic systems. They exploit the historical information to speculate on new offspring with expected improved performance. Since a GA works simultaneously on a set of solutions, it has very little chance to get struck to local optimum. Here, the resolution of the possible search space is increased by operating on potential solutions and not on the solutions themselves. Further, this search space need not be continuous. Recently, GAs have been applied in different areas like inventory [96, 181] numerical optimization [178], supply chain [127], neural network, traveling salesman, scheduling, etc.

The present supply chain framework differs from the previous studies as it involves cost discount and assumes fuzzy inference / fuzzy logic (FL) rules at two levels - one during initial purchase and another at final sale. Till now none has considered business period as random in a SCM with FL. Moreover, GA has been appropriately modified and successfully applied along with fuzzy inference criteria. Some interesting features in the above relations are also pointed out.

In this chapter, a SCM is considered with m suppliers and a wholesaler having n showrooms at different places for sale in the fuzzy-random environment. This is a single management SCM with respect to the wholesaler who purchases a single item from a set of m pre-determined suppliers with limited capacity of supply, stores the units in a warehouse and supplies those to a set of n- showrooms for sale against known fuzzy demand. The wholesaler decides the quantity amounts which are to be purchased from different suppliers based on their given cost discounts following some one parameter fuzzy rule. Quantity to be supplied to the showrooms by the wholesaler is decided using some two parameter fuzzy rules based on imprecise demand and price of the commodity. The total system has been formulated as a non-linear optimization problem for maximum profit. Here, SCM policies have been derived for the seasonal products, fashionable items, etc. for which the period of business are different over the time and follows a normal probability distribution. The appropriately modified GA has been used in finding the maximum profit incorporating the fuzzy inference. Here, a method has been presented to formulate fuzzy membership functions from the available raw data. The model is illustrated by a practical example and the behavior of profit and required quantity are plotted against selling price (mark-up).

The remainder of this chapter is organized as follows. Section -7.2 presents the basic notations and assumptions considered to formulate the SCM . Section -7.3 contains the basic SCM, formulates the profit function and the cost functions of the model. Section -7.4 describes the details about the rules and how fuzzy logic is introduced in this model through the fuzzy rules. A solution methodology has been proposed in section-7.5 to solve the model numerically. This section contains some ideas about GA and fuzzy inference module and their implementation to this problem including algorithms of FL, GA and working steps for the modal. Section -7.6 contains numerical illustration of a practical problem including raw data and optimum results. Discussion on the results and conclusion are presented in sections -7.7 and -7.8 respectively.

7.2 Notations and Assumptions

In the proposed model of SCM, the following notations and assumptions are made:

7.2.1 Notations

$\hat{H}$	random planning horizon that follows the normal distribution with mean $m_{\hat{H}}$ and variance $\sigma_{\hat{H}}$.
W	represents the wholesaler.
S_i	represents the $i-th$ supplier where, $i = 1, 2, \cdots, m$.
R_j	represents the $j-th$ showrooms where, $j = 1, 2, \cdots, n$.
Q_{si}	amount of quantity purchased by W from $i-th$ supplier.
C_{si}	per unit item selling price of S_i to W.
Q	total quantity required at W over the whole business period.
N_j	number of cycles taken by W to supply items to R_j.
T_j	length of replenishment cycle for R_j.
${Q^1}_{rj}$ [or ${Q^2}_{rj}$]	total quantity required at R_j over $\hat{H}$ [or, per cycle].
D_j	unit time demand at R_j satisfied by W.
c_m	mean purchasing cost of the wholesaler.
m_{rj}	mark-up used to fixed the selling price of R_j.
s_{rj}	selling price at R_j.
CA_{si}	supplying capacity of the S_i.
SU_{rj}	set-up cost per cycle at R_j.

SUR	total set-up cost of the showrooms.
H^1_w [or, $H^1{}_{rj}$]	holding cost at W[or at R_j] per unit item per unit time.
H^2_w [or, $H^2{}_{rj}$]	total holding cost at W[or at R_j].
HR	total holding cost of the showrooms.
$Tc^1{}_{si}$ [or, $Tc^1{}_{rj}$]	represents the transportation cost per lot from S_i to the W [or, from W to R_j].
TL_{si} [or, TL_{rj}]	represents the lot size, i.e the maximum amount of quantity which is transported in a single transportation from the S_i to W[or, W to R_j]
$Tc^2{}_{si}$ [or, $Tc^2{}_{rj}$]	total transportation cost from S_i to W[or, from W to R_j].
β, pr_j	two given real numbers where $\beta > 0$ and $0 \leq pr_j \leq 1$.
TF, SP, PW	the total profit, total earn and purchasing cost of the wholesaler respectively.

7.2.2 Assumptions

i) There are m pre-determined suppliers $S_i,\ i = 1, 2, ..., m$, a wholesaler W with its n showrooms $R_j,\ j = 1, 2, ..., n$ in a business of single item.

ii) The suppliers offer fuzzy cost discount depending on the amount of quantity in some imprecise language to the wholesaler such as
If Qunatity is **High** *Then Cost per unit item is* **Low**, which is handled by a fuzzy inference module FGL.

iii) The demands faced by the showrooms are imprecise in nature.

iv) The selling prices are different at the different showrooms which are calculated from per unit item mean purchasing cost imposing different mark-ups for different showrooms.

v) The quantities $Q^2{}_{rj}$ which have to be supplied by W to R_j in each cycle not only depends on the respective demand but also depends on the respective per unit quantity selling price s_{rj} (i.e. m_{rj}) at R_j under some fuzzy rules and a fuzzy inference module FGL1 is used to handle these rules which takes D_j and s_{rj} as inputs and gives $Q^2{}_{rj}$ as output.

vi) This is a profit maximization supply chain model with respect to wholesaler under a single management.

vii) Since the quantities are collected only one time at the wholesaler level, so we neglect the lead time between the supplier and wholesaler.

viii) In the proposed model, the holding cost considered at the showrooms are greater than holding cost at the wholesaler as the showrooms are situated in the market places.

ix) Inventory shortages are not allowed.

x) The transportation cost depends on the amount of quantity transported from the suppliers to wholesaler and from wholesaler to showrooms.

xi) To make the model more realistic, it is considered here that suppliers has a limitation on his/her supplying capacity.

xii) In the present problem the quantities have to be purchased from the suppliers is unknown. The number of cycles taken by the wholesaler to supply the quantities to the showrooms for sale are also unknown. In reality it is one of the main factors that what should be the selling price to get the maximum profit. So here it is considered that the mark-up, which is to be set by the wholesaler to get exact selling price is also unknown. The imprecise demand at the showrooms are known but the demand which originally occurs at the showrooms is unknown.

7.3 Formulation of the SCM

In this supply chain model (Figure-7.1), it is assumed that the imprecise data about selling

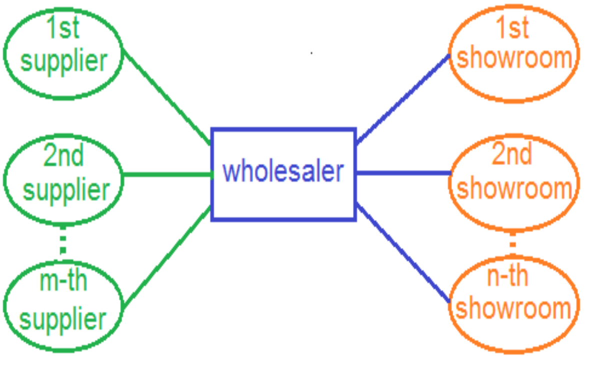


Figure 7.1: Model of the problem

price of the suppliers, demand at the showrooms and the mark-up are known previously but taking the most profitable decision is very difficult. For the optimal decision, the fuzzy inference procedure is used. The amount ${Q^2}_{rj}$ depends on D_j and m_{rj}, i.e.

$${Q^2}_{rj} = FGL2(D_j, m_{rj}, j) \text{ [c.f. sec.7.4]}$$

The number of replenishment cycles and the length of each cycle, N_j and T_j for the showroom R_j are unknown. The quantity to be supplied by the wholesaler to the $j - th$ showroom in each cycle is

$${Q^2}_{rj} = \frac{{Q^1}_{rj}}{N_j}$$

Where Q_j^1 is the total quantity required at the showroom R_j over the whole business period, $\hat{H}$ through N_j shipments. Then the total quantity to be purchased by the wholesaler from the suppliers is

$$Q = \sum_{j=1}^{n} Q^1{}_{rj}$$

Now the suppliers give some fuzzy cost discount depending upon the quantity to be purchased from the corresponding supplier. The fuzzy inference technique is used to calculate the amount of quantity Q_{si} which will give maximum cost discount to the wholesaler and as C_{si} is the required cost per unit item at the $i-th$ supplier therefore

$C_{si} = FGL1(Q_{si}, i)$ [c.f. sec.7.4].

and then we have

$$Q = \sum_{i=1}^{m} Q_{si} = \sum_{j=1}^{n} Q^1{}_{rj} \tag{7.1}$$

Thus the total purchasing cost PW to purchase the quantities from the suppliers is given by

$$PW = \textstyle\sum_{i=1}^{m} Q_{si} C_{si} \tag{7.2}$$

The supplying capacity of the i-th supplier is CA_{si}. Therefore

$$0 \leq Q_{si} \leq CA_{si}$$

As the wholesaler supplies quantities to the $j-th$ showroom in N_j cycles so the holding cost [c.f. Fig.7.2] at the wholesaler is

$$H^2{}_w = H^1{}_w . \sum_{j=1}^{n} T_j . (N_j - 1).(Q^1{}_{rj} - \frac{1}{2}. N_j . Q^2{}_{rj}) \tag{7.3}$$

The holding cost [c.f. Fig.7.2] for R_j over the business period is given by

$$H_{rj}^2 = \frac{1}{2} H_{rj}^1 Q_{rj}^2 N_j T_j, \qquad j = 1, 2, .., n \tag{7.4}$$

The total set-up cost at the showrooms is given by

$$SUR = \sum_{j=1}^{n} SU_{rj} . N_j \tag{7.5}$$

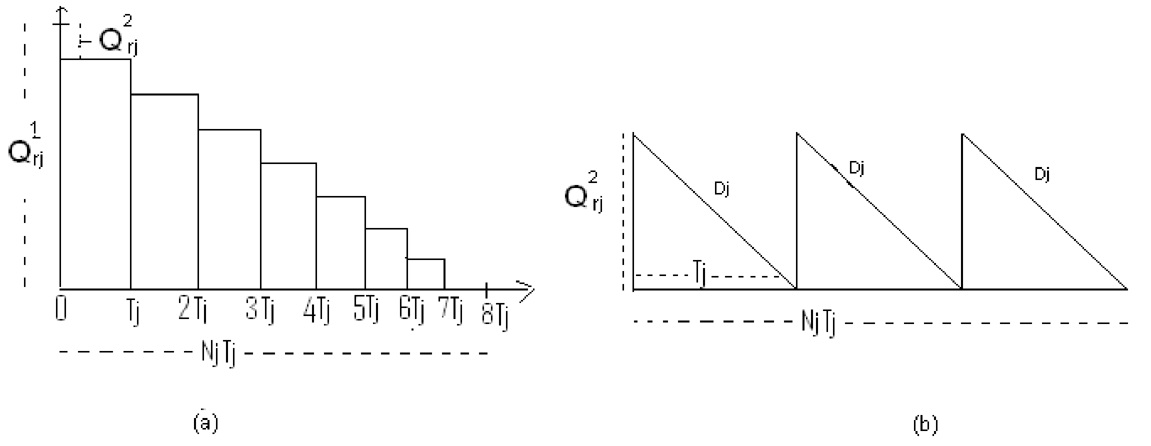


Figure 7.2: (a)Stock holding policy of W for R_j. (b)Stock holding policy for R_j.

The transportation cost of the wholesaler in transporting the quantities to the showrooms is as follows-

$$\begin{aligned}
&If\,(\,Q^2{}_{rj}\,\leq\,TL_{rj}\,)\\
&\qquad Then,\;Tc^2{}_{rj}=Tc^1{}_{rj},\quad \text{for any j=1,2,..n.}\\
&Else\,if\,(\,x.TL_{rj}<Q^2{}_{rj}<(x+1)TL_{rj}\,)\\
&\qquad Then,\,Tc^2_{rj}=(x+1)Tc^1_{rj},\quad \text{for some positive integer x}\\
&Else\;\;Tc^2{}_{rj}=\frac{Q^2{}_{rj}}{TL_{rj}}Tc^1{}_{rj}
\end{aligned}$$

The transportation cost of the wholesaler in purchasing quantities from the suppliers is calculated as follows-

$$\begin{aligned}
&If\,(\,Q_{si}\,\leq\,TL_{si}\,)\\
&\qquad Then,\;Tc^2_{si}=Tc^1_{si},\quad \text{for any i=1,2,..m.}\\
&Else\,if\,(\,xTL_{si}<Q_{si}<(x+1)TL_{si}\,)\\
&\qquad Then,\,Tc^2_{si}=(x+1)Tc^1_{si},\quad \text{for some positive integer x}\\
&Else\;\;Tc^2_{si}=\frac{Q_{si}}{TL_{si}}Tc^1_{si}
\end{aligned}$$

Again as the mean purchasing cost is c_m and the mark-up is m_{rj} then the unit selling price is

$$s_{rj}=m_{rj}c_m,\quad where,\quad c_m\;=\;\frac{\sum_{i=1}^{m}Q_{si}C_{si}}{\sum_{i=1}^{m}Q_{si}}$$

Therefore total revenue SP is given by

$$SP = \sum_{j=1}^{n} s_{rj} Q^1{}_{rj} \tag{7.6}$$

Thus the problem reduces to

$$Maximize\ TF$$
$$= SP - PW - H_w^2 - SUR - \sum_{j=1}^{n} H^2{}_{rj} - \sum_{i=1}^{m} Tc^2{}_{si} - \sum_{j=1}^{n} Tc^2{}_{rj} \tag{7.7}$$

where the constraints are

$$0 \le Q_{si} \le CA_{si}, \qquad i = 1, 2....., m \tag{7.8}$$
$$\sum_{i=1}^{m} Q_{si} = \sum_{j=1}^{n} Q^1{}_{rj} \tag{7.9}$$
$$Prob(|N_j.T_j - \hat{H}| \le \beta) \ge pr_j$$

which is reduced to [c.f. sec.2.1.3]

$$m_{\hat{H}} - \beta - \epsilon_j.\sigma_{\hat{H}} \le N_j.T_j \le m_{\hat{H}} + \beta - \epsilon_j.\sigma_{\hat{H}} \tag{7.10}$$
$$where,\ pr_j = F(\epsilon_j) = \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\epsilon_j} e^{-\frac{t^2}{2}} dt,\ for,\ j = 1, 2, \cdots, n$$
$$N_j \ge 1 \tag{7.11}$$

7.4 Supply Chain Model with Fuzzy Logic (SCMFL)

In this investigation the objective is to maximize the TF of SCM using the fuzzy logic. Here two types of fuzzy inference are used. First one is a single criterion used between the suppliers and wholesaler and second one is a double criterion used between wholesaler and showrooms.

The wholesaler knows the fuzzy demand of the item at the location of different showrooms from past experience. Here three types of fuzzy demands (Low, Medium, High) are considered. The wholesaler sets the Mark-up to get the selling price of the items which is also fuzzy in nature (Low, Medium, High). The wholesaler has to decide what amount to be stocked (Very low, Low, Medium, High, Very high) at the showrooms which depend on the market demand at different showrooms and the Mark-up which is to be set by the wholesaler himself. If we denote the membership functions of the fuzzy numbers Very low, Low, Medium, High, Very high respectively by $\mu_{\widetilde{VL}}(x)$, $\mu_{\widetilde{L}}(x)$, $\mu_{\widetilde{M}}(x)$, $\mu_{\widetilde{H}}(x)$ and $\mu_{\widetilde{VH}}(x)$, then these are taken in the following forms (c.f. Fig.7.3) -

$$\mu_{\widetilde{VL}}(x) = \begin{cases} 0 & \text{if } x \le a_0 \\ 1 & \text{if } a_0 \le x \le a_1 \\ \dfrac{a_2 - x}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ 0 & \text{if } x \ge a_2 \end{cases} \qquad \mu_{\widetilde{L}}(x) = \begin{cases} 0 & \text{if } x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \dfrac{a_2 - x}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{if } x \ge a_3 \end{cases}$$

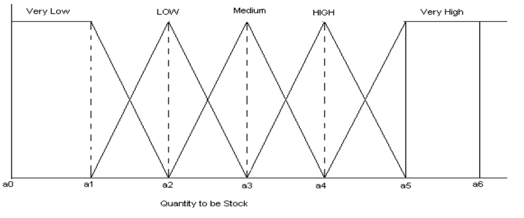


Figure 7.3: Representation of Fuzzy numbers for Quantity to be Stock

$$\mu_{\widetilde{M}}(x) = \begin{cases} 0 & \text{if } x \le a_2 \\ \dfrac{x - a_2}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{if } x \ge a_4 \end{cases} \qquad \mu_{\widetilde{H}}(x) = \begin{cases} 0 & \text{if } x \le a_3 \\ \dfrac{x - a_3}{a_4 - a_3} & \text{if } a_3 \le x \le a_4 \\ \dfrac{a_5 - x}{a_5 - a_4} & \text{if } a_4 \le x \le a_5 \\ 0 & \text{if } x \ge a_5 \end{cases}$$

$$\mu_{\widetilde{VH}}(x) = \begin{cases} 0 & \text{if } x \le a_4 \\ \dfrac{x - a_4}{a_5 - a_4} & \text{if } a_4 \le x \le a_5 \\ 1 & \text{if } a_5 \le x \le a_6 \\ 0 & \text{if } x \ge a_6 \end{cases}$$

Using these membership functions the following nine fuzzy rules are proposed the demand and selling price against the stock at showrooms:-

$$(A) -- \begin{cases} \text{If } Markup\ is\ \textbf{Low}\ and\ Demand\ is\ \textbf{Low}\ \text{Then } Stock\ is\ \textbf{Med} \\ \text{If } Markup\ is\ \textbf{Low}\ and\ Demand\ is\ \textbf{Med}\ \text{Then } Stock\ is\ \textbf{High} \\ \text{If } Markup\ is\ \textbf{Low}\ and\ Demand\ is\ \textbf{High}\ \text{Then } Stock\ is\ \textbf{V.High} \\ \text{If } Markup\ is\ \textbf{Med}\ and\ Demand\ is\ \textbf{Low}\ \text{Then } Stock\ is\ \textbf{Low} \\ \text{If } Markup\ is\ \textbf{Med}\ and\ Demand\ is\ \textbf{Med}\ \text{Then } Stock\ is\ \textbf{Med} \\ \text{If } Markup\ is\ \textbf{Med}\ and\ Demand\ is\ \textbf{High}\ \text{Then } Stock\ is\ \textbf{High} \\ \text{If } Markup\ is\ \textbf{High}\ and\ Demand\ is\ \textbf{Low}\ \text{Then } Stock\ is\ \textbf{V.Low} \\ \text{If } Markup\ is\ \textbf{High}\ and\ Demand\ is\ \textbf{Med}\ \text{Then } Stock\ is\ \textbf{Low} \\ \text{If } Markup\ is\ \textbf{High}\ and\ Demand\ is\ \textbf{High}\ \text{Then } Stock\ is\ \textbf{Med} \end{cases}$$

[where, the terms Med, V.High and V.Low mean Medium, Very High and Very Low respectively.]

