Synopsis for the research work towards Ph.D.

entitled

Modelling of Some Problems on Production Planning and Inventory Management

by

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Contents

1 Introduction

1.1 Abstract

The dissertation contains several recent studies on production policies and inventory controls, which is entitled as "MODELLING OF SOME PROBLEMS ON PRODUCTION PLANNING AND INVENTORY MANAGEMENT". The major issues of recent research is the products nature live when it can be deteriorated, when it can be used fully, when deterioration rate is random or when the products have fixed lifetime. These problems are solved by this research studies. Along with the product's nature, the optimal buffer inventory, replenishment rate, imperfect production and inspection errors during the shipment of production system from in-control-state to out-of-control state are efficiently studied by this research studies. The dissertation consists of six different studies on products produced by production system. The first model describes about two-warehouse inventory model with increasing demand where the deterioration rate is assumed as time-varying. The second model extends the idea of first model with probabilistic deterioration with inflation. The third model develops the optimal replenishment rate with variable deterioration where the products have fixed-lifetime. The fourth model extends the previous idea of deterioration with flexible setup cost. The fifth model develops the preventive maintenance and buffer inventory policy in a deteriorating production system. The sixth model considers an imperfects production system with inspection error when the system moves from *in-control* state to *out-of-control* state.

1.2 Literature review

The main aim of any company is to meet demand on time and for this purpose the company has to keep inventory in their own warehouse (OW). As we all know, the capacity of any warehouse is limited. In many practical situations there exist various factors (like price discounts for bulk purchases or if the items are seasonal product) under consideration that induce the decision maker of the inventory system to order more goods than can be stored in his warehouse. In this case, company will either rent (RW) other warehouse or rebuild a new warehouse. They usually choose RW because it is more economical for any manufacturing system. The company generally wants more profit with the help of production of different types of items which may keep in OW or RW. They use inventory management to get more profit. The basic two-warehouse inventory model was first introduced by Hartley (1976). In this direction, Sarma (1983) developed the inventory model with two levels of storage and optimum release rule. Murdeshwas and Sathe (1985) extended the model of Sarma (1983) with the production of non-perishable items. By considering constant demand, Sarma (1987) developed an inventory model with deterioration, shortage and two levels of storage. An EOQ models with two levels of storage was discussed by Dave (1988). By considering different stage production system, many researchers considered inventory models for deteriorating items. The term, deterioration is defined as damage, spoilage, dryness of an item. Research, in the field of deterioration began with the work of Whitin (1957) where he considered fashion goods deterioration at the end of a prescribed storage period. Deteriorating items with linear trend in demand was formulated by Chakraborty and Choudhuri (1997). Many researchers like Giri and Chaudhuri (1997), Hariga (1996), Khanra and Choudhuri (2003), Ghosh and Chaudhuri (2006) and others discussed about deterioration and different types of demand. Cárdenas -Barrón (2007) discussed a technical note on optimal manufacturing batch size with rework process at single-stage production system. Cárdenas -Barrón (2008) developed a simple derivation on optimal manufacturing batch size with rework in a single-stage production system. Furthermore, Cárdenas-Barrón (2009a) investigated the economic production quantity with rework process at a single stage manufacturing system with planned backorders. Sarkar (2012b) investigated an EOQ (Economic Order Quantity) model with delay-in-payments and time-varying deterioration rate. Pakkala and Acharya (1992) formulated a two-warehouse inventory model for deteriorating items with finite replenishment rate and shortage. Goswami and Chaudhuri (1992) found out a two-warehouse inventory model with linear trend in demand and shortage. Benkherouf (1997) proposed two-warehouse model with deterioration and continuously release pattern. C*á*rdenas -Barr*ó*n(2009b) analyzed a model on optimal batch sizing in a multi-stage production system with rework consideration.

Classical inventory model considers the demand rate as either constant, time- dependent or stock dependent demand instead of constant demand. However it is observed that the demand rate of electronic goods (e.g., hard disk, RAM, processer, mobile, etc.), new brand of consumer goods comes to the market, seasonal products (fruits, e.g., mango, orange, etc.,) increases linearly at the beginning up to a certain moment as time increases and then stabilizes to a constant rate until the end of the inventory cycle. To represent such type of demand pattern the term ramp-type is used. Wu *et al.* (2001) discussed an inventory model for deteriorating items with a ramp-type demand under stock-dependent consumption rate. A similar type model with partial backlogging was considered by Skouri *et al.* (2009). Several researchers have examined the inflationary effect on the inventory policy. Jaggi *et al.* (2006) considered a deteriorating inventory model under inflation induced demand over a finite planning horizon. Sarkar *et al.* (2012b, 2012c) developed two inventory models for imperfect production with inflation and time value of money. Wee (1997) formulated an optimal replenishment policy for deteriorating items with a linear price-function of demand.