The amount of quantity requires at a showroom is determined by the fuzzy inference process [c.f. sec.2.1.5] using the above defined membership functions and the fuzzy rules. Also we consider the fuzzy numbers $\mu_L(x)$, $\mu_M(x)$, $\mu_H(x)$ to represent the fuzzy quantities Q_{si}, C_{sj}, D_j, m_{rj}. The membership function for these fuzzy numbers are defined as

$$\mu_{\widetilde{L}}(x) = \begin{cases} 0 & \text{if } \; x \leq a_0 \\ 1 & \text{if } \; a_0 \leq x \leq a_1 \\ \dfrac{a_2 - x}{a_2 - a_1} & \text{if } \; a_1 \leq x \leq a_2 \\ 0 & \text{if } \; x \geq a_2 \end{cases} \qquad \mu_{\widetilde{M}}(x) = \begin{cases} 0 & \text{if } \; x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{if } \; a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } \; a_2 \leq x \leq a_3 \\ 0 & \text{if } \; x \geq a_3 \end{cases}$$

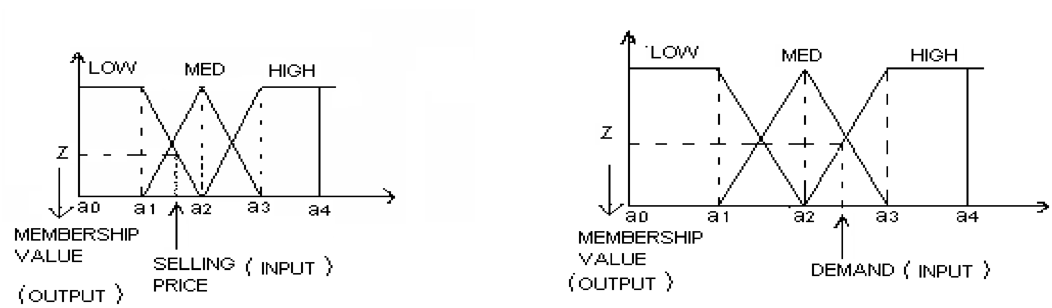


Figure 7.4: input and output calculation of model parameters

$$\mu_{\widetilde{H}}(x) = \begin{cases} 0 & \text{if } \; x \leq a_2 \\ \dfrac{x - a_2}{a_3 - a_2} & \text{if } \; a_2 \leq x \leq a_3 \\ 1 & \text{if } \; a_3 \leq x \leq a_4 \\ 0 & \text{if } \; x \geq a_4 \end{cases}$$

Also in this system, every supplier gives some deduction on purchasing cost depending upon the amount of purchased quantity. The rules proposed by the supplier S_i to the wholesaler are

$$(B) -- \begin{cases} \text{If } \; Purchased\,Amount \; is \; \textbf{High} \; \text{Then} \; cost \; is \; \textbf{Low} \\ \text{If } \; Purchased\,Amount \; is \; \textbf{Medium} \; \text{Then} \; cost \; is \; \textbf{Medium} \\ \text{If } \; Purchased\,Amount \; is \; \textbf{Low} \; \text{Then} \; cost \; is \; \textbf{High} \end{cases}$$

Thus using the rules and the corresponding membership function, one can find the amount of quantity to be purchased from the suppliers to get the maximum cost discount and here these concepts are used through GA process [described in sec.7.5], to get the best result.

7.5 Solution Methodology: A GA Process for the proposed model

A genetic algorithm contains three operators- reproduction, crossover and mutation. Initially, a population is selected and by means of above operators, the better of the population will remain, because of the survival of the fittest. For the present genetic algorithm, a number string specifying the discounts given by the wholesaler, cycle lengths and the number of cycles taken by the wholesalers to supply the quantities to the showrooms. The fitness function is defined by the profit of the wholesaler. Some discussions are made by following about the GA operators used in the present model.

Reproduction Operator: The principle object of a reproduction operator for using in GA is to make a population better than the previous one by replacing the bad solutions by duplicate copies of good solutions. There are many methods available such as tournament selection, proportionate selection and ranking selection [90].

In tournament selection, tournament are played between two solutions and the better solution is chosen and placed in mating pool. Similarly from other two solutions better one is chosen and placed in the matting pool. To carried out this process in similar manner we have to take each solution for tournament minimum two times. A best solution will win both times, thereby making two copies in the new population and in this way the worst solutions will be eliminated from the new population. It can also be mentioned that tournament selection revels better or same solutions than the other selection operators. Also it takes less computational time and has less complexity properties when compared to other reproduction operators. This is the cause why tournament selection operator is used here.

Algorithm for SBX: This method was developed by Deb and his students [77] which works with two parent solutions and creates two offsprings. As the name refers, the SBX operator simulates the working principle of the single point crossover operator on binary strings. The present crossover operator obeys the interval schemata processing, in the sense that common interval schemata between the parents are preserved in the offspring. First a random number u_i between 0 and 1 is created. After that from specified probability distribution function, the ordinate β_{q_i} is calculated so that the area under the probability distribution curve from 0 to β_{q_i} is equal to the chosen random number u_i.

If ${x_i}^{(1,t)}$ and ${x_i}^{(2,t)}$ are denotes two parent solutions at the t-th generation then their two offspring ${x_i}^{(1,t+1)}$, ${x_i}^{(2,t+1)}$ can be calculated by the following steps.

step-1: Choose a random number u_i in $[0, 1)$.

step-2: calculate a number say β_{q_i} using the equation

$$\beta_{q_i} = \begin{cases} (2u_i)^{\frac{1}{DISBX+1}} & , \ u_i \leq 0.5 \\ (\dfrac{1}{2(1-u_i)})^{\frac{1}{DISBX+1}} & , \ otherwise \end{cases}$$

where DISBX is any non-negative real number. A large value of DISBX give a higher probability for creating 'near-parent' solutions and small value allows distant solutions to be selected as offspring.

step-3: compute the offsprings by using the following two equations-

$$x_i{}^{(1,t+1)} = 0.5[(1+\beta_{q_i})x_i{}^{(1,t)} + (1-\beta_{q_i})x_i{}^{(2,t)}]$$
$$x_i{}^{(2,t+1)} = 0.5[(1-\beta_{q_i})x_i{}^{(1,t)} + (1+\beta_{q_i})x_i{}^{(2,t)}]$$

Polynomial Mutation: In polynomial mutation the polynomial function is used to represent the probability distribution. If $x_i{}^{(1,t+1)}$ be the offspring comes out after crossover and if $y_i{}^{(1,t+1)}$ be the muted copy then $y_i{}^{(1,t+1)} = x_i{}^{(1,t+1)} + (x_i{}^{(U)} - x_i{}^{(L)})\bar{\delta_i}$ where $x_i{}^{(U)}$,$x_i{}^{(L)}$ are the variable upper bound and lower bound and the parameter $\bar{\delta_i}$ is calculated from the polynomial probability distribution $P(\delta) = 0.5(DIMUT+1)(1-|\delta|)^{DIMUT}$.

$$\bar{\delta_i} = \begin{cases} (2r_i)^{\frac{1}{DIMUT+1}} - 1 & , \ r_i < 0.5 \\ 1 - [2(1-r_i)]^{\frac{1}{DIMUT+1}} & , \ r_i \geq 0.5 \end{cases}$$

Where r_i is a random number in $[0,1]$ and DIMUT is a positive real number. In this mutation operator, the shape of the probability distribution is directly controlled by the external parameter DIMUT and the distribution is not dynamically changed with generations.

7.5.1 GA parameters

The variable boundaries may be fixed or flexible. The best chromosome obtained by the proposed GA represents the optimum quantity to be purchased from the suppliers(Q_i), the optimum number of cycles(N_j), the demand(D_j) and selling price(i.e. the mark-up m_{rj}) for which the profit is maximum.

The different parameters of GA are generation number (MAXGEN), population size (POPSIZE), probability of cross-over (PXOVER), probability of mutation (PMUT), random seed (RSEED), distribution index for mutation (DIMU) and the others. As there is no clear indication as to how large a population should be, here with POPSIZE = no of variables$\times 10$, the expected result is obtained. Here a combination of real and natural number representation is used to structure a chromosome, where a chromosome is a string of genes which are the decision variables namely Q_{si}, N_j, D_j, m_{rj}.

At the initialization step, the GA module initializes the zero-th population by randomly generating the genes of the chromosomes between the imputed boundaries.

After initializing the population, using the fuzzy inference modules [FL1(Q_{si}, i) and FL2(D_j, m_{rj}, j)] it calculates Cs_i, $Q^1{}_{rj}$, the other dependent variables, the objective function value and the constraint values. An algorithm of FL1(Q_{si}, i)[or FL2(D_j, m_{rj}, j)] module for C_{si} and calculation of $Q^1{}_{rj}$ are given below.

7.5.2 Algorithm for FL1(Q_{si}, i)[or, FL2(D_j, m_{rj}, j)]

step 1 : Take the value of Q_{si} (or, m_{rj}, D_j) as input.

step 2 : Calculate the membership values to the fuzzy sets Low, $Medium$, $High$ of different variables.

step 3 :Evaluate the rules and find the rule strengths of each rule [c.f. Fig.7.4] by the min operator $min\{\mu_{\widetilde{A}}(x)\,, \mu_{\widetilde{B}}(y), ...\}$ where $\mu_{\widetilde{A}}(x)$ is the membership value for Q_{si} (or, m_{rj}, D_j).

step 4 : Calculates the membership functions of the different fuzzy numbers Low, $Medium$, $High$ (or, $Very\,low$, Low, $Medium$, $High$, $Very\,high$) for the parameter C_{si} (or, for the parameter ${Q^1}_{rj}$,) which are represented by the rules with non-zero strength.

step 5 : Apply fuzzy union operator to find the fuzzy output.

step 6 : Apply centroid method to find the difuzzyfied cost C_{si}(or stock ${Q^1}_{rj}$).

Using this algorithm, GA optimizes the proposed model.

7.5.3 Algorithm for proposed GA

Using the above algorithms, GA optimizes the proposed model. An algorithm of the proposed GA module is

```
Begin
initialize population
gen = 0
model evaluation for initial population
assign fitness
for (gen = 0 ; gen < maxgen ; gen ++)
{ print the result for current generation
  reproduction
  crossover
  mutation
  new population
  model evaluation for the new population
  assign fitness using fuzzy inference module
  gen=gen+1 }
  end.
```

7.5.4 Working steps for the model

step 1 :Collection of post raw data for purchased quantities and costs, selling prices, demands and stocks at the showrooms.

step 2 :Rearrange those as imprecise with linguistic terms data and formulate the linguistic relations between them. There may be two types of relations.(i) one input to one output and (ii) two inputs to one output.

step 3 :Formulate the model as profit maximization model w.r.t. wholesaler.

step 4 :Solve the model using GA and fuzzy inference module where the variables are Q_{si}, N_j, D_j, m_{rj}. Using the input for Q_{si}, GA gets the value of C_{si} from FL1(Q_{si}, i) and using the inputs D_j, m_{rj} GA gets the value of ${Q^1}_{rj}$ from FL2(D_j, m_{rj}, j) [c.f. sec.7.5.2].

step 5 :Analyze the results.

Table 7.1: *Raw Data collected from showrooms*

	mark-up	1.3	1.41	1.43	1.45	1.5	1.3	1.41	1.43	1.45	1.5	1.3	1.41	1.43	1.45	1.5
	demand	46	58	60	64	65	66	70	73	74	89	79	79	82	83	92
	stock	685	626	692	806	905	778	1005	1252	1028	957	1033	1151	1247	1247	1087
1st	mark-up	1.44	1.48	1.49	1.49	1.53	1.44	1.48	1.49	1.49	1.53	1.44	1.48	1.49	1.49	1.53
Show	demand	46	58	60	64	65	66	70	73	74	89	79	79	82	83	92
room	stock	465	564	615	600	857	685	626	692	806	905	778	1005	1252	1028	957
	mark-up	1.48	1.5	1.5	1.5	1.51	1.48	1.5	1.5	1.5	1.51	1.48	1.5	1.5	1.5	1.51
	demand	46	58	60	64	65	66	70	73	74	89	79	79	82	83	92
	stock	271	347	469	475	399	465	564	615	600	857	685	626	692	806	905
	mark-up	1.3	1.41	1.43	1.45	1.5	1.3	1.41	1.43	1.45	1.5	1.3	1.41	1.43	1.45	1.5
	demand	59	64	64	85	65	73	73	79	87	77	82	82	85	92	87
	stock	578	751	616	945	566	856	1067	1230	1207	1206	1202	1246	1309	1338	1298
2nd	mark-up	1.44	1.48	1.49	1.49	1.53	1.44	1.48	1.49	1.49	1.53	1.44	1.48	1.49	1.49	1.53
Show	demand	59	64	64	85	65	73	73	79	87	77	82	82	85	92	87
room	stock	520	572	616	945	566	578	751	940	940	818	856	1067	1230	1207	1206
	mark-up	1.48	1.5	1.5	1.5	1.51	1.48	1.5	1.5	1.5	1.51	1.48	1.5	1.5	1.5	1.51
	demand	59	64	64	85	65	73	73	79	87	77	82	82	85	92	87
	stock	278	475	532	604	512	520	572	616	945	566	578	751	940	940	818
	mark-up	1.3	1.41	1.43	1.45	1.5	1.3	1.41	1.43	1.45	1.5	1.3	1.41	1.43	1.45	1.5
	demand	63	69	69	85	70	76	76	80	85	83	70	86	86	92	88
	stock	803	803	831	1177	923	1025	1069	1152	1450	1191	1221	1280	1364	1425	1373
3rd	mark-up	1.44	1.48	1.49	1.49	1.53	1.44	1.48	1.49	1.49	1.53	1.44	1.48	1.49	1.49	1.53
Show	demand	63	69	69	85	70	76	76	80	85	83	70	86	86	92	88
room	stock	590	718	754	941	771	803	803	831	1177	923	1025	1069	1152	1450	1191
	mark-up	1.48	1.5	1.5	1.5	1.51	1.48	1.5	1.5	1.5	1.51	1.48	1.5	1.5	1.5	1.51
	demand	63	69	69	85	70	76	76	80	85	83	70	86	86	92	88
	stock	296	539	548	700	581	590	718	754	941	771	803	803	831	1177	923

7.6 Numerical Experiment

A Real Life Example: To illustrate the SCM numerically, we consider a practical example. A merchant in Jahalda, West Bengal, India makes a business of a food-grain (rice).The merchant collects rice from two village suppliers and sales those from three showrooms at Balda, Contai and Egra in West bengal, India, for a period of 12 months. The raw data regarding the selling price, purchase cost, demand at the showrooms etc. are collected during 2011-2012 and presented in Tables-7.1 and -7.2. The collected raw uncertain data for quantity, purchasing price, selling cost (i.e. mark-up), demands and stocks are rearranged in Tables-7.3 and -7.4 and from these data, some linguistic relations [c.f. sec.7.4] are derived.

7.6.1 Collected data

Table 7.2: *Raw Data collected from suppliers*

1st Supplier	Quantity	910	592	32	362	941	1213	1516	1603	1147	2587	2458	1043	2053	2109	2460
	Cost	17.3	18.2	23.8	18.3	16.9	16.1	15	14.6	16.1	10.6	11.36	12.5	13	12.5	11.36
2nd Supplier	Quantity	774	92	447	294	960	1554	1400	1085	1085	2377	1506	1733	2334	1993	2331
	Cost	15.22	22.74	15.25	16.02	15.21	13.8	14.51	16.11	14.98	13.8	13.21	13.22	10.31	12.48	12.14

Table 7.3: *rearranged data for suppliers and showrooms*

scm member	scm variable	Low					Medium					High				
1st Supplier	Quantity	910	592	32	362	941	1213	1516	1603	1147	2587	2458	1043	2053	2109	2460
	Cost	11.36	12.5	13	12.5	11.36	16.1	15	14.6	16.1	10.6	17.3	18.2	23.8	18.3	16.9
2nd Supplier	Quantity	774	92	447	294	960	1554	1400	1085	1085	2377	1506	1733	2334	1993	2331
	Cost	13.21	13.22	10.31	12.48	12.14	13.8	14.51	16.11	14.98	13.8	15.22	22.74	15.25	16.02	15.21
1st-show room	mark-up	1.3	1.41	1.43	1.45	1.5	1.44	1.48	1.49	1.49	1.53	1.48	1.5	1.5	1.5	1.51
	demand	46	58	60	64	65	66	70	73	74	89	79	79	82	83	92
2nd-show room	mark-up	1.3	1.41	1.43	1.45	1.5	1.44	1.48	1.49	1.49	1.53	1.48	1.5	1.5	1.5	1.51
	demand	59	64	64	85	65	73	73	79	87	77	82	82	85	92	87
3rd-show room	mark-up	1.3	1.41	1.43	1.45	1.5	1.44	1.48	1.49	1.49	1.53	1.48	1.5	1.5	1.5	1.51
	demand	63	69	69	85	70	76	76	80	85	83	70	86	86	92	88

Table 7.4: *rearranged data for stocks at showrooms*

linguistic variable	Quantity at 1st-showroom					Quantity at 2nd-showroom					Quantity at 3rd-showroom				
very-low	271	347	469	475	399	278	475	532	604	512	296	539	548	700	581
Low	465	564	615	600	857	520	572	616	945	566	590	718	754	941	771
Medium	685	626	692	806	905	578	751	940	940	818	803	803	831	1177	923
High	778	1005	1252	1028	957	856	1067	1230	1207	1206	1025	1069	1152	1450	1191
Very-high	1033	1151	1247	1247	1087	1202	1246	1309	1338	1298	1221	1289	1364	1425	1373

Table 7.5: *Mean deviations (σ) of fuzzy numbers and the ratio of left spread and right spread (ξ) of the fuzzy numbers*

scm members	fuzzy parameters	very low		low		medium		high		very high	
		σ	ξ	σ	ξ	σ	ξ	σ	ξ	σ	ξ
1st supplier	quantity			260	0.67	263.4	1.42	264.9	0.7		
	cost			0.99	1.32	0.96	0.78	0.95	1.33		
2nd supplier	quantity			252.4	0.56	265.9	1.37	256	0.77		
	cost			0.6	0.75	0.58	1.43	0.97	0.4		
1st show room	mark-up			0.04	2.8	0.014	5	0.003	1		
	demand			3.79	0.69	3.79	1.44	2.6	1.5		
	stock	57.8	0.67	57.3	1.56	75.04	0.47	207.6	1.72	70.3	0.64
2nd show room	mark-up			0.04	2.8	0.014	5	0.003	1		
	demand			3.33	0.5	3.47	1.5	2.49	1.33		
	stock	55.56	2	61.91	0.45	94.82	0.69	90.02	0.71	358.4	2
3rd show room	mark-up			0.04	2.8	0.014	5	0.003	1		
	demand			2.78	0.5	2.83	1.67	2.6	0.67		
	stock	56.97	0.62	56.52	1.69	77.87	0.42	81.85	0.59	53.33	0.67

From the above data, triangular fuzzy numbers of the from (a_1, a_2, a_3)[c.f. sec.2.1.6] can be formulated as below following Chang [46].

For Supplier-1 :
Quantity (Q_{s1}) : Low $=(0, 600, 1500)$, Medium $=(600, 1500, 2134)$, High $=(1500, 2134, 3041)$
Cost (Cs_1) : Low $=(9.2, 12.5, 15)$, Medium $=(12.5, 15, 18.2)$, High $=(15, 18.2, 20.6)$

For Supplier-2 :
Quantity (Q_{s2}) : Low $=(0, 500, 1400)$, Medium $=(500, 1400, 2058)$, High $=(1400, 2058, 2911)$
Cost (Cs_2) : Low $=(11, 12.5, 14.5)$, Medium $=(12.5, 14.5, 15.9)$, High $=(14.5, 15.9, 19.4)$

For Showroom-1:
Mark-up (M_{r1}) : Low = (1.3,1.44,1.49), Medium = (1.44,1.49,1.5), High = (1.49,1.5,1.51)
Demand (D_1) : Low = (51,60,73), Medium = (60,73,82), High = (73,82,88)
Quantity To Be Supplied $({Q^1}_{r1})$: Very Low = (266,400,600), Low = (400,600,728), Medium = (600,728,999), High = (728,999,1157), Very High = (999,1157,1402)

For Showroom-2:
Mark-up (M_{r2}) : Low = (1.3,1.44,1.49), Medium = (1.44,1.49,1.5), High = (1.49,1.5,1.51)
Demand (D_2) : Low = (59,65,77), Medium = (65,77,85), High = (77,85,91)
Quantity To Be Supplied $({Q^1}_{r2})$: Very Low = (300,500,600), Low = (500,600,824), Medium = (600,824,1150), High = (824,1150,1284), Very High = (1150,1284,1351)

For Showroom-3:
Mark-up (M_{r3}) : Low = (1.3,1.44,1.49) , Medium = (1.44,1.49,1.5) , High = (1.49,1.5,1.51)
Demand (D_3) : Low = (65,70,80) , Medium = (70,80,86) , High = (80,86,95)
Quantity To Be Supplied ($Q^1{}_{r3}$) : Very Low = (426,550,750) , Low = (550,750,868) , Medium = (750,868,1150) , High = (868,1150,1342) , Very High = (1150,1342,1438).

7.6.2 Other input data

supplying capacity (CA_{s1} **and** CA_{s2}) : 3000 units
Set-up cost at the showrooms : SU_{r1} = 55, SU_{r2} = 50, SU_{r3} = 52
Holding cost at the wholesaler and at the showrooms : H_w^1 = 0.2, $H^1{}_{r1}$ = 0.75, $H^1{}_{r2}$ = 0.7, $H^1{}_{r3}$ = 0.68
Lot size per unit transportation : TL_{si} = 200 quintal, TL_{rj} = 50 quintal
Transportation cost per lot from suppliers to wholesaler : $Tc^1{}_{s1}$ = 75, $Tc^1{}_{s2}$ = 50
Transportation cost per lot from wholesaler to showrooms : $Tc^1{}_{r1}$ = 15, $Tc^1{}_{r2}$ =18, $Tc^1{}_{r3}$ = 20
Range of cycles : $1 \leq N_1 \leq 12$, $1 \leq N_2 \leq 12$, $1 \leq N_3 \leq 12$.
Random Time Horizon :Mean $m_{\hat{H}}$=12 and variance $\sigma_{\hat{H}}$=$\sqrt{1.5}$ with $\beta = 0.2$.
The probabilities for the random time horizon are $pr_1 = 0.9032 = pr_2 = pr_3$.
GA parameters : MAXGEN=200, PXOVER=0.8, PMUT=0.2, RSEED=1.2, DIMU=100.

7.6.3 Optimum results

With the above input data, the non-linear optimization problem (7.7)-(7.11) are solved using GA with FL [c.f. sec.-7.4 and -7.5.2]. The optimum results are obtained and presented for different scenarios. Here the scenarios are given by linguistic variables and results are calculated by their corresponding numerical ranges. The scenarios are outlined in Table-7.6, the optimum results are presented in Table-7.7 and the information given by the merchant about the different parameters is given in Table-7.8.

Table 7.6: *Scenarios for the proposed model during computation*

Scenarios		1	2	3	4	5	6	7	8	9
Mark-up		Low (1.3,1.49)	Low (1.3,1.49)	Low (1.3,1.49)	Medium (1.44,1.5)	Medium (1.44,1.5)	Medium (1.44,1.5)	High (1.49,1.51)	High (1.49,1.51)	High (1.49,1.51)
Demand		Low	Medium	High	Low	Medium	High	Low	Medium	High
	1st-show room	(51,73)	(60,82)	(73,88)	(51,73)	(60,82)	(73,88)	(51,73)	(60,82)	(73,88)
	2nd-show room	(59,77)	(65,85)	(77,91)	(59,77)	(65,85)	(77,91)	(59,77)	(65,85)	(77,91)
	3rd-show room	(65,80)	(70,86)	(80,95)	(65,80)	(70,86)	(80,95)	(65,80)	(70,86)	(80,95)

Table 7.7: *Optimum Results for the proposed Model for Different Scenarios*

Scen ario	SCM Mem.	Quantity (quan.)	Mark up	Selling price($)	No. of cycle	Demand (quan.)	Total Holding Cost($)	Total Setup Cost($)	Total Trans. Cost($)	period length (month)	Total Profit ($)
	S_1	1167	-	16.443	-	-	-	-	450	-	
	S_2	1000	-	16.0554	-	-	-	-	250	-	
1	W	2167	-	-	-	-	1850.15	-	-	-	11276.85
	R_1	739	1.4879	24.1952	8	70	366.16	440	240	1.32	
	R_2	700	1.4886	24.2103	3	67	866.33	150	270	3.54	
	R_3	728	1.4894	24.2254	8	69	326.21	416	320	1.32	
	S_1	1199	-	16.331	-	-	-	-	450	-	
	S_2	1138	-	15.6215	-	-	-	-	300	-	
2	W	2337	-	-	-	-	2177.87	-		-	11980
	R_1	668	1.4867	23.7652	7	63	379.78	385	210	1.515	
	R_2	858	1.49	23.8186	7	78	478.03	350	378	1.6	
	R_3	811	1.49	23.8186	6	74	510.51	312	360	1.85	
	S_1	2222	-	11.5322	-	-	-	-	900	-	
	S_2	542	-	17.1821	-	-	-	-	150	-	
3	W	2764	-	-	-	-	2650.32	-		-	9647
	R_1	961	1.49	18.8341	7	88	569	385	315	1.58	
	R_2	880	1.49	18.8341	6	79	579	300	324	1.881	
	R_3	923	1.49	18.8341	7	81	517	364	420	1.648	
	S_1	1189	-	16.3673	-	-	-	-	450	-	
	S_2	1095	-	15.7677	-	-	-	-	300	-	
4	W	2284	-	-	-	-	1478	-		-	10457.67
	R_1	735	1.498	23.9725	1	72	281.4	55	225	10.21	
	R_2	711	1.4905	23.9659	10	67	264	500	360	1.061	
	R_3	838	1.4937	24.019	9	79	302	520	400	1.06	
	S_1	1380	-	15.6862	-	-	-	-	525	-	
	S_2	1199	-	15.391	-	-	-	-	300	-	
5	W	2579	-	-	-	-	2263	-		-	13329.13
	R_1	803	1.4928	23.2119	6	77	523.17	330	270	1.7377	
	R_2	887	1.4976	23.2855	6	84	546.61	300	324	1.7603	
	R_3	889	1.499	23.3085	6	85	533.5	312	360	1.7645	
	S_1	1400	-	15.6124	-	-	-	-	525	-	
	S_2	1335	-	14.7346	-	-	-	-	350	-	
6	W	2735	-	-	-	-	2368.25	-		-	13817
	R_1	888	1.4942	22.6881	6	87	566.95	330	270	1.702	
	R_2	878	1.4989	22.7587	6	86	522.77	300	324	1.701	
	R_3	969	1.4983	22.7498	7	93	490.45	364	420	1.489	
	S_1	1166	-	16.4496	-	-	-	-	450	-	
	S_2	936	-	16.2312	-	-	-	-	250	-	
7	W	2102	-	-	-	-	1767.56	-		-	11827.64
	R_1	750	1.4903	24.3691	5	72	585.89	275	225	2.08	
	R_2	806	1.4925	24.4065	9	76	332.54	450	324	1.18	
	R_3	545	1.49	24.6376	10	70	144.5	520	400	0.78	
	S_1	1267	-	16.0914	-	-	-	-	525	-	
	S_2	1141	-	15.6118	-	-	-	-	300	-	
8	W	2408	-	-	-	-	1989.74	-		-	12285.67
	R_1	796	1.4911	23.6557	6	75	527.42	330	270	1.77	
	R_2	827	1.4947	23.7129	6	79	511.97	300	324	1.77	
	R_3	785	1.4923	23.6747	3	74	943.71	156	360	3.54	
	S_1	1394	-	15.6353	-	-	-	-	525	-	
	S_2	1200	-	15.3871	-	-	-	-	350	-	
9	W	2594	-	-	-	-	2266.21	-		-	13579.55
	R_1	849	1.4932	23.1734	6	81	563.45	330	270	1.77	
	R_2	856	1.5076	23.3983	6	86	505.21	300	324	1.68	
	R_3	887	1.5041	23.3449	7	85	449.49	364	420	1.49	

7.6.4 Practical result(s)

Table 7.8: *Result obtained for the model with the inputs given by the merchant*

Scenario	SCM Mem.	Quantity (quan.)	Mark up	Selling price($)	No. of cycle	Demand (quan.)	Total Holding Cost($)	Total Setup Cost($)	Total Trans. Cost($)	period length (month)	Total Profit ($)
1	S_1	762	-	17.88	-	-	-	-	300	-	8904
	S_2	600	-	17.05	-	-	-	-	150	-	
	W	1362	-	-	-	-	764.77	-	-	-	
	R_1	390	1.51	26.45	4	55	259.42	110	120	1.77	
	R_2	427	1.51	26.45	3	63	337.12	150	162	2.3	
	R_3	545	1.51	26.45	6	68	247.92	312	240	1.17	
2	S_1	1320	-	15.90	-	-	-	-	525	-	12809.63
	S_2	1167	-	15.52	-	-	-	-	300	-	
	W	2487	-	-	-	-	2164.54	-		-	
	R_1	741	1.495	23.50	6	75	457.12	330	270	1.65	
	R_2	824	1.495	23.50	6	78	507.78	300	324	1.71	
	R_3	922	1.495	23.50	5	82	705.2	260	400	2.1	
3	S_1	1173	-	16.42	-	-	-	-	450	-	10457.17
	S_2	2676	-	12.32	-	-	-	-	700	-	
	W	3849	-	-	-	-	4896.1	-		-	
	R_1	1237	1.4	19	9	86	741.11	495	405	1.59	
	R_2	1278	1.4	19	9	90	706.2	450	486	1.57	
	R_3	1334	1.4	19	9	93	722.88	468	540	1.59	

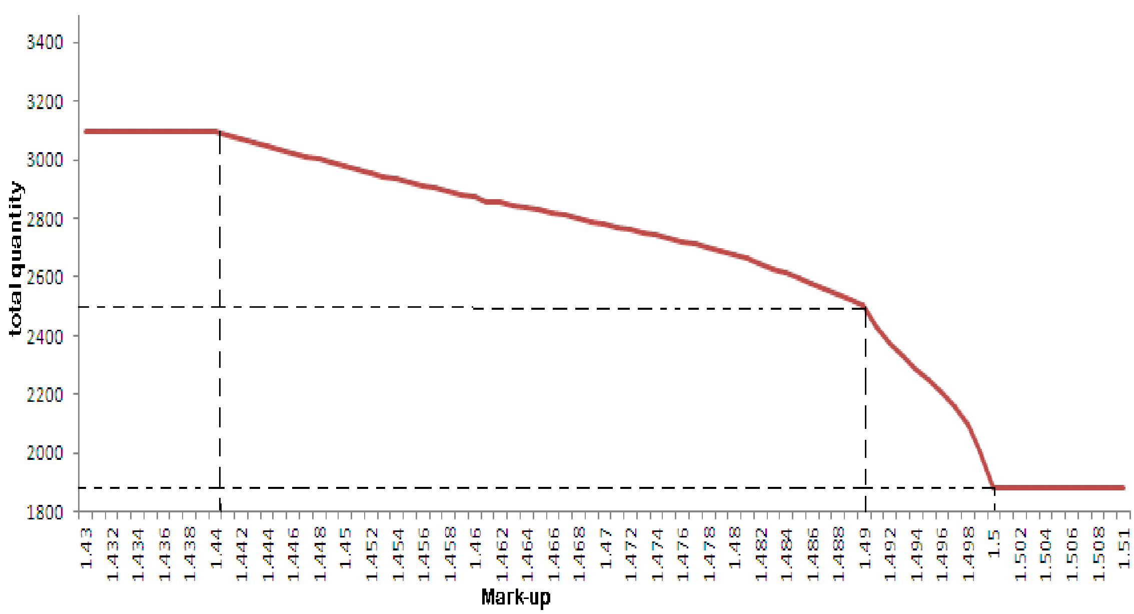


Figure 7.5: Change in Quantity Stocked by the wholesaler against mark-up

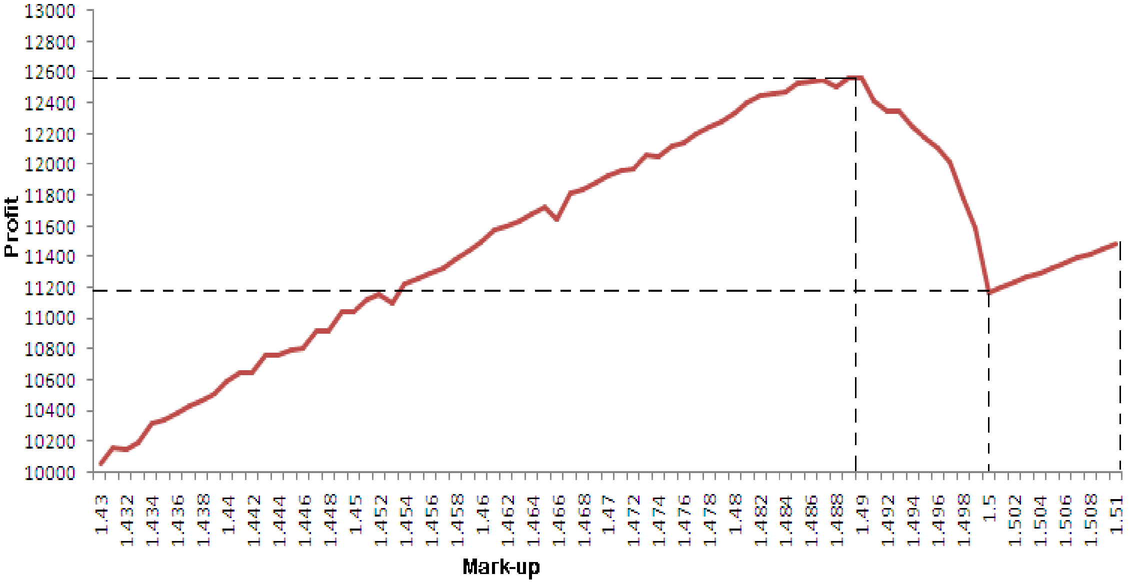


Figure 7.6: Change profit with respect to mark-up

7.7 Discussion:

Practically when a wholesaler supplies stocks to the showrooms against the fuzzy demands, then it may happen that the sales of the item decrease due to the high selling price or increase due to the low selling price and hence the supply of the item is affected. Even the wholesaler previously knows the fuzzy demand to choose the appropriate data i.e. for exact amount of supply to showrooms, fuzzy inference has a great utility. The target of the supply chain system is to maximize the profit of the wholesaler and for that, the related costs- purchasing cost, holding cost, etc. are obviously minimized. Here fuzzy inference rules are also applied successfully to reduce the purchasing cost of the wholesaler as the cost of the items charged by the suppliers varies with the amount of the quantities purchased by the wholesaler.

The Table-7.7 furnishes the optimum results of the mark-up, cycles and demands for the wholesaler for which his/her profit is maximum. Here optimum results are provided against nine sets of optimum fuzzy data i.e. for nine scenarios. Here, mark-up is low in the scenarios -1, -2 and -3, but the demand is low, medium and high in the respective scenarios -1, -2 and -3. Hence, as per expectation, both total procured amount and profit made by the wholesaler should be increase from scenarios -1 to -3 and maximum in scenario -3. The Table-7.7 agree with above observation. With medium mark-up in scenarios -4, -5 and -6 and demand as low, medium and high in the above scenarios, the behaviors of procured quantity and profit are same as before. These observations are also observed with the high mark-up in scenarios -7, -8 and -9.

Again, demand is low in scenarios -1,-4 and -7, but in these scenarios, mark-up is low, medium and high respectively. In our model formulation, mark-up and demand are taken as two independent parameters. Thus total stock at the wholesaler is lowest in scenario -1 following the rules (A)[c.f. 7.4]. But in scenario -4, stock takes the outer (higher) range of 'low stock' and in scenario -7, it assumes the lower range of 'medium stock'. As a result, the profit along-with the stock level are maximum in scenario -4. The above observations are observed for scenarios -2, -5 and -8 with medium demand and mark-up as low, medium and high respectively. Here, profit is maximum in scenario -5. Stock level in scenario -5 takes the outer range of 'medium stock' and lower range of 'high stock' in scenario -8 and thus stock along-with in scenario -5 are higher/more than those in scenario -8. similarly, for high demand and different types of mark-up, profit and stock are maximum for medium mark-up (i.e. scenario -6).

From the above findings, it may be calculated that high mark-up or high demand may not fetch always the maximum profit. It actually depends on the fuzzy data set.

In Table -7.8 three set of result are given which are obtained with known mark-up and demand given by the merchant. Which is near about the original profit made by the merchant. The difference arises between the merchants profit and profit obtained through the model depends on rounding off error, and the no. of cycles taken by the merchant. The difference between original profit and profit obtained from the model also caused by the total length of the business period. Table -7.8 reveals that high demand or high mark-up always may not a most profitable business policy.

The curves (Fig.-7.5 and Fig.-7.6) present an overview of the characteristics of the parameters-total quantity and profit against the mark-up with increment 0.001 with demand fixed at a particular value (75 units) with its range (Medium).

The normal relation between mark-up and required quantity at the showrooms (i.e. purchased quantity) is that requirement will decrease as mark-up increases. Fig.-7.5 more or less reflects this phenomena . In Fig.-7.5 total quantity is constant (3099 units) for mark-up 1.43 to 1.44 as the membership value of imprecise mark-up to the low fuzzy number is 1.0 and over this range demand remains constant. The total quantity decreases gradually from 3099 units to 2524 units as mark-up increases from 1.44 to 1.49 and then sharply decreases up-to 1.5. This is due to the nature of impreciseness of the mark-up over this range. After that total quantity is constant against mark-up as the membership value of mark-up again becomes 1.0 to the high fuzzy number.

As the fig.-7.6 drawn with fixed demand, the expected behavior of profit with respect to mark-up is that increment in mark-up will increase profit. The Fig.-7.7 more or less behaves initially in the above mentioned way but at end, it does not reflect so. The profit increases for mark-up up-to 1,489 with some small ups and down and is maximum at 1.489. From the fuzzy nature of mark-up, profit is maximum when mark-up is medium and then it decreases up-to 1.5. After that it increases with mark-up as mark-up is in high range with membership value 1.0 and the quantity required at the showrooms become fixed.

7.8 Conclusion

In this study, for the first time, a real life SCM with random planning horizon and fuzzy inference has been developed and solved by a soft computing method. Here we have developed a methodology to maximize the profit of a wholesaler who purchases quantities from suppliers and supplies the quantities to showrooms for sale in fuzzy-random environment. The selling price of the supplier varies inversely and imprecisely with the ordered quantities following one parameter fuzzy rule. Similarly, the amount of stock at sale depots varies inversely and directly with the selling price (i.e. mark-up) and demand respectively following two parameter fuzzy rules connecting mark-up, demand and stock. The whole planning horizon is taken as random with normal distribution. This has been expressed as a chance-constraint. A real coded genetic algorithm with fuzzy inference has been developed and used for solution. Examples imply that the proposed model outperforms conventional ones and is capable of developing a proper schedule to get the maximum profit. This study also gives the perfect quantity distribution by which one gets the maximum cost discount from the suppliers in limited capacity environment and makes maximum profit through proper distribution of stock in showrooms. This study can be useful for the inventory practitioners doing business of seasonal products such as warm garments, fruits, etc. Though the present model has been formulated under a single management system, but it can also be formulated under different management systems as a multi-objective problem treating the supplier, wholesaler and the retailers (against the showrooms) as different partners. The methodology described here can also be applied to different supply chain processes.

Chapter 8

A Deteriorating Multi-item Inventory Model with Price Discount and Variable Demands Via Fuzzy logic Under Resource Constraints [1]

8.1 Introduction

In the existing literature of inventory, most of the models are developed under infinite time horizon. As per Gurnani [100] , the life of a particular item is not infinite due to the change of design, technological development, variation of inventory costs, customers' changing taste, etc and this is very much true for the seasonal products in developing countries where preserving facilities are not available in plenty. For these seasonal products, even though the planning horizon is assumed as finite, in every season it fluctuates depending on some extraneous factors such as climatic conditions, etc. This time period may be assumed to be random with a probability distribution. In the literature Maiti *et al.* [169], Roy *et al.* [212] have solved some inventory problems with random planning horizon having exponential distribution. Also Moon and Lee [184] have presented an EOQ model under inflation and discounting with a random product life cycle.

In an inventory system, deterioration is an usual phenomenon. Mandal and Phaujder [174] presented an inventory model with deteriorating items. Roy *et al.* [213] have done a research work of deteriorating items with stock dependent demand over random planning horizon. Also Bhunia and Maiti [21], Mahapatra and Maiti [162] presented some inventory models for deteriorating items with time dependent demand and imprecise production time respectively.

In the present competitive market, the demand depends on the stock directly and also inversely on the selling price. Recently Widyadana *et al.* [259] presented a deteriorating inventory problem with constant demand via a simplified approach. Also Giri *et al.* [89], Mandal and Maiti [176] and others considered the demand as an indexed stock (i.e. $D = dq^{\beta}$, d and β are constants) dependent. But there are few research works with fuzzy demand depending on stock and selling price following

[1]This model has been published as a research paper in the journal **Computer and Industrial Engineering**

fuzzy inference. Recently, some inventory models with rework for the defective products [Jamal *et al.* [122], Cardenas-Barron [30–33], Sarker *et al.* [224], Cardenes-Barron *et al.* [34]] have been presented in the literature.