A Supply chain management (SCM) involves the movement and storage of raw materials and finished goods from point of origin to point of consumption. SCM obtains its importance in global market and network economy as organizations rely increasingly on effective supply chains or networks. Recently, C*á*rdenas -Barr*ó*n and Trevi*ñ*o-Garza(2014) developed an excellent model for an optimal solution to a three echelon supply chain network. Chung *et al.* (2014) discussed an inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit policy. Taleizadeh and Cárdenas -Barrón (2013) developed a met heuristic algorithm for supply chain management problems. Yan *et al.* (2011) extended the SCM model with a constant deterioration rate from Kim and Ha's (2002) model. Widyadana and Wee (2011) developed an EPQ model for deteriorating items with preventive maintenance policy and random machine breakdown. Teng *et al.* (2011) extended an EOQ model for buyer-distributor-vendor supply chain with backlogging without derivatives. Chung and Cárdenas -Barrón (2012) found out a complete solution procedure for the EOQ and EPQ inventory models with linear and fixed backorder costs. In real life situation, due to long run process, the manufacturing system shifts from *in-control* to *out-of-control* state, and then the manufacturing system produces perfect as well as imperfect (defective) quality items. This defective items may be reworked at a cost or sold at reduce price. Jamal *et al.* (2004) developed an inventory model with optimum batch quantity in a single-stage system in which rework was done by addressing two different operational policies to minimize the total system cost. A production-inventory model to determine the optimal product reliability and production rate that achieves the biggest total integrated profit for an imperfect manufacturing process was developed by Sana (2010a). Sana (2010b) presented an EPL (Economic Production Lotsize) model in an imperfect production system in which the manufacturing process may shift from an *in-control* state to an *out-of-control* state at any random time. Many researchers have investigated production models with unreliable machines. Rosenblatt and Lee (1986) initially studied the effects of process deterioration on the traditional (EMQ) model. In their model, they considered that the elapsed time to the *out-of-control* state is exponentially distributed and concluded that the presence of defective products generates smaller lot sizes than that of the classical (EPQ) inventory model. Porteus (1986) discussed an imperfect production process with significant relationship between quality and lot size and evaluated an optimal investment in process-quality improvement and setup cost reduction. Harriga and Ben-Daya (1998) extended Rosenblatt and Lee's (1986) model by assuming a more generalized assumption that an elapsed time, until process shift, is arbitrarily distributed, and provided distribution-based and distribution-free bounds on the optimal cost.

Full inspection policy results higher inspection cost and higher expected total cost. To reduce the inspection cost, Wang (2005) developed an inspection policy, where inspection was performed at the end of the production run. Using the same concept, Wang and Meng (2009) developed another model with online inspection policy of products. Hu and Zong (2009) proposed an extended product inspection policy for a deteriorating production system, where the product inspections are performed in the middle of a production cycle. There are many situations where the organization may have more than one objective functions to optimize. To solve this type of problem, Duffuaa and El-Ga'aly (2013) developed a multi-objective inventory model using 100% error-free inspection as a means of product control. Sarkar (2016) considered a supply chain coordination model with three-stage inspection to ensure perfect quality products where the products has special features as fixed lifetime.

Notation

H	total planning horizon
n	number of production cycles during the entire horizon H
d	demand rate of product (units per unit time)
p	constant production rate, where $p > d$
TC	total system cost during planning horizon
ψ	probabilistic deterioration rate
$\delta(t)$	backlogging rate
s	selling-price per unit
C_o	ordering cost per order
C_h	unit inventory holding cost per unit time
C_p	purchasing cost per unit purchase/production cost per unit product
C_b	backorder cost per unit backorder
C_l	lost sell cost per unit
C_d	deterioration cost
S_c	shortage cost
M_c	preventive maintenance cost
C_w	warranty cost
C_s	salvaged cost

C_r	rework cost
C_m	variable cost (labor cost, energy cost)
μ	parameter of the ramp-type demand function (break point)
$I(t)$	on-hand inventory level at time t
t_1	length of time in which the inventory level falls to zero
T	production run time/ length of each ordering cycle
Q	order quantity per cycle (units)/production lot size per batch-cycle (units)
$\alpha(t)$	time-dependent deterioration rate
α	constant deterioration rate
q	delivery lot size (units)
N	number of deliveries per production-batch, $N \geq 1$
R	reliability parameter
S	total shortage amount
S_o	initial setup cost for a production batch
S_1	variable setup cost for production batch
A_b	area under the buyer's inventory level
A_s	area under the supplier's inventory level
HC_s	holding cost for the supplier
HC_b	holding cost for the buyer

V_c	unit variable cost for order handling and receiving
t_p	production time duration for the supplier
t_n	non-production time duration for the supplier
t_d	duration between the two successive deliveries
X	time after which the production process shifts *in-control* to *out-of-control* state (random variable)
m_1	probability of Type I error (random variable)
m_2	probability of Type II error (random variable)
τ	preventive maintenance time (random variable)
$g(x)$	probability density function of X
$G(x)$	distribution function of X
$\overline{G}(x)$	survival function of X, i.e., $\overline{G}(x) = 1 - G(x)$
$g(m_1)$	probability density function of m_1
$g(m_2)$	probability density function of m_2
$g(\tau)$	probability density function of τ
θ_1	percentage of defective items when the production process is in the *in-control* state
θ_2	percentage of defective items when the production process is in the *out-of-control* state.
M_c	material cost

r	system restoration cost
η	system inspection cost
I_c	product inspection cost
B	buffer inventory
C_A	cost of falsely accepted defective item
C_R	cost of falsely rejected non-defective item

2 Chapter 1

A two-warehouse inventory model with increasing demand and time varying deterioration

Abstract

In this chapter, an inventory model of two-warehouse is considered with quadratically increasing demand and time varying deterioration. Comparing to the existing literature, the model is derived with finite replenishment rate and unequal length of the time cycle. The associated cost of the system is minimized. A numerical example, the graphical representation and sensitivity analysis are provided to illustrate the model.

2.1 Notation and assumptions

To derive the model, following notation, and assumptions are used.

Notation

$f(t)$ $a + bt + ct^2$ $(a, b, c > 0)$ with $0 < f(t) < p$, Here a is initial rate of demand, b is the rate with which the demand rate increases. The rate of change in the demand itself increases at a rate c

$\alpha(t)$ $\frac{1}{1+R_1-t}$= deteriorating rate of inventory items in OW, where R_1 is the maximum life time of an item in OW i.e., R_1 is always greater than or equal to t, thus, $\alpha(t) > 0$

$\beta(t)$ $\frac{1}{1+R_2-t}$=deteriorating rate of inventory items in RW, where R_2 is the maximum life time of an item in RW i.e., R_2 is always greater than or equal to t, thus, $\beta(t) > 0$

ω the storage capacity of OW

P_1 a class of production cycle when only OW is used

P_2 a class of production cycle when both OW and RW are used

t_{i0} the time at the beginning of the ith production cycle belonging to P_2

t_{i1} the time at which the inventory in OW first reaches ω units

t_{i2} the time at the end of production of the ith production cycle

t_{i3} the time at which all inventory units in RW are exhausted within the ith production cycle