Human knowledge is often represented imprecisely, vaguely and approximately. In our real life, some vague terms in the form of 'words' such as high, medium, low etc are used. The target of fuzzy inference process is to form it into natural language expressions of the type,

IF premise (antecedent) THEN conclusion (consequent).

There are two types of fuzzy inference systems: Mamdani- type [Mamdani and Assilina [173]] and Sugeno-type [Ban et.al. [11]]. These two types differ in the way by which output is determined. Mamdani's effort was based on Bellman and Zadeh's [15] paper developing fuzzy algorithms for complex systems and decision processes. The main difference between Mamdani and Sugeno is that Sugeno output membership functions are either linear or constant where the Mamdani output is a fuzzy set. Since performance or satisfaction level of a perfect order cannot be judged in terms of discrete values, a Mamdani-type inference system is selected here for evaluating and aggregating the fuzzy rules.

Among the recently used optimization techniques, Genetic Algorithm (GA) is the most popular one. Some advantages can be pointed out for acceptability of this method.

• GAs work with a population of solutions instead of a single solution and for this it gives more globalized solution.

• GAs do not require any auxiliary information except the objective function values. Also there are some classical direct search methods which work under the assumption that the function to be optimized is unimodal. GAs do not impose any such restriction.

• GAs use probabilistic guide lines for search where in most of the classical methods, fixed transition rules are used to move from one solution to another.

Some research works [Zydallis [279]] using GA process are available in the literature.

In-spite of several development in the area of supply-chain models, there are still some gaps in the literature.

i) There are very few supply-chain models for deteriorating items with fuzzy inference expressed verbally using 'words'.

ii) Till now, none has used three types of price discount (AUD, IQD, AUD in IQD) in a supply-chain model connecting through fuzzy inferences and sharing the part of the commission with customers.

iii) No supply-chain is available with MRP and commission on this following fuzzy rules.

iv) Use of random planning horizon is very limited and none has used it in connection with fuzzy inferences.

v) Ga is not yet developed connecting random planning horizon, fuzzy logic and price discount.

vi) For the first time, surprise function, possibility for resource constraints are used in a supply-chain model.

In this chapter an inventory model for some seasonal products is presented with a wholesaler and its m showrooms under a random planning horizon. The wholesaler purchases a number of items from a set of predetermined suppliers and supplies the items to the showrooms for sale in certain number of cycles to achieve the maximum profit. The suppliers offer some ranges of commission on MRP to the wholesaler and it is presented in three ways, in the forms of AUD, IQD or IQD in AUD. The wholesaler shares a part of this commission with his/her customers. Demands of the items at the open market depends on the discount given on MRPs of the items by some defined verbal fuzzy rules. Items considered here deteriorate at some fixed rates. Moreover the wholesaler has resource constraints in purchasing and storing the items due to limited budget and storage capacity. The model is formulated with a fuzzy space constraint in the form of possibility [Liu and Liu [154]] and a crisp budget constraint in the form of surprise function. In seasonal business, the business periods are uncertain in stochastic sense. So the time horizons considered in this model are random with normal distribution and are evaluated by chance-constraint method. The fuzzy relations are defuzzified following Mamdani technique. The profit function formulated with respect to the wholesaler is maximized using a real coded Genetic algorithm. The model is illustrated with numerical examples. Some raw data regarding the model parameters (such as demand, selling price etc.) in a developing country are collected and represented in the from of fuzzy number and an inventory policy is developed. The novelty of this investigation is that for the first time, a real-life supply chain / inventory model has been formulated and solved with price discount under random planning horizon and fuzzy inference concerning price and demand taking some imprecise and crisp resource constraints into account.

The rest of this discussion is organized as follows. Section-8.2 contains the notations and assumptions made for the proposed model. The development of the model is given in section-8.3. In section-8.4, there are some fuzzy rules considered for the problem. Section-8.5 contains the solution procedure using GA in favor of the proposed model. A numerical experiment is performed in section-8.6 and a real life application is presented in section-8.7. Discussion and conclusion are made in the sections-8.8 and -8.9 respectively.

8.2 Notations and Assumptions

8.2.1 Notations

$\hat{H}$	random planning horizon follows the normal distribution with mean $m_{\hat{H}}$ and variance $\sigma_{\hat{H}}$.
W	represents the wholesaler.
$i\,[or,\ j]$	index used for $i-th$ item [or, for $j-th$ showroom].
R_j	represents the $j-th$ showroom of the wholesaler W where, $j = 1, 2, \cdots, m$.
Q_i	amount of $i-th$ item purchased from the suppliers over the whole business period.

p_{wi}	purchasing cost of per unit of $i-th$ item.
QR_{ij}	amount of $i-th$ item required at R_j in each cycle.
N_j	number of cycles taken by W to supply items to R_j.
T_j	length of each cycle for R_j.
Q_{ijk}	status of the total amount of i-th item required to stock for R_j by the wholesaler in k-th cycle where, $k = 1, 2, \cdots, N_j$.
D_{ij}	per unit time demand of i-th item faced at R_j.
$IW_{ijt}\,[or,\, I_{ij}]$	inventory position of the required stock of i-th item for [or, of] the j-th showroom at time t at the wholesaler [or, at j-th showroom] in any cycle.
Qw_{il}	a certain amount of i-th item, which is used to define the ranges of commission offered by the suppliers where, l is a positive integer.
sr_{ij}	selling price of per unit i-th item for R_j.
m_{ij}	percentage of discount on sr_{ij}.
$hr_{ij}\,[or,\, hw_i]$	per unit item holding cost at $R_j\,[or,\, W]$ of i-th item.
HW_{ijk}	holding cost for the $i-th$ for the stock of $j-th$ showroom in $k-th$ cycle.
$HR_{ij}\,[or,\, THW_i]$	total holding cost at $R_j\,[or,\, W]$ of the $i-th$ item.
$sur_j\,[or,\, Tsur_j]$	per cycle [or, total] set-up cost at R_j.
$tr_{ij}\,[or,\, Ttr_{ij}]$	per lot [or, total] transportation cost in transporting $i-th$ item to R_j from W.
Lr_{ij}	lot size, i.e the maximum amount of $i-th$ item which can be transported to R_j from W at once.
$cw_{il}[or,\, cw_{i\infty}]$	percentage of commission on $i-th$ item in $l-th$ range [or, last range] of commission.
cw_i	final percentage of commission obtained by the wholesaler.
P_i	per unit item printed price (MRP) of i-th item.
$\tilde{a}_i = (a1_i, a2_i, a3_i)$	per unit i-th item fuzzy space required at the wholesaler.
$\tilde{S} = (s1, s2, s3)$	total fuzzy space available at the wholesaler.
$\tilde{B} = (B1, B2, B3)$	fuzzy budget amount of W over $\hat{H}$.
$\theta r_{ij}\,[or,\, \theta w_i]$	percentage of rate of deterioration of the $i-th$ item at R_j [or, W].
β, pr_j	two given real numbers where $\beta > 0$, $0 \leq pr_j \leq 1$.
ϵ_j	a real number whose standard normal value is pr_j.
PC, TC	respectively represent total purchasing cost and total cost of the wholesaler.
TE, TP	respectively represent total earn and total profit of the wholesaler.

8.2.2 Assumptions

i) The wholesaler, W maximizes his profit with its m showrooms R_j , $j = 1, 2, \cdots, m$, in a business of n deteriorating items over a random planning horizon, $\hat{H}$.

ii) The demand of $i-th$ item faced at R_j over the whole business period is imprecise in nature and depends on selling price under some fuzzy rules.

iii) Since the suppliers supply items under their own responsibility and by own transportation arrangement so the transportation cost is considered only for the transportation from wholesaler to showrooms which depends on the amount of items.

iv) In the proposed model it is considered that the wholesaler has limited space to store the items and also has a budget over the total time horizon which are fuzzy in nature.

v) Restriction on the budget amount is introduced in the profit function as surprise function.

vi) Some positive real no.s $\beta,\ A_1,\ A_2,\ A_3$ are considered to express the space constraint by possibility.

8.3 Formulation of The Model

The presented model is formulated as a single management problem of a wholesaler with m showrooms at different places over a random time horizon. After giving m_{ij} percent discount over the MRP of the i-th item at the j-th showroom to the customer, the selling price is

$$sr_{ij} \quad = \quad P_i.(1-\frac{m_{ij}}{100}) \quad where\,,\ i=1,2,\cdots,n\,,\ j=1,2,\cdots,m \tag{8.1}$$

8.3.1 Quantity at Showrooms

Let I_{ij} be the inventory position at the j-th showroom of i-th item and let θr_{ij} be the rate of deterioration of the i-th item at j-th showroom. Then

$$\frac{dI_{ij}}{dt} \quad = \quad -(D_{ij}+\theta r_{ij}\,.I_{ij}) \quad \text{where}\,,\ i=1,2,\cdots,n,\ \ j=1,2,\cdots,m \tag{8.2}$$

with the boundary conditions,

$$\begin{array}{ll} \text{for,} \quad t=0, & I_{ij}=QR_{ij} \ \text{ and} \\ \text{for,} \quad t=T_j, & I_{ij}=0 \end{array}$$

Solving this differential equation (8.2)using the corresponding boundary conditions the total quantity of i-th item required at the j-th showroom is obtained as

$$QR_{ij} \quad = \quad \frac{D_{ij}(e^{\theta r_{ij}}-1)}{\theta r_{ij}} \quad where\,,\ i=1,2,\cdots,n\,,\ j=1,2,\cdots,m$$

8.3.2 Holding Cost at Showrooms

If HR_{ij} be the total holding cost at the j-th showroom for the i-th item then according to figure-8.1,

$$HR_{ij} \quad = \quad hr_{ij}.(\frac{1}{2}.N_j.QR_{ij}.T_j) \quad for,\ i=1,2,\cdots,n,\ \ j=1,2,\cdots,m \tag{8.3}$$

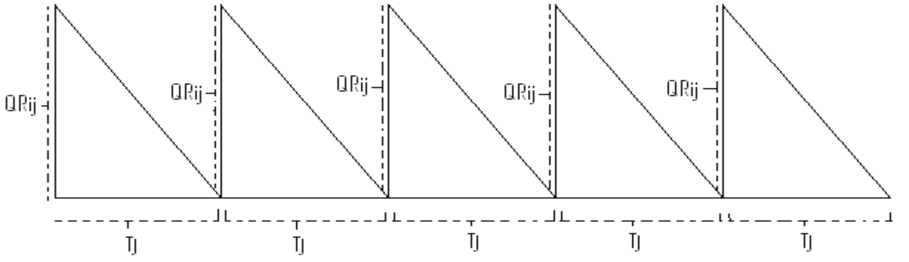


Figure 8.1: Inventory policy at the j-th showroom.

8.3.3 Set-up Cost at Showrooms

Let sur_j be the set-up cost per cycle at the j-th showroom then the total set-up cost $Tsur_j$ is

$$Tsur_j \quad = \quad N_j.sur_j \quad for, \;\; j = 1, 2, \cdots, m \tag{8.4}$$

8.3.4 Transportation Cost at Showrooms

Let tr_{ij} be the transportation cost per lot size for the j-th showroom of the i-th item. Then, as Lr_{ij} is the lot size for transportation of the i-th item to the j-th showroom then,

if $\quad x.Lr_{ij} < QR_{ij} \leq (x+1).Lr_{ij},$ $\qquad$ for some non-negative integer x,
then $\quad$ the total transportation cost,

$$Ttr_{ij} \quad = \quad (x+1).tr_{ij}, \text{ for each } i = 1, 2, \cdots, n\,,\; j = 1, 2, \cdots, m. \tag{8.5}$$

8.3.5 Model for the Wholesaler's Stock

Let θw_i be the rate of deterioration at the wholesaler and Q_{ij_k} stock for j-th showroom at the wholesaler in k-th cycle where, $k = 1, 2, 3, \cdots, N_j - 1$ and for the N_j-th cycle the stock is given by $Q_{ij_{N_j}} = QR_{ij}$

As IW_{ij_t} is the inventory position at time t in any cycle for the stock of $i-th$ item of $j-th$ retailer at the wholesaler then the corresponding differential equation is

$$\frac{dIW_{ij_t}}{dt} \quad = \quad -\theta w_i IW_{ij_t} \;\text{ for, } (k-1)T_j \leq t \leq kT_j,\; k = 1, 2, \cdots, (N_j - 1)$$

The policy of holding inventory and stock deduction policy at the wholesaler is depicted in figure-8.2. Thus according to figure-8.2,

$$\begin{aligned} IW_{ij_t} &= Q_{ij_k} - QR_{ij} \text{ for } t = (k-1)T_j \\ &= Q_{ij_{(k+1)}} \quad \text{for } t = kT_j \quad k = 1, 2, \cdots, (N_j - 1) \end{aligned}$$

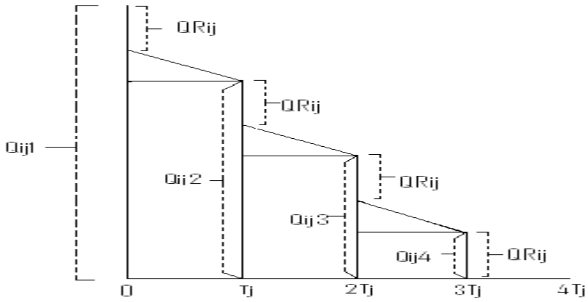


Figure 8.2: Representation of Inventory Policy of Wholesaler against different Showrooms.

Solving these differential equations the following results are obtained.

$$Q_{ij_1} = QR_{ij}\frac{e^{N_j.\theta w_i.T_j}-1}{e^{\theta w_i.T_j}-1}$$

and the other stocks are given by

$$\begin{aligned} Q_{ij_k} &= Q_{ij_1}e^{-(k-1)\theta w_i.T_j} - QR_{ij}\frac{e^{-(k-1)\theta w_i T_j}-1}{1-e^{\theta w_i T_j}},\ k = 2,3,\cdots,(N_j-1) \\ \text{and}\ Q_{ij_{N_j}} &= QR_{ij},\ i=1,2,\cdots,n,\ j=1,2,\cdots,m \end{aligned}$$

Therefore the total amount of $i-th$ item Q_i to be purchased by the wholesaler is given by

$$Q_i = \sum_{j=1}^{m} Q_{ij_1},\ \text{for}\ i=1,2,\cdots,n \tag{8.6}$$

8.3.6 Holding Cost at the Wholesaler

Let hw_i be the per unit time and per unit of $i-th$ item holding cost at the wholesaler. Then the holding cost for the $i-th$ item for the stock at $j-th$ showroom in $k-th$ cycle HW_{ij_k} is given by

$$\begin{aligned} HW_{ij_k} &= hw_i[Q_{ij_{(k+1)}}T_j + \frac{1}{2}T_j\{Q_{ij_k} - QR_{ij} - Q_{ij_{(k+1)}}\}] \\ & for,\ k=1,2,\cdots,(N_j-1) \end{aligned}$$

Therefore the total holding cost for the i-th item is

$$THW_i = hw_i\sum_{j=1}^{m}T_j\left[\frac{1}{2}\sum_{k=1}^{N_j-1}\{Q_{ij_{k+1}}+Q_{ij_k}\} - \sum_{k=1}^{N_j-1}QR_{ij}\right] \tag{8.7}$$

8.3.7 Commission for the Wholesaler

The percentage of commission cw_i obtained by the wholesaler depends on the amount of $i-th$ item supplied by the suppliers under AUD, IQD and IQD in AUD which are respectively given by the following.

Commission with AUD: Qw_{il} represents an amount of $i-th$ item such that,

$$cw_i = \begin{cases} cw_{i1} & if \;\; 0 < Q_i < Qw_{i1} \\ cw_{i2} & if \;\; Qw_{i1} \leq Q_i < Qw_{i2} \\ \ldots\ldots\ldots\ldots\ldots & \\ cw_{il} & if \;\; Qw_{i(l-1)} \leq Q_i < Qw_{il} \\ cw_{i\infty} & if \;\; Qw_{il} \leq Q_i < \infty \end{cases}$$

where cw_{il} is the percentage amount of $i-th$ item in the $l-th$ interval with the restriction $m_{ij} \leq cw_{i1} \leq cw_{i2} \leq \ldots.. \leq cw_{il} \leq cw_{i\infty}$ for each $l = 1, 2, ...,$ $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$.

Commission with IQD: Under incremental quantity discount (IQD) the commission process is given by the following

$$cw_i = \begin{cases} cw_{i1} & if \;\; 0 < Q_i < Qw_{i1} \\ [cw_{i1}Qw_{i1} + cw_{i2}(Q_i - Qw_{i1})]/Q_i & if \;\; Qw_{i1} \leq Q_i < Qw_{i2} \\ \ldots\ldots\ldots\ldots\ldots & \\ [cw_{i1}Qw_{i1} + ... + cw_{il}(Q_i - Qw_{i(l-1)})]/Q_i & if \;\; Qw_{i(l-1)} \leq Q_i < Qw_{il} \\ [cw_{i1}Qw_{i1} + ... + cw_{i\infty}(Q_i - Qw_{il})]/Q_i & if \;\; Qw_{il} \leq Q_i \end{cases}$$

here $m_{ij} \leq cw_{i1} \leq cw_{i2} \leq \ldots.. \leq cw_{il} \leq cw_{i\infty}$ for each $l = 1, 2, ...,$ $i = 1, 2, ..., n$ and $j = 1, 2, ..., m$.

Commission with IQD in AUD: The commission percentage cw_i under nested discount (IQD within AUD) scheme is taken as:

$$cw_i = \begin{cases} cw_{i1} & if \;\; 0 < Q_i < Qw_{i1} \\ cw_{i2} & if \;\; Q_i = Qw_{i1} \\ cw_{i2}.(1 + \dfrac{{Q_i}^{\delta_1}}{Qw_{i2}}) & if \;\; Qw_{i1} < Q_i < Qw_{i2} \\ \ldots\ldots\ldots\ldots\ldots & \\ cw_{il}.(1 + \dfrac{{Q_i}^{\delta_{l-1}}}{Qw_{il}}) & if \;\; Qw_{il-1} < Q_i < Qw_{il} \\ cw_{i\infty} & if \;\; Qw_{il} \leq Q_i < \infty \end{cases}$$

where $0 \leq cw_{i1} \leq cw_{i2}$, $0 \leq cw_{i2}.(1 + \frac{{Q_i}^{\delta_1}}{Qw_{i2}}) \leq cw_{i3}$,......, $0 \leq cw_{il-1}.(1 + \frac{{Q_i}^{\delta_{l-2}}}{Qw_{il-1}}) \leq cw_{il}$ and $cw_{il}.(1 + \frac{{Q_i}^{\delta_{l-1}}}{Qw_{il}}) \leq cw_{i\infty}$, where l is a positive integer.
$Qw_{i1}, Qw_{i2}, ..., Qw_{i\infty}$ are the amounts at which new discount is permitted and $0 < \delta_1, \delta_2, .., \delta_{l-1} < 1$.