I_{i1} inventory level in OW at time t, $t \in [t_{i0}, t_{i1}]$

I_{i2} inventory level in RW at time t, $t \in [t_{i1}, t_{i2}]$

I_{i3} inventory level in RW at time t, $t \in [t_{i2}, t_{i3}]$

I_{i4} inventory level in OW at time t, $t \in [t_{i3}, t_{i+1,0}]$

I_{i5} inventory level in OW at time t, $t \in [t_{i1}, t_{i3}]$

C_1 setup cost per production run

C_2 cost of deteriorated unit

D_i the quantity of deteriorated items during the ith production cycle

C_{OW} carrying cost per inventory unit per unit time in OW

C_{RW} carrying cost per inventory unit per unit time in RW

TC total system cost during H

Assumptions

(1) Demand is increasing quadratically with respect to time as $f(t) = a + bt + ct^2$, $a, b, c > 0$ and the production rate (p) is greater than the demand. Hence, there is no shortage.

(2) The OW has limited capacity of ω units and the RW has unlimited capacity.

(3) The inventory cost (including holding cost and deteriorating cost) in RW is higher than that of OW.

(4) Inventory decreases due to demand and deterioration.

(5) Deterioration rate is considered as time-dependent and the deteriorated units can not be repaired or replaced.

(6) The RW is located near the OW such that the transportation cost between them is negligible.

(7) Maximum life time (R_1) of an item in OW is greater than the maximum life time (R_2) of an item in RW i.e. after R_1 time, the items in OW are deteriorated and after R_2 time, the items in RW are deteriorated.

(8) The lead time is considered as negligible.

2.2 Model formulation

The inventory level in a production system with quadratic demand for deteriorating items is depicted in Fig.1 in which Fig.1a shows the inventory level during a production cycle when both OW and RW are used and Fig.1b shows when only OW is used. Any arbitrary production cycle i belonging to P_2 starts from t_{i0} and ends at $t_{i+1,0}$. Over the period $[t_{i0}, t_{i+1,0}]$, we can identify the points t_{i0}, t_{i1}, t_{i2}, t_{i3} and $t_{i+1,0}$. Production, demand and deterioration starts simultaneously at t_{i0}. During the period $[t_{i0}, t_{i1}]$ produced items accumulate from 0 up to ω units in OW. RW is used after time t_{i1} when production quantity exceeds ω units. The inventory level in RW begins to decrease at t_{i2} and finally reaches at 0 unit at t_{i3} due to demand and deterioration. The inventory level in OW comes to decrease at t_{i1} and falls below ω units up to time t_{i3} only for deterioration and

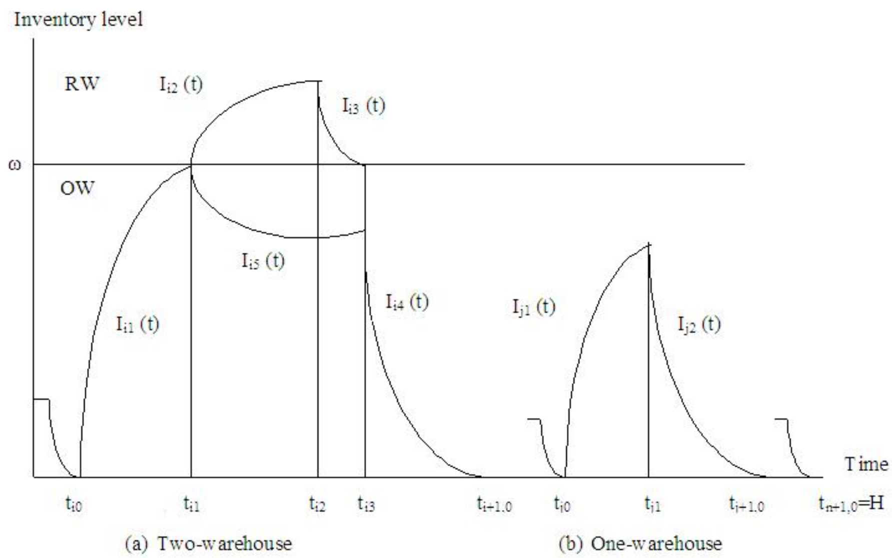

Figure 1: Inventory level versus time horizon for two warehouse and one warehouse system.

the remaining quantity in OW is fully exhausted at $t_{i+1,0}$. Any arbitrary production cycle j belonging to P_1, starts from t_{j0} and ends at $t_{j+1,0}$. Here, we can identify a point t_{j1}, the time at the end of production. During $[t_{j0}, t_{j1}]$ the inventory level in OW gradually decreases but it is always less than ω units. During $[t_{j1}, t_{j+1,0}]$, the stocks in OW gradually decreases due to demand and deterioration as well as it is exhausted at $t_{j+1,0}$.
The governing differential equations stating the inventory levels within the ith cycle are given as follows:

$$\begin{aligned}
\frac{dI_{i1}(t)}{dt} + \alpha(t)I_{i1}(t) &= p - f(t); \quad t_{i0} \leq t \leq t_{i1}, \\
\frac{dI_{i2}(t)}{dt} + \beta(t)I_{i2}(t) &= p - f(t); \quad t_{i1} \leq t \leq t_{i2}, \\
\frac{dI_{i3}(t)}{dt} + \beta(t)I_{i3}(t) &= -f(t); \quad t_{i2} \leq t \leq t_{i3}, \\
\frac{dI_{i4}(t)}{dt} + \alpha(t)I_{i4}(t) &= -f(t); \quad t_{i3} \leq t \leq t_{i+1,0},
\end{aligned}$$

$$\frac{dI_{i5}(t)}{dt} + \alpha(t)I_{i5}(t) \;=\; 0; \quad t_{i1} \le t \le t_{i3},$$