8.3.8 Space constraint for the Wholesaler

Let $\widetilde{a_i} = (a1_i, a2_i, a3_i)$ be the fuzzy space required for per unit of $i-th$ item and $\widetilde{S} = (s1, s2, s3)$ is the total fuzzy space available at the wholesaler. Then the space constraint is

$$\sum_{i=1}^{n} \widetilde{a_i} Q_i \quad \leq \quad \widetilde{S}$$

The membership functions are defined by

$$\mu_{\widetilde{a_i}}(x) = \begin{cases} 0 & \text{if } x \leq a1_i \\ \dfrac{x - a1_i}{a2_i - a1_i} & \text{if } a1_i \leq x \leq a2_i \\ \dfrac{a3_i - x}{a3_i - a2_i} & \text{if } a2_i \leq x \leq a3_i \\ 0 & \text{if } x \geq a3_i \end{cases} \qquad \mu_{\widetilde{S}}(x) = \begin{cases} 0 & \text{if } x \leq s_1 \\ \dfrac{x - s_1}{s_2 - s_1} & \text{if } s_1 \leq x \leq s_2 \\ \dfrac{s_3 - x}{s_3 - s_2} & \text{if } s_2 \leq x \leq s_3 \\ 0 & \text{if } x \geq s_3 \end{cases}$$

Then for some positive real number η the

$$Pos\left(\sum_{i=1}^{n} \widetilde{a_i}.Q_i \leq \widetilde{S}\right) > \eta \; iff \; \frac{s_3 - A_1}{s_3 - s_2 + A_2 - A_1} > \eta \tag{8.8}$$

$$\text{where, } A_1 = \sum_{i=1}^{n} a1_i Q_i, \qquad A_2 = \sum_{i=1}^{n} a2_i Q_i, \qquad A_3 = \sum_{i=1}^{n} a3_i Q_i$$

8.3.9 Budget constraint for the Wholesaler

If pw_i be the purchasing cost per unit of $i - th$ item of the wholesaler then

$$pw_i = P_i(1 - \frac{cw_i}{100}), \; for \; i = 1, 2, \cdots, n$$

Then the purchasing cost (PC) to purchase Q_i (given by8.6) amount of $i - th$ item of the wholesaler is given by

$$PC = \sum_{i=1}^{n} pw_i Q_i \tag{8.9}$$

As $\widetilde{B} = (B_1, B_2, B_3)$ is the fuzzy budget amount of the wholesaler then

$$0 \leq \mu_{\widetilde{B}}(PC) \leq 1$$

Thus the surprise function is given by

$$S(PC) = \begin{cases} 0 & \text{if } B_1 \leq PC \leq B_2 \\ \dfrac{PC - B_2}{B_3 - PC} & \text{if } B_2 < PC < B_3 \\ \infty & \text{if } PC \geq B_3 \end{cases} \tag{8.10}$$

8.3.10 Chance Constraint for Random Time Horizon

Here the time horizon $\hat{H}$ is considered as random but the total time horizon is partitioned into N_j cycle with length T_j for the $j - th$ showroom. Therefore the corresponding Chance Constraint for the $j - th$ showroom is given by

$$Prob(|N_j.T_j - \hat{H}| \leq \beta) \geq pr_j \qquad \text{where, } j = 1, 2, \cdots, m$$

which can be reduced to [c.f. 2.1.3],

$$m_{\hat{H}} - \beta - \epsilon_j \sigma_{\hat{H}} \leq \quad N_j T_j \quad \leq m_{\hat{H}} + \beta - \epsilon_j . \sigma_{\hat{H}} \tag{8.11}$$

$$\text{where } pr_j \;=\; F(\epsilon_j) \;=\; \frac{1}{\sqrt{2\Pi}} \int_{-\infty}^{\epsilon_j} e^{-\frac{t^2}{2}} dt \;\; for, \; j = 1, 2, \cdots, m$$

is the cumulative probability $P[t \leq \epsilon_j]$ of standard normal variate available in standard statistical table for different values of ϵ_j.

8.3.11 Maximization Problem w.r.t. the Wholesaler

For selling items in the demand rate D_{ij} per unit time per unit of $i-th$ item in N_j cycles with cycle length T_j at the $j-th$ showroom, the total earn TE is given by

$$TE \;=\; \sum_{i=1}^{n} \sum_{j=1}^{m} D_{ij} N_j T_j sr_{ij} \tag{8.12}$$

where sr_{ij} is given by the equation (8.1) and the total cost TC is given by

$$TC \;=\; \sum_{i=1}^{n} THW_i + \sum_{i=1}^{n} \sum_{j=1}^{m} HR_{ij} + \sum_{i=1}^{n} \sum_{j=1}^{m} Ttr_{ij} + PC + \sum_{j=1}^{m} Tsur_j \tag{8.13}$$

where $HR_{ij}, Tsur_j, Ttr_{ij}, THW_i$ and PC are given by the expressions (8.3), (8.4), (8.5), (8.7) and (8.9) respectively. Thus if TP be the total profit of the wholesaler then the problem is

$$\text{maximize } TP \;=\; TE - TC - S(PC) \tag{8.14}$$

whereTE, TC and $S(PC)$ are given by (8.12), (8.13) and (8.10) respectively. Subject to the constraints

$$\frac{s_3 - A_1}{s_3 - s_2 + A_2 - A_1} \;>\; \eta \tag{8.15}$$

$$m_{\hat{H}} - \beta - \epsilon_j \sigma_{\hat{H}} \leq \quad N_j T_j \quad \leq m_{\hat{H}} + \beta - \epsilon_j \sigma_{\hat{H}} \tag{8.16}$$

8.4 Fuzzy Inference Rules

The demand of the i-th item faced by the j-th showroom depends on the selling price under the following fuzzy rules-

IF (Selling Price is $High$) THEN (Customers' Demand is Low).
IF (Selling Price is $Medium$) THEN (Customers' Demand is $Medium$).
IF (Selling Price is Low) THEN (Customers' Demand is $High$).

where Low, $Medium$ and $High$ represented by triangular fuzzy numbers of the form $(a_1, \, a_2, \, a_3)$.

8.5 Solution Methodology: A Routine Framework for GA

The constrained single objective non-linear problem is evaluated numerically using Genetic Algorithm (GA). The discussion about the Genetic Algorithm process and algorithms are given in the sections-2.2.4 and -7.5.

At the beginning of the GA module, the different parameters of GA i.e. generation number (MAXGEN), population size (POPSIZE), probability of cross-over (PXOVER), probability of mutation (PMUT), random seed (RSEED), distribution index for SBX (DISBX) and for mutation (DIMUT) and the others input data have to be supplied. As there is no clear indication as to how large a population should be, here with $POPSIZE = no\,of\ variables \times 15$, the expected result is obtained. Here a combination of real and natural number representation is used to structure a chromosome, where a chromosome is a string of genes which are specified by the decision variables of the problem namely- percentage of commission (m_{ij}), length of each cycle (T_j), and the no. of cycles taken by the wholesaler to supply the items to i-th retailer (N_j). The variable boundaries may be fixed or flexible. The fitness function is the profit function (TP) and the constraints are defined by the wholesaler given by the equations 8.14, 8.10, 8.11 respectively.

8.6 Numerical Experiment

A supply chain model with respect to a wholesaler with its two showrooms in a business of two items is considered with following data.

8.6.1 Input Data

Crisp Data

$pr_i = 50,\ \theta w_i = 0.02,\ hw_i = 0.1.\ sur_j = 300,\ \epsilon_j = 1.3.\ hr_{ij} = 0.2,\ \theta r_{ij} = 0.01,\ tr_{ij} = 50,\ Lr_{ij} = 500,\ where,\ i = 1,2,\ j = 1,2.\ eta = 0.9,\ \beta = 0.01,\ m_H = 12,\ \sigma_H = 0.9.$

Commission with AUD: $Qw_{il} = 5000.l,\ \ where\ l = 1,2,3,4.$

$$cw_i = \begin{cases} 30 & if\ 0 < Q_i < Qw_{i1} \\ 40 & if\ Qw_{i1} \le Q_i < Qw_{i2} \\ 50 & if\ Qw_{i2} \le Q_i < Qw_{i3} \\ 55 & if\ Qw_{i3} \le Q_i < Qw_{i4} \\ 60 & if\ Qw_{i4} \le Q_i < \infty \end{cases}$$

Commission with IQD: $Qw_{il} = 5000.l, \;\; where \; l = 1, 2, 3.$

$$cw_i = \begin{cases} 30 & if \;\; 0 < Q_i < Qw_{i1} \\ \dfrac{30Qw_{i1} + 40(Q_i - Qw_{i1})}{Q_i} & if \;\; Qw_{i1} \leq Q_i < Qw_{i2} \\ \dfrac{30Qw_{i1} + 40(Qw_{i2} - Qw_{i1}) + 60(Q_i - Qw_{i2})}{Q_i} & if \;\; Qw_{i2} \leq \; Q_i \; < Qw_{i3} \\ \dfrac{30Qw_{i1} + 40(Qw_{i2} - Qw_{i1}) + 60(Q_i - Qw_{i2}) + 80(Q_i - Qw_{i3})}{Q_i} & if \; Qw_{i3} \leq \; Q_i \; < \infty \end{cases}$$

Commission with IQD in AUD: $Qw_{il} = 5000.l, \;\; where \; l = 1, 2, 3, 4.$

$$cw_i = \begin{cases} 30 & if \;\; 0 < Q_i < Qw_{i1} \\ 40 & if \;\; Q_i = Qw_{i1} \\ 40(1 + \dfrac{{Q_i}^{0.5}}{Qw_{i3}}) & if \;\; Qw_{i1} < Q_i < Qw_{i3} \\ 45 & if \;\; Q_i = Qw_{i3} \\ 45(\dfrac{1 + {Q_i}^{0.7}}{Qw_{i4}}) & if \;\; Qw_{i3} < \; Q_i \; < \; Qw_{i4} \\ 60 & if \;\; Qw_{i4} \leq \; Q_i \; < \infty \end{cases}$$

Fuzzy Data

Selling Price:
$Low = (25, 32.5, 40), \; Med = (32.5, 40, 47.5), \; High = (40, 47.5, 50).$

Demand:
$Low = (0, 400, 600), \; Med = (400, 600, 800), \; High = (600, 800, 1400).$

Unit Fuzzy Space : $\tilde{a}_1 \, , \, \tilde{a}_2 = (8, 10, 12).$

Total Fuzzy Space : $\tilde{S} = (100000, 200000, 300000).$

Total Fuzzy Budget : $\tilde{B} = (600000, 700000, 800000).$

GA Parameters : $POPSIZE = 80 \; MAXGEN = 200$, $PXOVER = 0.8$, $PMUT = 0.2$, $RSEED = 1.2$, $DISBX = 2$, $DIMUT = 100$.

The variables are ranges as m_{ij}-(0.0 to 0.5), T_j-(1 to 12) and N_j-(1 to 12).

8.6.2 Optimum Results

The expression (8.14) is evaluated using GA for different discounts policies with the above input data and two sets of near-optimum solutions are presented in the Table-8.1. The total profit (8.14) is also evaluated allowing a fixed discount on MRP and given in Table-8.2.

Table 8.1: *Near-optimum Results for Different Discount Process*

disc ount pro cess	sol set	scm mem ber	am- ount		comm- ission (per cent)		deteri- oration		Selling price		Dem- and		no of cycle	cycle len gth (m)	total fuzzy space required	total purch asing cost	total profit
			1st item	2nd item	1st item	2nd item	1st item	2nd item	1st item	2nd item	1st item	2nd item					
AUD	1	W	13780	10489	50	50	640.6	487.5	-	-	-	-	-	-	(19416.3	606757	171440
		R1	1579.8	1352	10.08	7.07	69.2	60	44.96	46.47	332.3	284.3	2	4.65	242702.9,		
		R2	4990.1	3649.2	46.63	22.42	237.5	173.7	26.69	38.79	1047.6	766.1	2	4.65	291243.5)		
	2	W	5703.6	11571	40	50	457.2	923.3	-	-	-	-	-	-	(138197.2,	460384	185018
		R1	295.2	262.9	7.08	3.9	6.6	18	46.46	48.05	284.4	253.3	9	1.03	172746.5,		
		R2	323.7	1035.2	6.56	25.8	22.75	73.44	46.72	37.10	276.6	884.5	8	1.16	207295.8)		
IQD	1	W	13907	10678	42	36.6	852.75	654.53	-	-	-	-	-	-	(196680.6,	741701	142709
		R1	2590.4	2473	23.8	22.9	114	109.2	38.1	38.57	823.4	786	3	3.1	245850.7,		
		R2	1761.1	868.1	19.2	6.49	85.3	42.2	40.43	46.76	558.9	275.5	3	3.1	295020.9)		
	2	W	11823	11400	38.85	38.1	845.5	800.1	-	-	-	-	-	-	(185780.8,	714451.9	130605
		R1	939.7	1719.8	13.7	21.7	37.3	69.1	43.16	39.13	399.3	730.7	4	2.33	232226,		
		R2	1443.7	744.1	22.5	13.5	68.7	35.8	38.8	43.2	768.8	396.2	5	1.86	278671.2)		
IQD in AUD	1	W	11823	11400	40.3	40.3	845.7	800.1	-	-	-	-	-	-	(185780.8,	693341	151716
		R1	939.6	1719.8	13.7	21.7	37.6	68.7	43.2	39.1	399.2	39.1	4	2.33	232226,		
		R2	1443.7	744.1	22.5	13.5	68.5	31.7	38.8	43.2	768.8	396.2	5	1.86	27871.2)		
	2	W	10257	6217	40.3	40.2	544.3	373.2	-	-	-	-	-	-	(131792.1,	492182	148391
		R1	475.6	591.2	2.6	9.02	21.9	27.2	48.7	45.5	253.3	591.2	5	1.86	164740.1,		
		R2	3667.3	1443.9	22.5	8.31	162.2	63.9	38.7	45.8	771.2	303.6	2	4.65	197688.2)		

Table 8.2: *Near-optimum Results for fixed Discount*

scm mem ber	am- ount		comm- ission (per cent)		deteri- oration		Selling price		Dem- and		no of cycle	cycle len gth (m)	total fuzzy space required	total purch asing cost	total profit
	1st item	2nd item	1st item	2nd item	1st item	2nd item	1st item	2nd item	1st item	2nd item					
W	12688	8817	7	30	30	658.8	-	-	-	-	-	-	(172048,	752709	20018
R1	720.4	1338.5	8.47	19.37	4.4	8.4	45.77	40.32	306.1	568.7	4	2.35	215060,		
R2	4573.9	1463.1	29.3	8.5	218.8	70.2	35.35	45.73	960.1	307.1	2	4.65	258071)		

8.7 Practical Implications and Application

A merchant in West Bengal, India sells food-grains- rice and wheat from two showrooms and allows AUD discounts to the customers. For a period of 12 months. From customers' point of view, following data regarding different system parameters- selling price , demand are collected.

8.7.1 Collected Data

Table 8.3: *Data collected for selling price, Demand of Rice and Wheat*

		Low					Medium					High				
Rice	s_{1j}	15	22.1	20.8	20.7	22	20	23.8	25.1	23.2	25	23	26.2	28.1	28	26.8
	D_{11}	398	481	354	200	480	800	542	639	668	548	738	871	766	600	854
	D_{12}	344	232	223	304	445	250	397	434	486	546	424	752	756	571	640
wheat	s_{2j}	7	7.5	6.8	7.5	5.5	8.5	9.5	7.8	7.9	8.6	9	9.4	10	9.4	9
	D_{21}	270	161	166	218	206	299	263	235	212	302	250	416	440	352	338
	D_{22}	310	421	435	153	347	668	494	477	632	573	850	655	715	652	705

Table 8.4: *Data collected for per unit item fuzzy space, total fuzzy budget, total fuzzy space.*

$\tilde{a}_1$	3.7	5.2	3.5	4.1	4
$\tilde{a}_2$	4.7	6.2	4.5	5.1	5
$\tilde{B}$	352753	258594	449999	305613	252702
$\tilde{S}$	117286	96085	92562	108949	103532

Using these data and using the method mentioned in section-2.1.6, ratio of left spread and right spread (ξ) and the deviation(σ) of the fuzzy numbers from mode are calculated and the fuzzy numbers are computed as follows.

Table 8.5: *values of σ and ξ for different fuzzy numbers*

		Rice			Wheat						
		selling price	- D_{11}	- D_{12}	selling price	- D_{21}	- D_{22}	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{B}$	$\tilde{S}$
Low	σ	1.02	66.59	57.8	0.41	28.65	63.43				
	ξ	0.75	0.71	1.5	0.71	1.56	0.69				
Medium	σ	1.02	65.51	60.2	0.41	28.58	61.9	0.29	0.29	49744	6672
	ξ	0.72	1.52	0.64	1.55	0.64	1.65	0.49	1.55	0.49	1.61
High	σ	1.03	66.56	89.4	0.27	47	38.65				
	ξ	0.72	0.65	0.68	1.51	0.6	1.42				

For Rice

Selling Price: $Low = (18, 21, 24)$, $Med = (21, 24, 27)$, $High = (24, 27, 30)$.

Demand:
D_{11} : $Low = (239, 400, 627)$, $Med = (400, 627, 777)$, $High = (627, 777, 1000)$.
D_{12} : $Low = (100, 300, 434)$, $Med = (300, 434, 644)$, $High = (434, 644, 962)$.

Per Unit Fuzzy Space: $\tilde{a}_1 = (3, 4, 4.7)$.

For Wheat

Selling Price: $Low = (6, 7, 8.4)$, $Med = (7, 8.4, 9.3)$, $High = (8.4, 9.3, 9.9)$.

Demand:
D_{21} : $Low = (100, 200, 264)$, $Med = (200, 264, 364)$, $High = (264, 364, 530)$.
D_{22} : $Low = (200, 350, 568)$, $Med = (350, 568, 700)$, $High = (568, 700, 793)$.

Per Unit Fuzzy Space : $\tilde{a}_2 = (4, 5, 5.7)$.

Total Fuzzy Space : $\tilde{S} = (79797, 103202, 117773)$.

Total Fuzzy Budget : $\tilde{B} = (222994, 311497, 490720)$.

8.7.2 Other Inputs

Variable ranges: $0 \le m_{ij} \le 40$, $1 \le N_j \le 12$, $1 \le T_j \le 12$ where $i = 1, 2.$, $j = 1, 2$.
Commission with AUD:
$Qw_{il} = 5000.l, \ where\ l = 1, 2.$

$$cw_i = \begin{cases} 20 & if\ 0 < Q_i < Qw_{i1} \\ 30 & if\ Qw_{i1} \le Q_i < Qw_{i2} \\ 40 & if\ Qw_{i2} \le Q_i < \infty \end{cases}$$

$pr_1 = 30$, $pr_2 = 10$, $\theta w_i = 0.02$, $hw_i = 0.1$. $sur_j = 300$, $\epsilon_j = 0.1$. $hr_{ij} = 0.2$, $\theta r_{ij} = 0.01$, $tr_{ij} = 50$, $Lr_{ij} = 500$, $where, i = 1, 2, \ \ j = 1, 2.$
$eta = 0.1$, $\beta = 0.01$, $m_H = 12$, $\sigma_H = 0.9$.
The result obtained using the above collected is given in Table-8.6.

Table 8.6: *Result obtained from collected data*

scm mem	amount (unit)		comm (%)		sel. price($)		demand (unit)		No. of cycle	cycle length	profit
W	11694	8874	40	30	-	-	-	-	-	-	
R1	1326	694	12.2	12.4	26.34	8.76	439	230	4	2.98	44851.5
R2	2766	2743	19.65	12.09	24.1	8.79	451	447	2	5.96	

8.8 Discussion

Results of Table-8.1 reveal that the system with AUD gives the maximum profit, than the systems with IQD in AUD and IQD. Comparing the results of Tables-8.1 and -8.2, as per expectation, the system with fixed discount (equal to the initial value of the AUD, IQD and IQD in AUD systems) fetches maximum profit, more than other systems. Some results of maximum profit, optimum quantities for first and second items and random time period are graphically presented for different values of normal variate ϵ_j and for the commission shared with the customers m_{ij} in Figs- 8.3, 8.4, 8.5, 8.6, 8.7 and 8.8 respectively. As the total time period is random, the fig-8.6 depicts that when $\epsilon_j = 0$, the time period is 12, equal to the mean value of the given normal distribution and decreases gradually with the increase of ϵ_j. This is as per the usual expectation. The Figs- 8.3, 8.4, 8.5 give respectively the variations of the maximum total profit and the optimum amounts for the first and second items.