Solving the above differential equations, we obtained the different inventories levels in the different time intervals. Then, the different inventory levels in OW and RW are separately derived. Total quantity of deteriorating items D_i during the production cycle i is found out here. The total system cost within the planning horizon H, consisting of setup cost, carrying cost and deteriorating cost can be expressed as follows

$$TC = nC_1 + C_{RW}\sum_i I_{RW,i} + C_{OW}\sum_i I_{OW,i} + C_{OW}\sum_j I_{OW,j} + C_2\sum_i D_i + C_2\sum_j D_j$$

We have expressed the relations between different time intervals by satisfying boundary conditions. The values of t_{i1}, t_{i2}, t_{i3} has obtained by giving the values of t_{i0} and $t_{i+1,0}$. Once the values of t_{10}, t_{20}, t_{30}...... $t_{n-1,0}$ are determined, the value of t_{n0} is obtained. Therefore, the decision is to obtain the optimal values of the decision variables so that TC is minimum. Since, n is an integer and the optimization of TC is a desecrate optimization as well as the expression of TC is highly non-linear, an algorithm is proposed to find the optimal solution. A numerical example and sensitivity analysis are used to illustrate the model.

2.3 Conclusions

In this study, an effort has been made to develop a two-warehouse inventory model with quadratically increasing demand and time varying deterioration. Since the deterioration depends on preserving facility available in a warehouse, the different warehouses may have different deterioration rates. The model is formulated by considering time-dependent deterioration rate for different warehouses. We assume that the inventory cost (including holding cost and deteriorating cost) in RW is higher than that in OW. The cost of the whole system is derived analytically. The cost function is highly nonlinear, thus it cannot be solved analytically. Therefore, the total cost of the whole system is minimized by a proposed solution. Most of researchers developed their works completely ignoring the time variation of either demand or deterioration. Time varying demand and deterioration, by considering two warehouse models, has not yet been considered. Therefore, our model has a new managerial insight that helps a manufacturing system/industry to reduce the total system cost at the optimum level.

3 Chapter 2

Mitigation of high-tech products with probabilistic deterioration and inflations

Abstract

This model describes probabilistic deterioration rate with ramp-type demand pattern under stock-dependent consumption rate. Proposed model assumes partially back order where the back order rate follows a negative exponential with the waiting time. The effects of inflation and time value of money are incorporated into the model. The purpose of this study is to develop an optimal replenishment policy so that the total profit per unit time is maximum.

3.1 Assumptions

To derive the model, following assumptions are made:

1. The model is considered for a single item.

2. Deterioration rate ϕ is probabilistic and there is no replacement or repair of deteriorated units during the period under consideration.

3. The demand rate $d(t)$ is assumed to be a ramp-type function of time, i.e.,

   $$d(t) = d_0[t - (t-\mu)H(t-\mu)], \qquad d_0 > 0$$

   where $H(t-\mu)$ is the Heaviside's function as follows:

   $$H(t-\mu) = \begin{cases} 1 & \text{if} \quad t \geq \mu \\ 0 & \text{if} \quad t < \mu \end{cases}$$

   .

4. $S(t)$ is the selling rate at time t, and it is influenced by the demand rate and the

on-hand inventory according to relation

$$S(t) = \begin{cases} d(t) + \gamma I(t), & I(t) > 0 \\ d(t), & I(t) \leq 0 \end{cases}$$

where γ is positive constant and $I(t)$ is the on-hand inventory level at time t.

5. Shortages are allowed and partially backlogged at a rate $\delta(t)$, which is a decreasing function of time with $0 \leq \delta(t) \leq 1, \delta(0) = 1$ and $\lim_{t\to\infty} \delta(t) = 0$. The cases with $\delta(t) = 1$ (or) 0 for all t correspond to complete backlogging (or complete lost sales) models.

6. The effects of inflation and time-value of money is considered.

7. Lead time is considered as negligible.

3.2 Model formulation

The model considers an inventory model for deteriorating items with ramp-type demand and stock-dependent selling rate. The replenishment at the beginning of the cycle brings the inventory level up to I_{max}. The inventory level decreases during the time interval $[0, t_1]$ due to demand and deterioration of items, and falls to zero at $t = t_1$. Thereafter shortages occur during the period (t_1, T), which are partially backlogged. The inventory level, $I(t), 0 \leq t \leq T$ satisfies the following differential equations

$$\begin{aligned} \frac{dI(t)}{dt} + \psi I(t) &= -S(t), \quad 0 \leq t \leq t_1, \qquad I(0) = I_{max} \\ \frac{dI(t)}{dt} &= -S(t)\delta(T-t), \quad t_1 \leq t \leq T, \quad I(t_1) = 0 \end{aligned}$$

The solutions of these differential equations depend on the selling rate. There are two cases considering in this model: (a) $t_1 \leq \mu$, (b) $t_1 \geq \mu$.

Model 1: $t_1 \le \mu$ In this case, the selling rate $S(t)$ is

$$S(t) = \begin{cases} d_0 t + \gamma I(t), & 0 \le t \le t_1 \\ d_0 t, & t_1 \le t \le \mu \\ d_0 \mu, & \mu \le t \le T \end{cases}$$

So, the inventory level satisfies the following differential equations

$$\begin{aligned} \frac{dI(t)}{dt} + \psi I(t) &= -[d_0 t + \gamma I(t)], \quad 0 \le t \le t_1 \qquad \text{with } I(0) = I_{max} \\ \frac{dI(t)}{dt} &= -d_0 t \delta(T - t), \quad t_1 \le t \le \mu \qquad \text{with } I(t_1) = 0 \\ \frac{dI(t)}{dt} &= -d_0 \mu \delta(T - t), \quad \mu \le t \le T \qquad \text{with } -I(T) = S \end{aligned}$$

Using the boundary conditions, I_{max} and S can be found. Thus, the order quantity Q is

$$\begin{aligned} Q = I_{max} + S &= \frac{d_0}{(\psi+\gamma)^2}\left[(\psi+\gamma)t_1 e^{(\psi+\gamma)t_1} - e^{(\psi+\gamma)t_1} + 1\right] + \frac{d_0\mu}{\sigma e^{\sigma T}}\left[e^{\sigma T} - e^{\sigma\mu}\right] \\ &- \frac{d_0}{\sigma^2 e^{\sigma T}}\left[e^{\sigma t_1}(\sigma t_1 - 1) - e^{\sigma\mu}(\sigma\mu - 1)\right] \end{aligned}$$

Therefore, the total profit per unit time under the effect of inflation and time-value of money is

$$\begin{aligned} &= Z_1(t_1) = \frac{1}{T}\left[RV - (OC + PC + HC + BC + LC)\right] \\ &= \frac{d_0}{T}\Bigg[\frac{(\gamma s - C_h)}{(\psi+\gamma)^2}\Bigg\{\frac{[(\psi+\gamma)t_1 - 1][e^{(\psi+\gamma+\rho)t_1} - 1]}{(\psi+\gamma+\rho)e^{\rho t_1}} - \frac{(\psi+\gamma)\left[1 - (1+\rho t_1)e^{-\rho t_1}\right]}{\rho^2} + \frac{[1 - e^{-\rho t_1}]}{\rho}\Bigg\} \\ &+ s\left\{\frac{1 - e^{-\rho t_1}[1 + \rho t_1]}{\rho^2} + \frac{e^{(\sigma-\rho)t_1} - e^{(\sigma-\rho)\mu} + (\sigma-\rho)[\mu e^{(\sigma-\rho)T} - t_1 e^{(\sigma-\rho)t_1}]}{e^{\sigma T}(\sigma-\rho)^2}\right\} - \frac{C_o[1 - e^{-\rho T}]}{d_0\rho} \\ &- C_l\Bigg\{\frac{(\rho-\sigma)[\mu e^{(\sigma-\rho)T} - t_1 e^{(\sigma-\rho)t_1}] + [e^{(\sigma-\rho)\mu} - e^{(\sigma-\rho)t_1}]}{e^{\sigma T}(\sigma-\rho)^2} + \frac{e^{-\rho t_1}[1 + \rho t_1] - \rho\mu e^{-\rho T} - e^{-\rho\mu}}{\rho^2}\Bigg\} \\ &- \frac{C_p Q[1 - e^{-\rho T}]}{d_0\rho} - \frac{C_b\mu}{\sigma^2 e^{\sigma T}}\Bigg\{\frac{\sigma[e^{\mu(\sigma-\rho)} - e^{T(\sigma-\rho)}]}{\rho-\sigma} + \frac{[\sigma t_1 - 1]e^{(\sigma-\rho)t_1} - [\mu\sigma - 1]e^{(\sigma-\rho)\mu}}{(\rho-\sigma)\mu} \\ &+ \frac{\sigma[e^{(\sigma-\rho)t_1} - e^{(\sigma-\rho)\mu}]}{\mu(\sigma-\rho)^2} + \frac{(1 - \sigma t_1)[e^{(\sigma-\rho)t_1} - e^{-\rho T}e^{\sigma t_1}] + [e^{-\rho T} - e^{-\rho\mu}]e^{\sigma\mu}}{\rho\mu}\Bigg\}\Bigg] \end{aligned}$$

Similarly, for **Model 2:** $t_1 \geq \mu$, we obtained the total profit per unit time under the effect of inflation and time-value of money as

$$\begin{aligned}
&= Z_2(t_1) = \frac{1}{T}\left[SR - (OC + PC + HC + BC + LC)\right] \\
&= \frac{d_0}{T}\Bigg[s\left\{\frac{1 - e^{-\rho\mu} - \rho\mu e^{-\rho t_1}}{\rho^2} + \frac{\mu e^{-\sigma T}[e^{(\sigma-\rho)T} - e^{(\sigma-\rho)t_1}]}{(\sigma-\rho)}\right\} + C_l\mu\left\{\frac{e^{(\sigma-\rho)T} - e^{(\sigma-\rho)t_1}}{e^{\sigma T}(\sigma-\rho)}\right. \\
&- \left.\frac{e^{-\rho t_1} - e^{-\rho T}}{\rho}\right\} - \frac{C_b\mu}{\sigma e^{\sigma T}}\left\{\frac{e^{\sigma t_1}[e^{-\rho T} - e^{-\rho t_1}]}{\rho} + \frac{e^{(\sigma-\rho)T} - e^{(\sigma-\rho)t_1}}{(\sigma-\rho)}\right\} + \frac{\gamma s - C_h}{(\psi+\gamma)^2} \\
&\quad \left\{\frac{\rho\mu(\psi+\gamma)e^{-\rho t_1} - [1 - e^{-\rho\mu}](\gamma+\psi-\rho)}{\rho^2} + \frac{\mu(\psi+\gamma)[e^{(\psi+\gamma)t_1} - e^{-\rho t_1}] - e^{(\psi+\gamma)\mu} + e^{-\rho\mu}}{(\gamma+\psi+\rho)}\right\} \\
&- \frac{[1 - e^{-\rho T}]}{d_0\rho}(C_0 + C_pQ)\Bigg]
\end{aligned}$$

Finally, necessary and sufficient conditions of the existence and uniqueness of the optimal solution are derived, which maximize the total profit. Some numerical examples along with graphical representations are provided to illustrate the proposed model. Sensitivity analysis of the optimal solution with respect to key parameters of the model has been carried out and the implications are discussed.