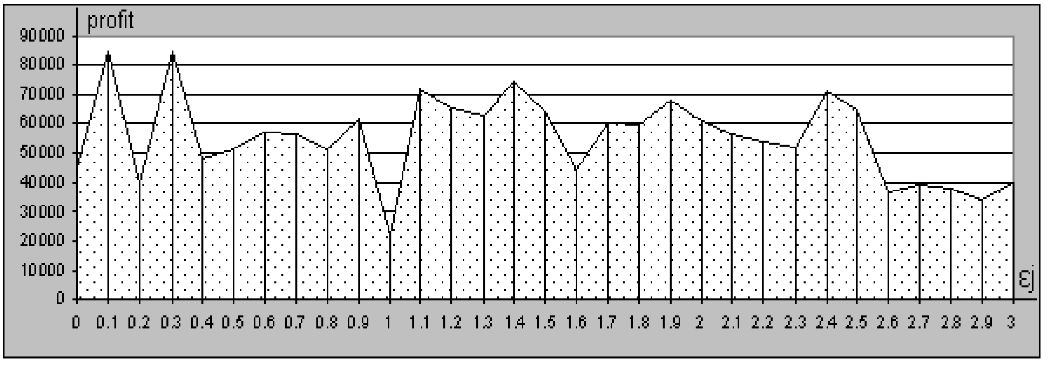


Figure 8.3: Profit / Random Variate

In the present model, the wholesaler gets a commission from the suppliers on the items' MRPs and shares a part of this with the customers. In Fig-8.7 and Fig-8.8, the commission cw_i and total profit of the wholesaler are respectively plotted against the commission offered to the customers m_{ij}. Here, the market demand depends on the selling price of the commodities i.e. on the commission enjoyed by the customer by some fuzzy rules. From Fig-8.8, it is seen that wholesaler's commission remains constant when customers' commission gradually increases from 1 to 5. This is for the following reason.

As the selling price is a fuzzy quantity and depends on the commission offered to customers by

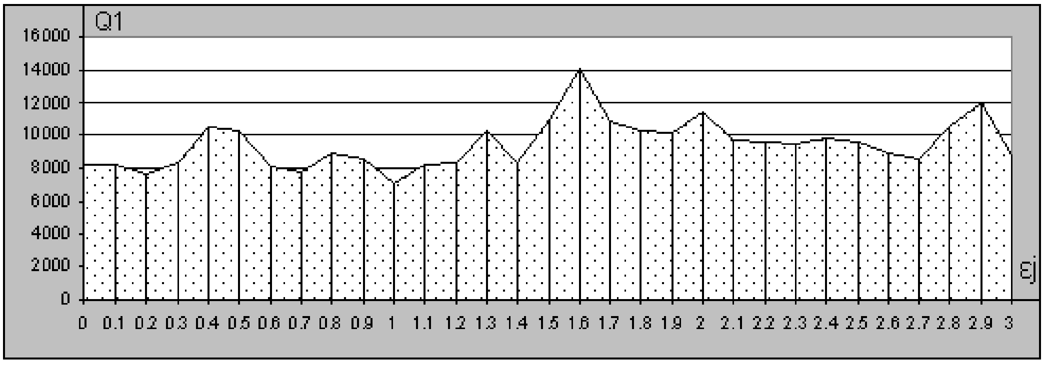


Figure 8.4: Total Amount of 1st item / Random Variate

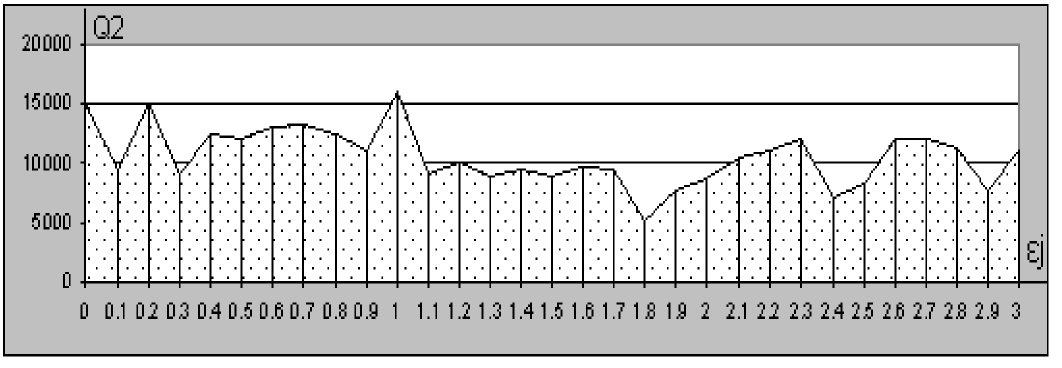


Figure 8.5: Total Amount of 2nd item / Random Variate

the relation (8.1), so during the evaluation of fuzzy rules for demand, values of the commission i.e. m_{ij} within 1 to 5 generates the values of sr_{ij} within 47.5 to 50 units with membership value 1 corresponding to high selling price and so the demand for the value of m_{ij} within 1 to 5 remains constant which again leads to constant total procured quantity and thus the wholesaler receives the constant commission from suppliers for the same quantity. The corresponding part of this explanation on total profit is given in Fig-8.8. Here, though purchasing price remains constant, the selling price gradually goes down by the relation (8.1) and hence the profit reduces linearly for m_{ij} within 1 to 5, when the value of m_{ij} gradually increases beyond 5, the selling price sr_{ij} decreases gradually and hence customers' demand goes up. For higher demand, quantity of procured quantities are more and hence the wholesaler enjoys the higher commission on MRP and as a result, the total profit also increases. These phenomena are reflected in Fig-8.7 and -8.8.

From a real-life practical case study, data are collected from different surveys and corresponding triangular fuzzy numbers are formed. With these numbers and fuzzy rules, the optimum profit is calculated with AUD discount. In this case, demands at the showrooms are low-medium, for both cases and at the 2nd showroom these demands are medium-high and low-medium for 1st and 2nd items respectively.

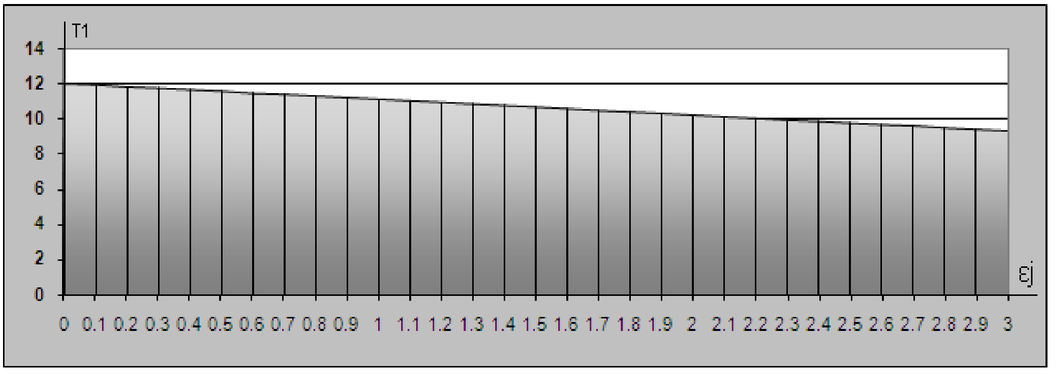


Figure 8.6: Total Time for R1 / Random Variate

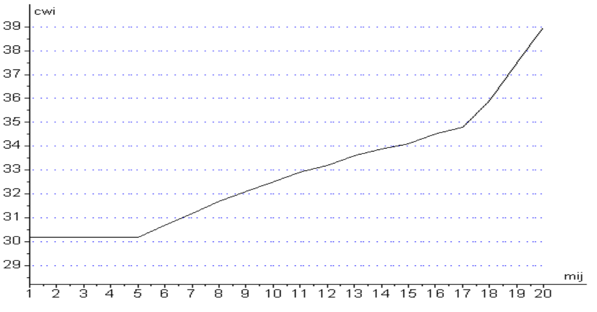


Figure 8.7: Commission achieved by W / Commission given to the end customers

8.9 Conclusion

In this research, a realistic supply-chain/inventory model is depicted for a wholesaler with showrooms at different places selling multi-seasonal products and allowing different systems of discount on MRP, in which selling price and market demand are connected by fuzzy logic. Here, the time period of business is random having a distribution with mean and standard deviation. For the first time, randomness of the time horizon has been introduced in the from of a chance constraint and implemented in a supply chain/inventory problem. This analysis will help the practitioners of seasonal products such as fashionable goods, warm cloths, medicines, etc. A methodology is presented to formulate the fuzzy data from some practical real-life survey data and this to solve the model following fuzzy rules and discounts The formulation and analysis presented here are quite general and can be extended to include different fuzzy logic relations connecting different parameters such as demand, price and exhibited quantity, etc. fully or partially backlogged shortages, fuzzy time period, etc.

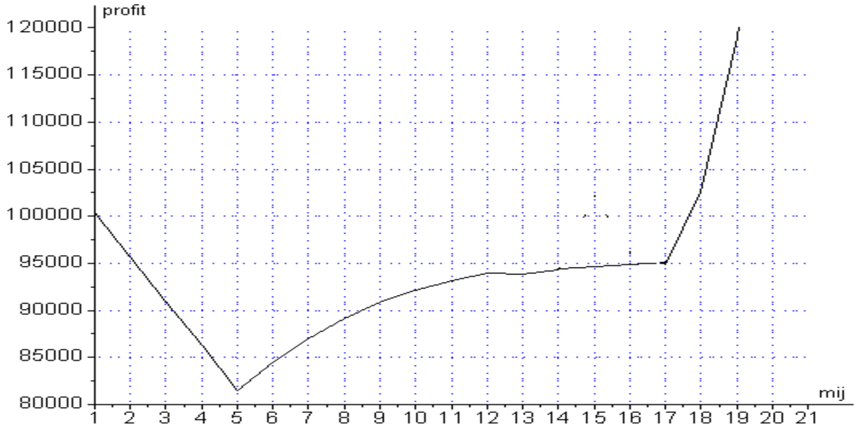


Figure 8.8: Profit / Commission given to the end customers

Chapter 9

An EPQ Model for Deteriorating items under Random Planning Horizon with some Linguistic Relations between Demand, Selling Price and Trade Credit, Ordered Quantity[1]

9.1 Introduction

The concept of EPQ model with production center and sale counter together is to determine the optimum produced quantity against the customers' demand so that total cost involved in the system is minimum. In this process, normally the production firm pays the supplier for the raw materials as and when these are purchased. Now-a-days, with the advent of multi-national in the markets of developing countries like India, Nepal, China, etc, competition between the traders / suppliers is very stiff and they take up different promotional ventures / tools to push the sale. In real practice, a supplier provides forward financing to the retailers i.e. offers credit period for payment to attract more customers. In this systems, relaxed period for payment is given to the firm management if the outstanding dues are paid within a given credit period. Here credit period is treated as a promotional tool as it is one kind of price discount because paying later circuitously reduces the purchasing cost and motivates the firms to increase the ordered quantity and to go for more production.

Goyal [92] firstly explored an EPQ model under the conditions of permissible delay in payment. Chung *et al.* [62] derived the optimal pricing and ordering policy for an integrated inventory model when trade credit is linked to order quantity. Then Chen and Ouyang [52] considered permissible delay in payment in a fuzzy inventory model for deteriorating items. Chen *et al.* [54] developed an EPQ model for deteriorating items with up-stream full credit and down-stream partial credit. These models are developed under the assumption that suppliers offers a credit period to the wholesaler. This policy is named as single level credit system where in two level credit system, retailer gets a

[1]This model has been published as a research paper in the journal of Mathematics and Informatics

part of credit achieved by the wholesaler. Ho [109] presented integrated inventory model with price and credit linked demand under two level trade credit system. In the last two decades, the inventory models with trade credit have been widely studied by several researchers. Recently Das *et al.* [72] developed an integrated production inventory model under interactive fuzzy trade credit policy.

Deterioration of units is one of the most crucial factor in inventory problems for deteriorating items. Over the years, there are some investigations on inventory control / supply chain of deteriorating items with permissible delay in payment. Aggarwal and Jaggi [2] and Chu *et al.* [59] presented the ordering policies for deteriorating item with trade credit. Jamal *et al.* [120] allowed shortages in the model of Aggarwal and Jaggi. Chang and Dye [42] allowed the partial backlogged shortages with time dependent variation in deterioration rate in Jamal *et al.* model. A finite time horizon with deterioration and monetary time-value was developed by Chang *et al.* [43]. Ouyang *et al.* [194] developed two inventory models for deteriorating items with permissible delay in payment. Some notable research papers of deteriorating items incorporating various types of assumptions are due to Bhunia *et al.* [23], Sana [219], etc. Most of the above inventory models are developed with constant deterioration. Recently Sarkar *et al.* [228] developed an integrated inventory model with variable lead time, defective items and delay in payment.

In the existing literature, most of the inventory models are generally developed with the assumption of infinite planning horizon.Jaggi and Khanna [117] highlights on a Supply chain model for deteriorating items with stock-dependent consumption rate and shortages under inflation and permissible delay in payment. Gurnani [99] pointed out that an infinite planning horizon is of rare occurrence because with the passage of time, the inventory cost is likely to vary disproportionately, product specifications may be changed, etc. Here it is assumed that for a long period in future, all the assumptions of the model such as nature of demand, types of production, inventory cost, etc. will remain valid. But there are many real-life situations where this assumption is not valid, i.e. the time periods of seasonal / fashionable products are normally finite and these are of single period only. Moreover, the demands of customers change with time, production process improves with the improvement of technology over time, etc.

In decision making problems like inventory control systems being connected with the available data / possible values of the system parameters can not be always specified exactly i.e. deterministically. There are several reasons for that like lack of input information, multiple sources of data, fluctuating nature of parameter values, noise in data, bad statistical analysis, etc. For example, it is well known that demand of a commodity depends on its price. Now-a-days, in the volatile market, the price changes very often and thus it is almost impossible to give an exact mathematical relation between price and demand. Similar is the case with order quantity and credit period though it is a fact that offered credit period varies with order amount. But, in the society, these imprecise information / relations have been fairly communicated through human words such as high, low, large, medium, small etc. Commonly these relations are expressed as IF premise (antecedent) THEN conclusion (Consequent). These type of fuzzy relations are handled by fuzzy inference technique. The commonly used fuzzy inference techniques are- Mamdani type [173] and Takagi-Sugeno type. These two methods are differ in the way by which the output is calculated. There are several research papers using fuzzy logic in different areas of investigation. Ban *et al.* [11] discussed the stability of a simplest Takagi-Sugeno fuzzy control system. Recently Chakraborty *et al.* [38] used Mamdani fuzzy inference technique to solved an inventory model of deteriorating seasonal products with different price discounts.

Among the optimization techniques for the models with fuzzy logic, the evolutionary techniques are more useful. In the literature, There are several evolutionary methods such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), etc. Maiti and Maiti [168] used simulated annealing (SA) and contractive mapping GA to solve a production inventory model. Gupta *et al.* [98] used rank based selection process in a real coded GA for solving an inventory model with interval valued inventory cost. Recently Chakraborty *et al.* [38] developed a real coded GA to solve an inventory model of deteriorating multi-items with price discount and variable demand defined by fuzzy inference under resource constraints.

To derive the relations between fuzzy parameters (such as demand and selling price) and to get the membership function of their different fuzzy values it requires one's sufficient experience about the market. Normally, these values are expressed verbally by human languages and due to the complexity of human language, it is difficult to derive the image idea from the above market data. Again ideas about a fact vary from man to man. Chang [46] presented a methodology of construction membership function for group opinion aggregation based on a gradation process.

In spite of the above developments, there are some inventory control problems yet to be investigated such as till now, none has developed inventory models with trade credit defined with the help of fuzzy inference which is more realistic.

The present model defers from the others for incorporating the following new ideas.

1. Normally, in the inventory models of trade credit, amount of trade credit is given deterministically through a numerical value. A relation is presented by a mathematical expression in crisp way. In practice, often this relation is expressed by "words" linguistically. Here, for the first time, linguistic relations between (price, demand) and (ordered quantity, credit period) are considered.

2. A new method of payment of dues of retailer to supplier is presented and a lemma is presented which assures the validity of the new method. A comparative study has been done with the conventional method.

3. The business period of the seasonal products are finite and varies every year. Thus the time period of these products are assumed as random having a probability distribution.

4. The construction of membership function (MF) from the market / business data is very important for the model with fuzzy inferences. Here, a methodology is presented for the construction of MF from the marketing experts opinions.

5. GA is very appropriate for the models with fuzzy logic. Here a GA has been developed for this purpose.

In this chapter an EPQ model is considered in which the business starts with shortage. The manufacturer purchases raw material from the raw material supplier with delay in payment. The trade credit offered by the raw material supplier depends on the amount of raw material purchased through some fuzzy rules. After the end of offered credit period, the raw material supplier charges a high rate of interest on the unpaid amount. So, at the end of credit period, the cash in hand is paid to the supplier. Two methods are considered for the payment of the dues. Conventionally, dues are cleared at the end of time period. The proposed new one is clear the dues when the gross earn becomes equal to the rest unpaid amount with interest. The total raw material, required in a cycle and the produced quantities both are stored upto a certain time and their deterioration are taken into account. It is also assumed that it requires $\delta\,(>\ 1)$ amount of raw material to produce a single finished unit. An environment protection cost is added with the production cost in order to reduce the carbon emission during production. Also some accessory costs due to production like laborer cost, wear & tear cost are considered. The per unit item selling price is fixed by imposing a mark-up on per unit item raw material cost and per unit time demand depends on the selling price. Also the time horizon is taken as random which follows the random distribution with known mean and standard deviation. Randomness of the cycle is removed using chance constraint technique. The whole problem is formulated to maximize the profit of the manufacturer and a real coded GA developed for this purpose. The model is illustrated numerically. For practical implication, raw data for the model parameters are collected from a manufacturing firm in India and these are represented as fuzzy numbers by constructing their membership functions. With these data, optimum inventory policy is derived for maximum profit with fuzzy selling price, demand, trade credit and ordering quantity. The difference between the conventional and new methods of clearing dues in graphically presented. Some useful relations between model parameters are also graphically depicted. The rest of the chapter arranged in the following manner.

The notations and assumptions for this model is given in section -9.2. The formulation of the model and the effect of trade credit on it are represented in the sections -9.3, -9.4 and -9.5 respectively. The fuzzy relations used in the model are given in the section -9.6. Section -9.7 contain some discussion about GA process and the optimum results with sensitivity analysis are made in section -9.8. Some discussion about the model and the conclusion are made in sections -9.9, -9.10 respectively.

9.2 Notations and Assumptions

In the proposed model, the following notations and assumptions are used.

9.2.1 Notations

T	length of each cycle.
$t_1\,/\,t_2\,/\,t_3$	time from beginning of the cycle when production starts / the shortage is fully back-logged / production end.
M	length of credit period in each cycle.
$\hat{H}$	random time horizon which follows normal distribution with mean $m_{\hat{H}}$ and the standard deviation $\sigma_{\hat{H}}$.
K	per unit time rate of production.
D	per unit time demand.
$Q_r\ (or,\ Q_p)$	total purchased raw material (or, quantity produced) in a cycle.
$Q_1\ (or,\ Q_2)$	total amount of shortage (or, total amount of stock) in each cycle.
$q(t)$	inventory position at any time t.
(r_0) *or* RC	(per unit item) or total raw material cost in a cycle.

r_1	laborer cost.
r_2	wear & tear cost and α is a given real numbers with $0 \le \alpha \le 1$.
r_3	environment protection cost.
$p_3(\,or,\,SC)$	per unit item (or, total) shortage cost.
$hc_1(\,or,\,HC)$	per unit item per unit time (or, total) holding cost.
S	per unit item selling price.
m_s	mark-up imposed upon the per unit item raw material cost to fix up selling price (S).
$N\ (\ge 1)$	number of cycles.
$i_e(or,\,i_p)$	percentage of interest earn (or, interest payed).
$\theta_r\,(or,\,\theta_p)$	rate of deterioration of raw material (or, produced quantity).
x	time from the starting of a cycle when the total due raw material cost(DRC) is paid to the raw material supplier.
$RE_m/RE_x/RE_T$	amount of revenue earned upto time $t \le M\,/\,t \le x\,/\,t \le T$ in a cycle.
$IE_m/IE_x/IE_T$	amount of revenue earned upto time $t \le M\,/\,t \le x\,/\,t \le T$ in a cycle.
$IP_x\,(or,\,IP_T)$	interest have to pay on DRC at the time $t = x\,(or,\,t = T)$.
$\delta\,(>\,1)$	rate of usefulness of raw material to produce finished goods.

9.2.2 Assumptions

i) In the proposed model it is considered that Demand depends on the selling price (i.e. mark-up) and the length of credit period depends on total amount of raw material purchased following some fuzzy rules.

ii) It is also considered in the model that the manufacturer does not gives any penalty for shortage and the shortage amount is fully back-logged.

iii) It is assumed that $x > M$.