3.3 Conclusions

In this marketing environment, when a new brand of consumer goods are launched, the demand of goods increases quickly to a certain moment and after sometime it stabilizes. Finally, it becomes almost constant. Keeping in mind this type of demand pattern, we consider the demand as a ramp-type function of time. To make the research a more realistic one, four different types of continuous probabilistic deterioration functions are considered in numerical example. We found that the associated profit function is maximized at the optimal values of decision variables. We provide a solution procedure to find the optimal solution. Some numerical examples, graphical representations, special cases, and sensitivity analysis are given to illustrate the model. There are several extensions of this work that could constitute future research related in this field. This model can be extended in several ways, like multi-item inventory models, reliability of the items, variable deterioration, etc. The research can also be extended to consider fuzzy demand case.

4 Chapter 3

Optimal replenishment policy with variable deterioration for fixed-lifetime products

Abstract

Although numerous researchers have developed different inventory models for deteriorating items, very few of them have taken the maximum lifetime of a deteriorating item into consideration. This model illustrate a mathematical model to obtain an optimal replenishment policy for deteriorating items with maximum lifetime, ramp-type demand, and shortages. Holding cost and deterioration function both are linear function of time which are treated as constants in most of the deteriorating inventory model. A simple solution procedure is provided to obtain the optimal solutions.

4.1 Assumptions

To derive the model, following assumptions are used.

Assumptions

1. The model is considered for a single type of item.

2. The deterioration rate is assumed as $\alpha(t) = \frac{1}{1+\rho-t}$, where $\rho > t$ and ρ is the maximum lifetime of products at which the total on-hand inventory deteriorates. When t increases, $\alpha(t)$ increases and $\text{Lim}_{t\to\rho}\alpha(t) \to 1$.

3. The demand rate $d(t)$ is assumed to be a ramp-type function of time, i.e.,

$$d(t) = d_0[t - (t-\mu)H(t-\mu)], \qquad d_0 > 0$$

where $H(t-\mu)$ is the Heaviside's function as follows:

$$H(t-\mu) = \begin{cases} 1 & \text{if } \ t \geq \mu \\ 0 & \text{if } \ t < \mu \end{cases}$$

.

4. Holding cost is linear function of time i.e., $C_h(t) = h_0 + h_1 t$, where $h_0, h_1 > 0$.

5. Shortages are allowed and fully backlogged.

6. Lead time is assumed as negligible and the replenishment rate is infinite.

4.2 Model formulation

The model considers an inventory model for deteriorating items with ramp-type demand and stock-dependent selling rate. The replenishment at the beginning of the cycle brings the inventory level up to I_{max}. The inventory level decreases during the time interval $[0, t_1]$ due to combined effects of the demand, deterioration, and falls to zero at $t = t_1$. Shortages occur during the period (t_1, T), which are fully backlogged.
During the replenishment cycle $[0, T]$, the inventory level, $I(t)$, satisfies the following differential equations

$$\begin{aligned}
\frac{dI(t)}{dt} &= -d(t) - \alpha(t)I(t), \quad 0 \le t \le t_1, \qquad \text{with } I(0) = I_{max} \\
\frac{dI(t)}{dt} &= -d(t), \quad t_1 \le t \le T, \quad \text{with } I(t_1) = 0
\end{aligned}$$

To solve the above differential equations, two cases are considered as (a) $\mu \ge t_1$, (b) $\mu \le t_1$.
For **Model 1** when $\mu \ge t_1$, the demand rate $d(t)$ is

$$d(t) = \begin{cases} d_0 t, & 0 \le t \le t_1 \\ d_0 t, & t_1 \le t \le \mu \\ d_0 \mu, & \mu \le t \le T \end{cases}$$

Inventory levels satisfies

$$\frac{dI(t)}{dt} = -d_0 t - \alpha(t)I(t), \qquad 0 \le t \le t_1 \quad \text{with } I(0) = I_{max}$$

$$
\begin{aligned}
\frac{dI(t)}{dt} &= -d_0 t, && t_1 \le t \le \mu \quad \text{with } I(t_1) = 0 \\
\frac{dI(t)}{dt} &= -d_0 \mu, && \mu \le t \le T \quad \text{with } -I(T) = S
\end{aligned}
$$

Then, average total cost per unit time under this condition is

$$
\begin{aligned}
TC_1(t_1) &= \frac{1}{T}\left[OC + PC + HC + DC + SC\right] \\
&= \frac{d_0}{T}\left[\frac{C_o}{d_0} + h_0\left\{\frac{(1+\rho)^3}{2}\ln\left(\frac{1+\rho}{1+\rho-t_1}\right) + \frac{t_1}{12}\left[2t_1^2 - 3t_1(1+\rho) - 6(1+\rho)^2\right]\right\}\right. \\
&+ \left\{\frac{(1+\rho)^4}{6}\ln\left(\frac{1+\rho}{1+\rho-t_1}\right) + \frac{1}{36}\left[3t_1^4 - 6t_1(1+\rho)^3 - 3t_1^2(1+\rho)^2\right.\right. \\
&- \left.\left. 2t_1^3(1+\rho)\right]\right\}h_1 + (C_p + C_d)(1+\rho)\left[(1+\rho)\ln\left(\frac{1+\rho}{1+\rho-t_1}\right) - t_1\right] \\
&+ \left. C_p\left(\mu T - \frac{\mu^2}{2} - \frac{t_1^2}{2}\right) - \frac{C_d t_1^2}{2} + C_s\mu\left(\frac{\mu^2}{6} + \frac{t_1^3}{3\mu} - \frac{\mu T}{2} - \frac{T t_1^2}{2\mu} + \frac{T^2}{2}\right)\right]
\end{aligned}
$$

Similarly, for **Model 2** when $\mu \le t_1$, the average total cost per unit time is

$$
\begin{aligned}
TC_2(t_1) &= \frac{1}{T}\left[OC + PC + HC + DC + SC\right] \\
&= \frac{d_0}{T}\left\{\frac{C_o}{d_0} + h_0\left[\frac{1}{4}(1+\rho)(1+\rho-\mu)^2 - \frac{1}{4}(1+\rho)^3 + \frac{\mu}{12}\left[3t_1^2 - \mu^2 - 6t_1(1+\rho)\right]\right]\right. \\
&+ \frac{(1+\rho)^2}{6}\left[h_1(1+\rho) + 3h_0\right]\left[\mu\ln\left(\frac{1+\rho-\mu}{1+\rho-t_1}\right) + (1+\rho)\ln\left(\frac{1+\rho}{1+\rho-\mu}\right)\right] \\
&+ \frac{h_1\mu}{36}\left[(3\mu - 6t_1)(1+\rho)^2 + (\mu^2 - 18\rho - 3t_1^2)(1+\rho) - 6(1+\rho^3) - \mu^3 + 4t_1^3\right] \\
&+ (C_p + C_d)(1+\rho)\left[\mu\left\{\ln\left(\frac{1+\rho-\mu}{1+\rho-t_1}\right) - 1\right\} - (1+\rho)\ln\left(\frac{1+\rho-\mu}{1+\rho}\right)\right] \\
&+ \left. C_p\mu(T - t_1) - C_d\left[\frac{\mu^2}{2} + \mu(t_1 - \mu)\right]\right\} + \left.\frac{C_s\mu}{2}(T - t_1)^2\right\}
\end{aligned}
$$

Solution procedure is given, which can ensure the existence of a unique t_1 to minimize the average total cost for the model. Numerical examples, the graphical representation and sensitivity analysis are provided to illustrate the model.

4.3 Conclusions

In modern marketing environment, a practical problem is to control the deterioration of items. Some products (e.g., fruits, vegetables, pharmaceutical, volatile liquids, and others) not only deteriorate continuously due to evaporation, obsolescence, spoilage, etc., but also it deteriorates with increase in time (i.e., a deteriorating item has its maximum lifetime). In existing literature, very few researcher considered maximum lifetime of deteriorating items in their model. In this regard, this study considered an inventory model for products with maximum lifetime, time-varying deterioration rate, and ramp-type demand. A simple solution procedure was given and existence and uniqueness of the optimal solutions were obtained analytically. This model minimized the associated cost function at the optimal values of the decision variable. Finally, the sensitivity analysis on the optimal solution with respect to key parameters were studied to illustrate the model and some managerial insights were provided.

5 Chapter 4

Flexible setup cost and deterioration of products in a supply chain model

Abstract

Product reliability is of significant importance in today's technological world. People rely more and more upon the sustained functioning of machinery and complex equipments for purposes such as health, economic welfare, safety, to name just a few. Thus in a business arena it is critical to assess the reliability of new product. In this model, a two echelon supply chain model with variable setup cost and deterioration cost is analyzed. We assume that the setup cost is directly proportional and the deterioration rate is inversely proportional to reliability. We use algebraical procedure to obtain the optimal closed-form solution of our model. The objective is to minimize the total cost of the entire system by considering reliability as the decision variable.

Decision variables

q delivery lot size (units)

N number of deliveries per production-batch, $N \geq 1$

R reliability parameter

5.1 Assumptions

We consider the following assumptions to develop the model.

1. Single type of item is produced by the production-inventory system.
2. Setup cost S_0 and deterioration cost C_d depend on the reliability parameter R.
3. Information regarding the inventory position and demand of the buyer are given to the supplier.

4. Production rate is greater than demand, i.e., $p > d$.

5. Handling and transportation costs are paid by the buyer.

6. Shortage and backlogging are not considered.

5.2 Model formulation

A single-setup-multiple-delivery (SSMD) production is considered in this article. The quantity ordered by the buyer is manufactured at a time and the manufactured products are delivered after a fixed time interval over multiple deliveries in an equal amount. The splitting of the ordered quantity into multiple lots is consistent with JIT implementation. The average total cost of the production-inventory model is developed for the buyer's and the supplier's which is then minimized. Without any loss of generality, we consider that the products ordered arrives at the exact time when the items from the previous delivery has just been depleted. Here, total time span T is divided into two components: t_1, the production time duration for the supplier and t_2, the non-production time duration for the supplier. t_3 is considered as the time duration between the two successive deliveries. Firstly, separate calculations for buyer's and the supplier's inventory cost are carried out.

Inventory cost for the buyer

becomes

$$TC_b = \left(\frac{d}{Nq} + \frac{\sigma}{2NR}\right)(C_o + NK + V_cNq) + \frac{q}{2}\left[HC_b + \frac{\sigma C_d}{R}\right]$$

Inventory cost for the supplier

$$TC_s \;=\; \frac{1}{T}\left(S_o + \rho R + HC_sA_s + \frac{\sigma C_d}{R}A_s\right)$$

Integrated inventory cost for the entire SCM

The total average cost for the entire SCM is $TC(q, N, R) = TC_b + TC_s$

$$TC(q,N,R) \;=\; \left(\frac{d}{Nq} + \frac{\sigma}{2NR}\right)[C_o + (S_o + \rho R) + NK + V_cNq] + \frac{q}{2}\left(HC_b + \frac{\sigma C_d}{R}\right)$$

$$+ \quad \left(HC_s + \frac{\sigma C_d}{R}\right)\left\{\frac{(2-N)d}{p} + N - 1\right\}$$

Minimum order quantity

The required ordered quantity that makes the SSMD policy superior to single-delivery policy is obtained from the savings.

$$\begin{aligned} SV(q,N,R) &= \left(\frac{d}{q} + \frac{\sigma}{2R}\right)\{C_o + (S_o + \rho R)\}\left(1 - \frac{1}{N}\right) \\ &+ \frac{q}{2}\left(HC_s + \frac{\sigma C_d}{R}\right)(N-1)\left(\frac{d}{p} - 1\right) \end{aligned}$$

When N and R are fixed, TC can be written in the symbolic form as

$$TC(q) = x_1 q + \frac{x_2}{q} + x_3 = \frac{x_1}{q}\left(q - \sqrt{\frac{x_2}{x_1}}\right)^2 + 2\sqrt{x_1 x_2} + x_3$$

When q and R are fixed, TC can be written in the symbolic form as

$$TC(N) = x_4 N + \frac{x_5}{N} + x_6$$

When q and N are fixed, TC can be written in the form as

$$TC(R) = x_7 R + \frac{x_8}{R} + x_9$$

Then minimum values of q, N, and R are obtained by using algebraic method, which minimizes the cost functions $TC(q)$, $TC(N)$ and $TC(R)$ respectively.