9.3 Formulation of the Model

In this model the business horizon $\hat{H}$ is considered as random which follows the normal distribution with parameters $(m_{\hat{H}},\,\sigma_{\hat{H}})$ and in deterministic form the whole planning horizon is divided into N cycles each of length T. Therefore, the chance constraint is given by

$$Prob(|NT - \hat{H}| \le \beta) \quad \ge \quad pr \tag{9.1}$$

And according to the chance constraint method (section-3) it can be reduced as

$$\begin{aligned} m_{\hat{H}} - \beta - \epsilon\sigma_{\hat{H}} \le \quad NT \quad &\le m_{\hat{H}} + \beta - \epsilon\sigma_{\hat{H}} \qquad (9.2)\\ \text{where, } pr = F(\epsilon) \quad = \quad &\frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{\epsilon} e^{-\frac{t^2}{2}}\,dt \end{aligned}$$

In this business plain every cycle starts with shortage and ends with ending of stock. The manufacturer starts production at time $t = t_1$ from the starting of each cycle and at first shortage is

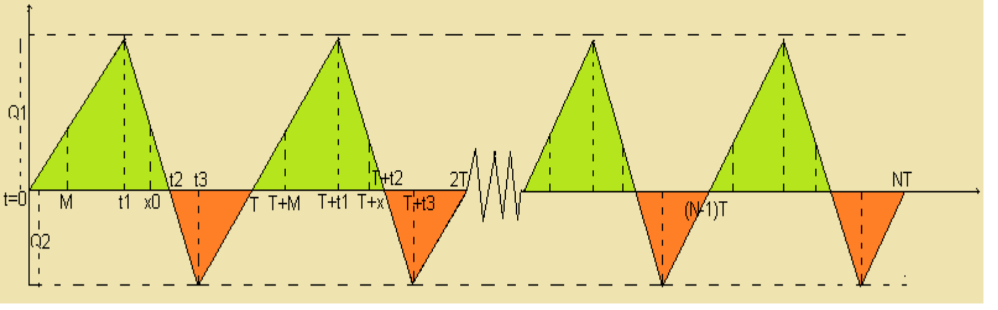


Figure 9.1: Business planning

fully back-logged (in time $t = t_2$) so the corresponding differential equation for each cycle is

$$\frac{dq(t)}{dt} = \begin{cases} K - D - \theta_p q(t) & for,\ t_2 \le t \le t_3 \\ -D - \theta_p q(t) & for,\ t_3 \le t \le T \end{cases} \tag{9.3}$$

$$\text{where, } q(t) = \begin{cases} 0 & for,\ t = t_2 \\ Q_2 & for,\ t = t_3 \\ 0 & for,\ t = T \end{cases}$$

Also the total raw material (Q_r) decreases due to deterioration (at a rate θ_r) and for production (at a rate $\delta.K$), so the corresponding differential equation is

$$\frac{dq(t)}{dt} = -\theta_r q(t) - \delta.K,\ t_2 \le t \le t_3 \tag{9.4}$$

$$\text{where, } q(t) = \begin{cases} Q_r & for,\ t = t_1 \\ 0 & for,\ t = t_3 \end{cases}$$

Now every cycle contains the following time intervals

9.3.1 Shortage Period $[0 \le t \le t_2]$

As from the starting of each cycle the shortage continuously increases up-to $t = t_1$ at demand rate and the production starts at $t = t_1$, so the total amount of shortage,

$$Q_1 = t_1.D$$

The shortage is fully back-logged with in the time $t = t_2$ in a rate $(K - D)$ so,

$$Q_1 = (t_2 - t_1)(K - D).$$

$$\text{therefore, } t_2 = t_1 + \frac{Q1}{(K - D)}$$

9.3.2 Period from end of Shortage to end of Production $[t_2 \le t \le t_3]$

In this case the corresponding differential equation is

$$\frac{dq(t)}{dt} = K - D - \theta_p.q(t)$$

Now using the conditions of equation (9.3), the inventory position in this time interval and the total amount of stock are reduced as follows

$$
\begin{aligned}
q(t) &= \frac{(K-D)}{\theta_p}[1-e^{\theta_p(t-t_2)}] \qquad (9.5)\\
\text{and} \quad Q_2 &= \frac{(K-D)}{\theta_p}[1-e^{\theta_p(t_3-t_2)}] \text{ where, } t_2 \le t \le t_3\\
\text{therefore,} \quad t_3 &= t_2 + \frac{1}{\theta_p}\log\left(\frac{K-D}{K-D-\theta_p.Q_2}\right)
\end{aligned}
$$

Solving the differential equation (9.4) and using the corresponding boundary conditions, the required amount of required raw material in a cycle is reduced as

$$Q_r = \frac{\delta.K}{\theta_r}[e^{\theta_r(t_3-t_1)}-1]$$

Also the total produced amount is

$$Q_p = \frac{K}{\theta_r}\log\left(1+\frac{Q_r\theta_r}{\delta.K}\right)$$

9.3.3 Period from end of Production to end of Cycle[$t_3 \le t \le T$]

In this case the differential equation is

$$\frac{dq(t)}{dt} = -D-\theta_p q(t)$$

and from this differential equation the following expressions for the inventory position for this time interval and the length of the cycle are reduced using the conditions of equation (9.3)

$$
\begin{aligned}
q(t) &= Q_2.e^{\theta_p(t-t_3)} - \frac{D}{\theta_p}[1-e^{\theta_p(t-t_3)}] \qquad (9.6)\\
T &= t_3+\frac{1}{\theta_p}log\left(1+\frac{\theta_p.Q_2}{D}\right)
\end{aligned}
$$

The costs arises in the model are given by the following-

9.3.4 Holding Cost

The manufacturer holds quantities from $t = t_2$ to $t = T$ in every cycle. So using the expressions given by (9.5) and (9.6) the equation for total holding cost in cycle is reduced as follows

$$
\begin{aligned}
HC &= hc_1\left[\int_{t_2}^{t_3} q(t)dt + \int_{t_3}^{T} q(t)dt\right]\\
&= \frac{hc_1}{\theta_p}[(K-D)\{(t_3-t_2)-(1-e^{-\theta_p(t_3-t_2)})\} + (Q_2+\frac{D}{\theta_p})\{1-e^{-\theta_p(T-t_3)}\}\\
&\quad -D(T-t_3)] \qquad (9.7)
\end{aligned}
$$

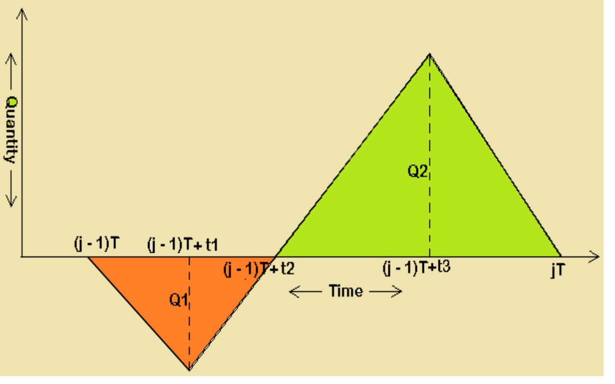


Figure 9.2: j-th cycle

9.3.5 Costs due to Production

There are many accessories cost exists rather than raw material cost (RC) . Some of them namely-laborer cost, wear and tear cost and environment protection cost are considered in the proposed model under the name other cost (OC), where per unit laborer cost is inversely proportional to the rate of production, per unit wear and tear cost proportional to the rate of production and per unit item environment protection cost proportion to the rate of production up-to a certain degree α where $0 \leq \alpha \leq 1$. Thus the total raw material in a cycle,

$$RC = Q_r.r_0 \tag{9.8}$$

where r_0 is the per unit item raw material cost. The other cost is

$$OC = (r_1.k^{-1} + r_2.k + r_3.k^{\alpha})Q_p \text{ where, } 0 \leq \alpha \leq 1 \tag{9.9}$$

9.3.6 Shortage Cost

As the total shortage amount is Q_1 and p_3 is per unit item shortage cost therefore per cycle shortage cost

$$SC = Q_1.p_3 \tag{9.10}$$

9.3.7 Set-up Cost

The set-up cost is considered in two part, 1st one is constant and the 2nd one is proportion to the total quantity produced with a degree γ . Thus per cycle set-up cost is

$$SUC = su_1 + su_2.Q^{\gamma} \text{ where, } 0 < \gamma < 1 \tag{9.11}$$

9.3.8 Selling Price

A mark-up is imposed upon the per unit item raw material cost to fix the selling price. Therefore the selling price

$$S = m_s.r_0 \tag{9.12}$$

9.4 Credit Period (Case 1: $t_1 \le M \le T$)

The raw material supplier takes interest on due payment in a rate $i_p\%$ from the producer after given some time gap of length M (credit period) from purchase and the producer earns some interest on earned revenue at a rate $i_e\%$. So, in any cycle the revenue earned by the producer in between $t = t_1$ to $t = M$ is

$$RE_m = S.K(t_2 - t_1) + S.D(M - t_2)$$

and the earned interest on RE_m

$$\begin{aligned} IE_m &= i_e.S \int_{t_1}^{t_2} K(t_2 - t)dt + i_e.S \int_{t_2}^{M} D(M - t)dt \\ &= \frac{i_e.S}{2}[K(t_2 - t_1)^2 + D(M - t_2)^2] \end{aligned}$$

Therefore after end credit period the due raw material cost(DRC) is

$$\begin{aligned} DRC &= RC - (RE_m + IE_m) \\ &= p_1.K.(t_3 - t_1) - \left[S.K.(t_2 - t_1)\left\{1 + \frac{i_e.(t_2 - t_1)}{2}\right\} + S.D.(M - t_2) \times \right. \\ &\qquad \left. \left\{1 + \frac{i_e.(t_2 - t_1)}{2}\right\}\right] \end{aligned} \tag{9.13}$$

9.4.1 A New Strategy of Due Payment

As the raw material supplier offers a strategy of payment by which the buyer can pay DRC [given by (9.13)] at instant when earn is equal to DRC. Here x is the time from the starting of a cycle when Producer pays total DRC therefore, earned revenue(RE_x), earned interest (IE_x) and interest have to pay (IP_x) in between the time range $t = M$ to $t = x$ are

$$\begin{aligned} RE_x &= S.D(x - M) \\ IE_x &= i_e.S.D\,\frac{(x - M)^2}{2} \\ IP_x &= i_p.DRC.(x - M) \end{aligned}$$

Now according to the strategy of payment

$$RE_x + IE_x = DRC + IP_x$$

From this relation the following quadratic equation is reduced.

$$A.(x - M)^2 + B.(x - M) + C = 0 \tag{9.14}$$

$$\text{where,} \quad A = \frac{i_e.S.D}{2}, \qquad B = (S.D - i_p.DRC), \qquad C = -DRC \tag{9.15}$$

before finding the roots of the above quadratic equation a lemma relating to the problem circumstances with the problem variables is proved below.

Lemma

If i_e, i_p, DRC, S and D all are positive with $i_p > i_e$ then the positive root of the quadratic equation

$$A.x^2 + B.x + C = 0 \; where, \;\; A = \frac{i_e.S.D}{2}, \;\; B = (S.D - i_p.DRC), \;\; C = -DRC$$
$$is \quad x = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

Poof: Now, $\sqrt{B^2 - 4AC} \; = \; (S.D - i_p.DRC)^2 - 4.\frac{i_e.S.D}{2}.(-DRC)$
$$= \{S.D - (i_p - i_e)(DRC)\}^2 + (DRC)^2(2i_p - i_e)i_e$$

As $i_p > i_e > 0$ therefore, $B^2 - 4AC > 0$.
Also it is given that $DRC > 0$ therefore $C < 0$. Again since i_e, S, $D > 0$ therefore $A > 0$. Hence $\sqrt{B^2 - 4AC} > |B|$ and for any value of B (positive or negative) $-B - \sqrt{B^2 - 4AC} < 0$ and $-B + \sqrt{B^2 - 4AC} > 0$ Hence the lemma.

As $x > M$, using the expressions (9.15) and the above lemma the solution of the equation (9.14) is

$$x = M + \frac{-B + \sqrt{B^2 - 4AC}}{2A} \tag{9.16}$$

After Full Payment of DRC [$x \le t \le T$]

After payment of dRC earned revenue(RE_T) and earned interest (IE_T) on it are given by

$$RE_T = S.D.(T - x) \tag{9.17}$$
$$IE_T = i_e.S.D \int_x^T (T - t)dt$$
$$= \frac{i_e.S.D.(T - x)^2}{2} \tag{9.18}$$

where, x is given by equation (9.16).

Objective Function

The whole time horizon is divided into N cycles of equal length, so the total profit(TP) is given by

$$TP = N(RE_T + IE_T - HC - OC - SC - SUC) \tag{9.19}$$

Where RE_T, IE_T, HC, OC, SC and SUC are given by the equations (9.17), (9.18), (9.7), (9.9), (9.10) and (9.11) respectively.

9.4.2 Old Strategy of Due Payment

In this case payment of due raw material cost is paid at the end of cycle. Therefore, the interest have to pay (IP_T) on DRC for the time range M to T is given by

$$IP_T = i_p DRC(T-M), \quad \text{where DRC is given by equation (9.13)} \tag{9.20}$$

Since the producer doesn't pay any amount of cash during the time gap t=M to t=T, so he gets some interest on the earned revenue. Thus the total earn in the time gap M to T (RE_T) and interest earned (IE_T) in this time gap is given by,

$$RE_T = s.D(T-M) \quad \text{and} \tag{9.21}$$

$$IE_T = i_e.s.D\frac{(T-M)^2}{2} \tag{9.22}$$

Objective Function

Thus the profit function in this case is given by

$$TP = N(RE_T + IE_T - DRC - IP_T - HC - OC - SC - SUC) \tag{9.23}$$

Where RE_T, IE_T, DRC, IP_T, HC, OC, SC and SUC are given by the equations (9.21), (9.22), (9.13), (9.20), (9.7), (9.9), (9.10) and (9.11) respectively.

9.5 Credit Period (Case-2: $T \leq M \leq T+t_1$)

In this case it is considered that the raw material supplier offers that manufacturer may place the payment at time before the placement of next order without any extra charge. As the manufacturer can earn interest on unpaid raw material cost as much as possible, so the earned revenue (RE_m) and earned interest on RE_m are given by the following.

$$RE_m = SK(t_2-t_1) + SD(T-t_2) \tag{9.24}$$

$$\begin{aligned} IE_m &= i_e S\Big\{\int_{t_1}^{t_2} K(t_2-t)dt + \int_{t_2}^{T} D(T-t)dt + K(t_2-t_1)(M-t_2) \\ &\qquad + D(T-t_2)(M-T)\Big\} \\ &= i_e S\Big[\frac{1}{2}\{K(t_2-t_1)^2 + D(T-t_2)^2\} + K(t_2-t_1)(M-t_2) \\ &\qquad + D(T-t_2)(M-T)\Big] \end{aligned} \tag{9.25}$$

9.5.1 Objective Function

Thus in this case the total profit(TP)

$$TP = N(RE_m + IE_m - RC - HC - SC - OC - SUC) \tag{9.26}$$

Where RE_m, IE_m, HC, RC, OC, SC and SUC are given by the equations (9.24), (9.25), (9.7), (9.8), (9.9), (9.10) and (9.11) respectively.

9.6 Fuzzy Rules used in the Model

The linguistic values considered for the problem variables selling price (S) and demand (D) are Low, $Medium$ and $High$. Also the The linguistic values considered for the problem variables raw material (Q_r) and credit period (M) are $Small$, $Medium$ and $Large$. All the linguistic values Low (or, $Small$), $Medium$, $High$ (or, $Large$) are handled by taking them as triangular fuzzy number of the from (l, m, u).

The model parameter demand (D) depends on the selling price (S) by the following fuzzy rules-
R-1: If (S is **Low**) Then (D is **High**).
R-2: If (S is **Medium**) Then (D is **Medium**).
R-3: If (S is **High**) Then (D is **Low**).

Also the length of credit period (M) is depend on the purchased amount of raw material (Q_r) by the following fuzzy rules-
R-4: If (Q_r is **Small**) Then (M is **Small**).
R-5: If (Q_r is **Medium**) Then (M is **Medium**).
R-6: If (Q_r is **Large**) Then (M is **Large**).

9.7 Solution Methodology: A Routine Framework for GA

The problem considered in this discussion is solved using heuristic search method Genetic Algorithm (GA). A brief discussion about GA are given in the sections-2.2.4 and -7.5. At the beginning of the GA module, the different parameters of GA i.e. generation number (MAXGEN), population size (POPSIZE), probability of cross-over (PXOVER), probability of mutation (PMUT), random seed (RSEED), distribution index for SBX (DISBX) and for mutation (DIMUT) and the others have to be supplied. As there is no clear indication as to how large a population should be, here with $POPSIZE = no\,of\,variables \times 10$, the expected result is obtained. Here a combination of real and natural number representation is used to structure a chromosome, where a chromosome is a string of genes which are specified by the decision variables of the problem namely- length of the shortage period (t_1), mark-up (m_s) to fix up the selling price, Production rate (K), maximum amount of stock (Q_2) and the no. of cycles taken by the manufacturer (N). The variable boundaries may be fixed or flexible. The fitness function is the profit function (TP) defined by the manufacturer. An overall process of GA is given by the following algorithm.

Algorithm for the proposed GA

Step 1: Population initialization:-Initializes the zero-th population.

Step 2: Run function_Model (see Appendix-1)- Calculates the values of different model parameters for 0-th population.

Step 3: Set gen=0.

Step 4: Check **if** $(gen < Maxgen)$, **then**

$\Big\{$ Run Selection operator $\rightarrow$ Run Crossover operator $\rightarrow$ Run Mutation operator.

Create new population $\rightarrow$ Run Function_Model.

if (all the constraints are satisfied) **then**,

{ print the result for the current generation.

set, gen = gen+1.

repeat step 4 }.

else repeat step 4 }.

else Stop.

9.8 Numerical Experiment: Illustration with practical Data

A rice mill, Mahabir Rice Mill Company in Midnapore, West Bengal, India produces rice from raw paddy and sale to the retailers. Here, both the raw paddy and produced rice deteriorate and normally season oriented. The data from the said mill are collected and given below. For the construction of fuzzy MF, the opinions of experts / business managers in this field are taken into account.

9.8.1 Input Data

Crisp Data: $\omega_r = 0.01$, $\omega_p = 0.005$, $\delta = 1.1$, $r_0 = 10$, $p_3 = 2$, $hc_1 = 0.15$, $r_1 = 0.5$, $r_2 = 0.005$, $r_3 = 0.5$, $su_1 = 5$, $su_2 = 2.5$, $\gamma = 0.01$, $\alpha = 0.4$, $i_e = 0.08$, $i_p = 0.12$, $\beta = 0.01$, $m_h = 48$, $\sigma = 0.16$, $\epsilon = 1.3$.
Raw and Fuzzy Data: The raw data are collected from the market (expert's opinion) considering that the demand depends on selling price and credit period depends on total purchased amount of raw material. The data regarding the parameters (selling price, demand etc.) are given by the Table-9.1 and arranged maintaining the relations. The triangular fuzzy numbers Low (or, Small), Medium, High (or, Large) which are constructed from the collected raw data are given in the Table-9.1 and the membership functions are depicted in the Figures-9.3, -9.4.

GA Parameters: POPSIZE=50 MAXGEN=200, PXOVER=0.8, PMUT=0.2, RSEED=1.2, DISBX=2, DIMUT=100, t_1-(0.0 to 4), m_s-(1.6 to 2.3), K-(120 to 240), Q_2-(0 to 500) and N-(1 to 24).

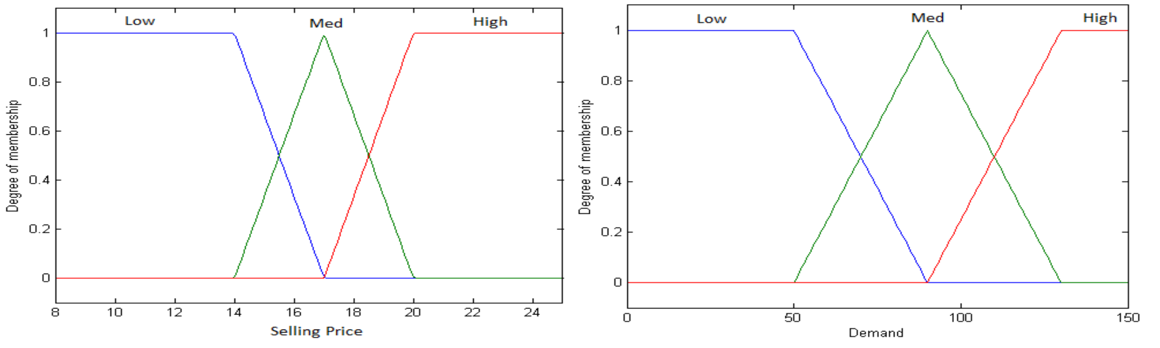


Figure 9.3: Membership functions of Selling price and Demand

Table 9.1: *Collected raw data and the corresponding fuzzy numbers*

name of linguistic fuzzy variable	raw	data number	from is	which made	fuzzy	name of fuzzy number	range	of number	fuzzy
Selling Price Demand	14.1 147	14.6 154	6.3 97	13.7 122	18.9 146	Low High	(8, (90,	14, 130,	17) 150)
Selling Price demand	22.8 79	16 104	17.3 57	17.3 104	16.2 71	Medium Medium	(14, (50,	17, 90,	20) 130)
Selling Price demand	17.7 50	18.7 81	19.9 64	19 33	28.2 50	High Low	(17, (0,	20, 50,	25) 90)
Total Quantity Credit period	627 8	571 8	1034 9.6	403 5.1	271 1.9	Small Small	(0, (0,	550, 7,	895) 10)
Total Quantity Credit Period	1187 10.4	168 10.6	882 8.7	871 9.6	1014 15.8	Medium Medium	(550, (7,	895, 10,	1421) 13)
Total Quantity Credit Period	520 22.8	1692 11.9	1350 13	1386 12	1677 10.9	Large Large	(895, (10,	1421, 13,	2166) 20)

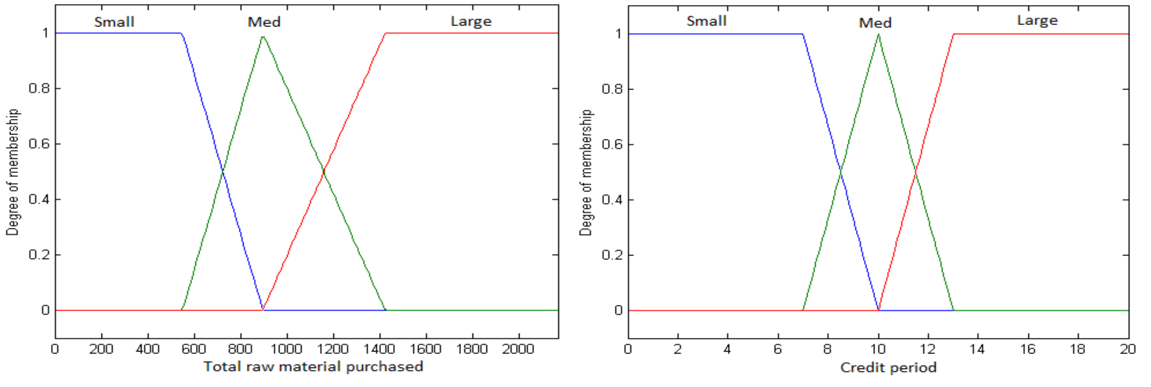


Figure 9.4: Membership functions of Total purchased raw material and Credit period

9.8.2 Optimum Result

Table 9.2: *Result Obtained using new methodology for the collected data*

problem variable	case-1	case-2	case-3	case-4	problem variable	case-1	case-2	case-3	case-4
m_s	1.9	1.857	1.884	1.752	Q_r	559	635	738	1051
t_1	3.186	1.79	2.032	5.414	Q_d	498	563	652	929
t_2	4.835	2.781	2.719	5.414	Q_p	500	568	661	935
t_3	6.458	5.222	5.092	7.78	RC ($)	5592	6353	7383	10507
T	9.56	9.557	11.95	11.95	SC ($)	332	211	222	540
M	4.379	5.126	6.345	12.789	HC ($)	58	131	263	160
x	5.396	5.48	6.579	–	OC ($)	647	770	1064	1510
N	5	5	4	4	SUC ($)	35	37	39	44
D	52	59	55	78	RDP ($)	934	376	236	-
K	153	165	216	217	RE_T ($)	4122	4464	5524	16285 (RE_M)
Q_1	166	106	111	270	IE_T ($)	687	728	1187	7863 (IE_M)
Q_2	163	258	381	328	profit ($)	18686	20221	20498	45548

Table 9.3: *Result obtained using old methodology for the collected data*

problem variable	case-1	case-2	case-3	case-4	problem variable	case-1	case-2	case-3	case-4
m_s	1.9	1.857	1.884	1.752	Q_d	498	563	652	929
t_1	3.186	1.79	2.032	5.414	Q_p	500	568	661	935
t_2	4.835	2.781	2.719	5.414	RC ($)	5592	6353	7383	10507
t_3	6.458	5.222	5.092	7.78	SC ($)	332	211	222	540
T	9.56	9.558	11.95	11.95	HC ($)	58	131	263	160
M	4.379	5.127	6.345	12.789	OC ($)	647	770	1064	1510
N	5	5	4	4	SUC ($)	35	37	39	44
D	52	59	55	78	RDP ($)	934	376	236	-
K	153	165	216	217	IP_T ($)	581	200	159	–
Q_1	166	106	111	270	RE_T ($)	5129	4851	5765	16285 (RE_M)
Q_2	163	258	381	328	IE_T ($)	1063	860	1293	7863 (IE_M)
Q_r	559	635	738	1051	profit ($)	18030	19931	20302	45548

9.9 Discussion

In the numerical experiment, some real life data are collected from a firm and presented in Table-9.1. Following the method in section-2.1.6, the membership functions for the different parameters are drawn and presented in Figs. -9.3, -9.4. From these data it can be easily verified that the relations between demand is inversely proportional to selling price and purchased amount of raw material is proportional to credit period which support the rules given in section-9.6.

Optimum results given in Table-9.2 obtained using the new methodology of payment gives more profit than the results given in Table-9.3 obtained using the old payment policy in all cases. Also larger credit period gives more profit and from the results given in the Tables -9.2 and -9.3, it can be seen that the profit increases in the cases ($t_1 \leq M \leq t_2$, $t_2 \leq M \leq t_3$, $t_3 \leq M \leq T\, and\, T \leq M \leq T + t_1$) in an ascending order of the time intervals of M. This is as per expectation. For new method of payment, it is considered that the time of payment of due cost (x) is always greater than the credit period (M). As a result increment in M reduces the gap between the time of payment for the new method (x) and the old method (T). Also the difference of profit in these results decreases as the credit period M becomes larger which is reflected in Fig.9.5(a).

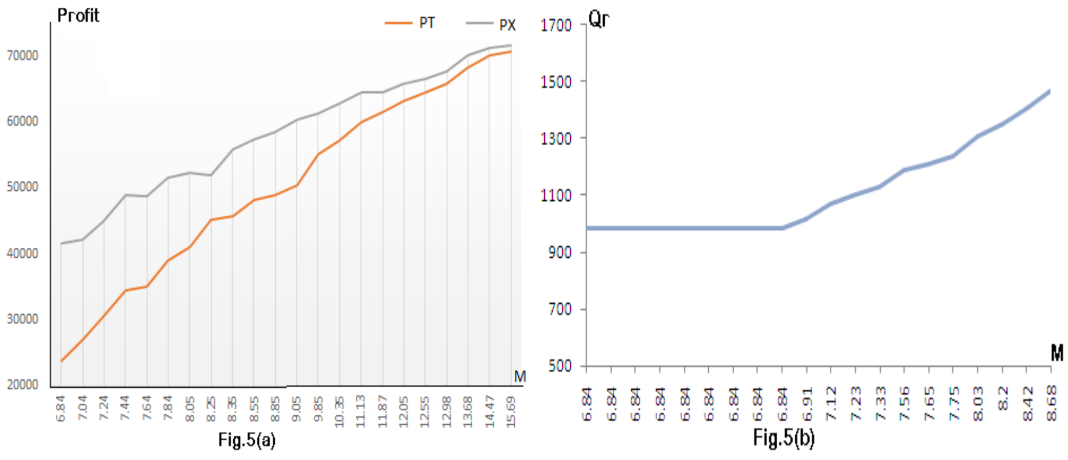


Figure 9.5: (a): Length of credit period(M) / Difference Between Profit obtained using old method(PT) and new method(PX), (b): Length of credit period / Total raw material amount.

From Tables-9.2 and -9.3 it can be seen that the purchased raw material amount (Q_r) increases with credit period (M). This is the effect of fuzzy relations ($R_4 - R_6$) which is also reflected in Fig.9.5(b). In this figure the curve of Q_r remains unchanged for each value of the M less than 7 as each value of credit period less than 7 takes a membership value 1 (c.f. Fig. 9.4), to the fuzzy number "Small" and therefore the rule strength of the rule R_4 (c.f. section-9.7) becomes 1. For this the purchased amount of raw material (Q_r) gets a constant value. Then for the next values of credit period (> 7) the amount of raw material increases as per expectation.

Figs.9.6(a) and 9.6(b) depict the variation in profit and demand with respect to the change in mark-up because the selling price is fixed by imposing a mark-up to a fixed number [given by (9.12)], so the change in selling price will make a same impression as the change in mark-up. Here, with the values of selling price $S(i.e. m_s)$, the demand (D) changes inversely as per the relations ($R_1 - R_3$). This is also depicted in Fig.9.6(a). In this figure, demand decreases as selling price (i.e. mark-up) increases and when selling price takes the value 21 (i.e. $m_s = 2.1$), the demand becomes constant as value of the fuzzy membership function for mark-up becomes 1 to the fuzzy number "High" (according to Fig.9.3 mark-up takes a constant membership value 1 in the range 2 - 2.5). This is also reflected in the Fig.9.6(b).

In the optimum results, profit decreases as mark-up increases. Normally, profit linearly related to mark-up and demand. It increases with the increase of selling price (i.e. mark-up) and / or demand. Here, with the fuzzy rules ($R_1 - R_3$), demand decreases with selling price (i.e. mark-up). Thus selling price increases the profit and at the same time, decreases the demand which, in turn decreases the profit. On the profit, there is mixed effect due to selling price (i.e. mark-up) and demand. From Fig.9.6(a), it is seen that the effect of demand on profit dominates over the effect of mark-up (selling price) and for this reason, as mark-up (selling price) increases, profit decreases along with demand and for the value of m_s=2.1 (S=21), as demand becomes constant, there is only effect of mark-up (selling price) on profit and as a result, profit increases with mark-up.

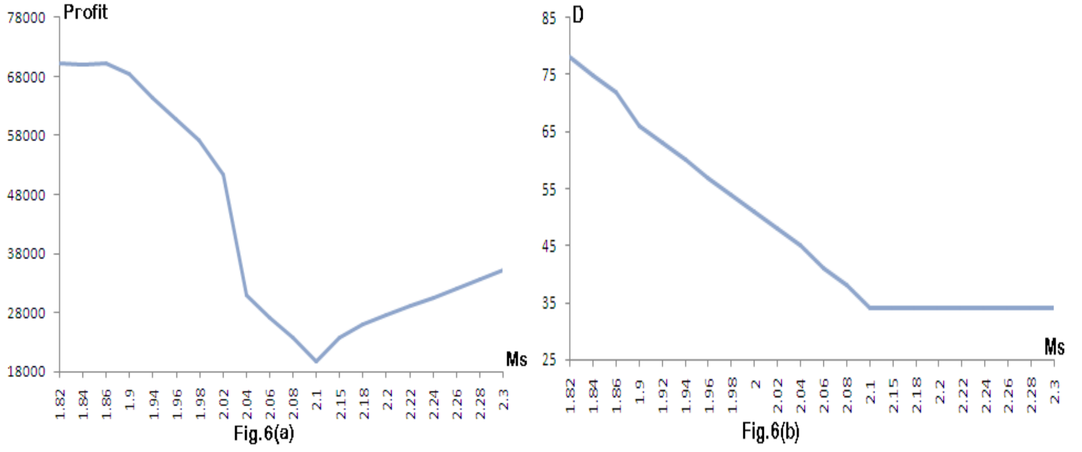


Figure 9.6: (a): Mark-up(m_s) / Profit, (b): Mark-up / Demand.

9.10 Conclusion

In this investigation, a practical problem for the inventory control system with trade credit is considered with some fuzzy relations between the decision variables and solved. For the first time. the membership functions for the parameters of the fuzzy relations, are formulated from some collected practical data and using fuzzy inference at two stage, optimum profits are determined and presented in Tabular and graphical form. A new method for repayment of dues is presented and compared with the conventional method. Here, fuzzy relations with single input and output have been used. Other forms of fuzzy relations can also be used. The model can be extended to include the promotional cost, profit sharing etc. among the supply chain partners.

Part V

Summary and Future Extension

Chapter 10

Summary and Future Extension

10.1 Summary of the Thesis

In this thesis, total seven virgin inventory /production-inventory models, of which one is in crisp, three in random / fuzzy-random and three in fuzzy environments are formulated and solved.

- The models are formulated with linguistic relations, in-control and out-control states, advanced payment, trade credit policy, inflation of money and many more criteria which are visible in recent management system.
- The models are developed for different types of demands like stock dependent demand, time dependent demand, price dependent demand, credit period dependent dynamic demand and fuzzy demand.
- The models are transformed to deterministic ones by using method of Fuzzy Inference, Possibility, Necessity, Credibility measures, method of chance constraint, etc.
- For the solution of single and multi-objective models with/without constraints, different optimization techniques such as Genetic Algorithm (GA), Multi-objective GA (MOGA), Generalized Reduced Gradient method (GRG) etc. are developed / modified and used. The appropriate solution methods are developed for different models.
- The models are illustrated with appropriate numerical examples and the optimum results are presented numerically and graphically. Moreover, the obtained results are discussed as managerial insights.
- In practice, all available past data / experts' opinions are deterministic. An appropriate method is presented in the thesis to formulate fuzzy membership functions from available crisp data / experts' opinions.
- Several inventory / production-inventory models for seasonal products with random business periods, imprecise resource constraints, conditional trade credit, etc. have been developed and solved.

10.2 Future Extension

- Each model presented in the thesis can be extended to include realistic features of inventory / production-inventory systems. The presented models can also be formulated in other types of uncertain environments, such as: rough, fuzzy-rough, rough-fuzzy, bi-random, bi-fuzzy, etc.
- In the inventory models only some specific types of resource constraints are used as random, fuzzy etc. The limited resource can also be taken as intuitionstic fuzzy, type-2 fuzzy, etc.
- There are various optimization techniques such as Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Geometric Programming (GP), etc which are not applied to the models in this dissertation. So, these methods can be developed for the present models, if possible and applied for optimal solutions.
- For the models with fuzzy inferences, only Mamdani's method has been used. Other inference methods (such as, Sugeno type) can also be used to handle fuzzy relations by changing the models appropriately.
- Here, one method for formation of fuzzy membership function from raw crisp data has been presented. Other appropriate methods can also be developed for this purpose.
- In the thesis, two supply-chain models are developed with two level trade credit period. For the supplier-wholesaler-retailer supply chain, three level trad credit period can be conceived and applied.

Therefore, there is a huge scope to extend the research works presented in this thesis.

Part VI

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A deteriorating multi-item inventory model with price discount and variable demands via fuzzy logic under resource constraints ☆

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ABSTRACT

An inventory model of deteriorating seasonal products with Maximum Retail Price (MRP) for a wholesaler having showrooms at different places under a single management system is considered under random business periods with fuzzy resource constraints. The wholesaler replenishes the products instantaneously and earns commissions on MRP which vary with the ordered quantities following All Unit Discount (AUD), Incremental Quantity Discount (IQD) or IQD in AUD policy. Demand at showrooms are imprecise and related to selling prices by 'verbal words' following fuzzy logic. The wholesaler shares a part of commission with customers. The business periods follows normal distribution and converted to deterministic ones through chance constraint technique. The fuzzy space and budget constraints and fuzzy relations are defuzzified using possibility measures, surprise function and Mumdani fuzzy inference technique. The model is formulated as profit maximization for the wholesaler and solved using a real coded Genetic Algorithm (GA) and illustrated through some numerical examples and some sensitivity analysis. A real-life problem of a developing country is presented, solved using the above mentioned procedures and an appropriate inventory policy is suggested.

1. Introduction

In the existing literature of inventory, most of the models are developed under infinite time horizon. As per Gurnani (1985), the life of a particular item is not infinite due to the change of design, technological development, variation of inventory costs, customers' changing taste, etc. and this is very much true for the seasonal products in developing countries where preserving facilities are not available in plenty. For these seasonal products, even though the planning horizon is assumed as finite, in every season it fluctuates depending on some extraneous factors such as climatic conditions. This time period may be assumed to be random with a probability distribution. In the literature Maiti, Maiti, and Maiti (2006) and Roy, Pal, and Maiti (2009) have solved some inventory problems with random planning horizon having exponential distribution. Also Moon and Lee (2000) have presented an EOQ model under inflation and discounting with a random product life cycle.

In an inventory system, deterioration is an usual phenomenon. Mandal and Phaujdar (1989) presented an inventory model with deteriorating items. Roy, Maiti, Kar, and Maiti (2009) have done a research work of deteriorating items with stock dependent demand over random planning horizon. Also Bhunia and Maiti (1997) and Mahapatra and Maiti (2006) presented some inventory models for deteriorating items with time dependent demand and imprecise production time respectively.

In the present competitive market, the demand depends on the stock directly and also inversely on the selling price. Recently Widyadana, Cardenas-Barron, and Wee (2011) presented a deteriorating inventory problem with constant demand via a simplified approach. Also Giri, Pal, Goswami, and Chaudhuri (1996), Mandal and Maiti (2000) and others considered the demand as an indexed stock (i.e. $D = dq^{\beta}$, d and β are constants) dependent. But there are few research works with fuzzy demand depending on stock and selling price following fuzzy inference. Recently, some inventory models with rework for the defective products (Jamal, Sarker, & Mondal, 2004; Cardenas-Barron, 2007, 2008, 2009a, 2009b; Sarker, Jamal, & Mondal, 2008; Cardenas-Barron, Trevino-Garza, & Wee, 2012) have been presented in the literature.

Human knowledge is often represented imprecisely, vaguely and approximately. In our real life, some vague terms in the form of 'words' such as high, medium, and low, are used. The target of fuzzy inference process is to form it into natural language expressions of the type,

IF premise (antecedent) THEN conclusion (consequent).

There are two types of fuzzy inference systems: Mamdani-type (Mamdani & Assilina, 1975) and Sugeno-type (Ban, Gao, Huang, & Yin, 2007). These two types differ in the way by which output is determined. Mamdani's effort was based on Bellman and Zadeh's

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An EPQ Model for Deteriorating Items under Random Planning Horizon with Some Linguistic Relations Between Demand, Selling Price and Trade Credit, Ordered Quantity

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Abstract. In this paper, an environment friendly Economic Production Quantity (EPQ) model of a single item is considered in which the business in each cycle starts with shortage and ends with the end of stock. The whole problem is formulated to maximize profit of the manufacturer with random business period and the randomness is removed by chance constrained method. This model involves selling price dependent demand and purchased raw material dependent credit period which are described by two sets of linguistic relations under fuzzy logic. In addition, a new method of payment of due raw material cost (DRC) (DRC is paid as soon as it can possible) is prescribed with supported lemma and a comparative study has been done between the new method of payment and the old method of payment (DRC is paid at the end of cycle). The model is optimized by a real coded genetic algorithm (GA) developed for this purpose with tournament selection, arithmetic crossover and polynomial mutation. The model is illustrated with different sets of numerical examples for different scenarios. A practical application has also been demonstrated with real world data. Some sensitivity analysis are presented graphically.

Keywords: Fuzzy logic; genetic algorithm; construction of membership function; delay in payment; chance constrained technique

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1. Introduction

The concept of EPQ model with production center and sale counter together is to determine the optimum produced quantity against the customers' demand so that total cost involved in the system is minimum. In this process, normally the production firm pays the supplier for the raw materials as and when these are purchased. Now-a-days, with the advent of multi-national in the markets of developing countries like India, Nepal, China, etc, competition between the traders / suppliers is very stiff and they take up different promotional ventures / tools to push the sale. In real practice, a supplier provides forward financing to the retailers i.e. offers credit period for payment to attract more customers. In this systems, relaxed period for payment is given to the firm management if the outstanding