Optimal interval of the lot size

By assumptions of N, the number of deliveries per production batch-cycle, must be greater than or equal to 1 and from the expression for optimum q, N attains its upper bound at $N = 1$, i.e.,

$$q \leq \sqrt{\frac{2pdR(C_o + S_o + \rho R + K)}{R\{pHC_b + dHC_s\} + \sigma\{C_d(d+p) + V_c p\}}}$$

As N, the number of deliveries per production batch-cycle increases, the corresponding lot size value q decreases hence, from the equation of optimum lot size, one can obtained

$$q \geq \sqrt{\frac{2pdR(C_o + S_o + \rho R + NK)}{\{HC_b R + \sigma(C_d + V_c)\}\{2d + N(p-d)\}N}}$$

The total cost TC, the delivery lot size q, the number of deliveries per production batch N, and the reliability parameter R are obtained in this model. If the value of N is not an integer, then one can choose N in such a way, which gives $\min\{TC(N^+), TC(N^-)\}$ for the model where N^+ and N^- represent the closest integers larger or smaller than the optimal N^*. Optimal minimal cost, given by TC, is obtained by substituting the values of N^*, q^* and R^* in $TC(q^*, N^*, R^*)$.

Numerical examples, the graphical representation and sensitivity analysis are provided to illustrate the model. This model is compared with that of Sarkar (2013) by using the same parametric values.

5.3 Conclusions

In this proposed model, we discussed the effect of the reliability parameter on the setup cost and the deterioration rate. With the help of an algebraical procedure we minimized the cost for the entire SCM model and obtained a closed-form solution. The main contribution of the model was to obtain the minimum cost with integer number of deliveries, optimal lot size, and the reliability parameter using algebraical procedure. The proposed procedure for the computation of the total cost of the SCM can be easily done without any tedious calculation.

6 Chapter 5

Optimal buffer inventory and inspection errors in an imperfect production system with preventive maintenance

Abstract

This model considers an imperfect production system with preventive maintenance to obtain the optimal buffer inventory and inspection policy for sold products with free minimal repair warranty. The production system is subject to a random movement from an *in-control* to an *out-of-control* state, where some proportion of defective items are produced by the production system during both the *in-control* and *out-of-control* states. Online inspection is continuing after a time variable during the production process. Another offline human-based inspection policy is considered at the end of the production cycle to identify the defective items. Defective items found by the inspector are salvaged at some fixed cost before being shipped and the non-inspected items are passed to the customer with free minimal repair warranty. During human-based inspection, some misclassifications may arise from the inspector's side. Thus, two types of inspection errors (Type I and Type II) are considered to make the model more realistic rather than the existing models. A numerical example along with graphical representations are provided to illustrate the proposed model. Sensitivity analysis of the optimal solution with respect to major parameters of the system has been carried out and the implications are discussed.

6.1 Problem definition, and assumptions

This section contains problem definition and several assumptions.

Problem definition

The main focus of this model is to produce a single-type of items in a single-stage production system. During production, the machinery system may shift *in-control* to *out-of-control* state at any random time, which may follow any specific distribution. In both

in-control and *out-of-control* state, the machinery system produces defective items, but the production rate of defective items in *in-control* state is less than that of *out-of-control* state. As production processes are prone to fail at the end of a production run, on-line inspection is considered after some time of the production and another human-based inspection policy is considered at the end of the production process to detect the defective items. Product inspection starts from $(p\lambda t)$th item until the end of the production lot, and the defective items by the inspector are salvaged at some fixed cost in a parallel system before shipped. The human-based inspection is considered to assure the quality of products, even though the inspection are with errors. The non-inspected items are sold with free minimal warranty. After the end of production run time, the preventive maintenance starts and continues up to a certain time, which follows a specific distribution. Shortages occur, when the buffer inventory goes to zero, but the preventive maintenance of machine is not completed. The management system uses two types of inspections, but as they finds inspection errors by human inspection, thus that outcomes is not used for the rework or warranty, but they use some funds for this human based inspection, thus this cost is incorporated within the system cost.
Decision variables in this study are

B buffer inventory (units) and

λ non-inspected fraction in a batch $(0 \leq \lambda \leq 1)$

Assumptions

The following assumptions are considered to develop this model.

1. At any time, production system, for single-type of products, may shifts *in-control* state to *out-of-control* state until the end of the production-run.

2. Production rate of defective items in *out-of-control* state is grater than *in-control* state.

3. Online inspection is started after some time of production to detect defective units and human-based inspection policy is considered to identify the defective items, where human inspectors are not properly skilled, thus, two types of inspection errors (Type I and Type II) are considered during product inspection.

4. The free minimal repair warranty (FRW) policy is adopted for non-inspected defective items.

5. After the preventive maintenance, it is grunted that the probability of a breakdown of the manufacturing system during the production run time t is zero.

6. At the beginning of any preventive maintenance, the buffer is not subjected to deterioration or obsolescence.

7. Shortages are allowed and are fully backlogged.

6.2 Model formulation

In this model, it is assumed that at the initial stage, the production system is in *in-control* state and after a random time X it moves to an *out-of-control* state. In this system, three cases may arise: **Case I** when $X \geq t$, **Case II** when $\lambda t < X < t$ and **Case III** when $X \leq \lambda t$.

Total post-sale warranty and salvage cost within the time interval $[0, t]$ is obtained as

$$C_d = \begin{cases} W_c\theta_1 p\lambda t + V_c\theta_1 p(t - \lambda t), & \text{if } X \geq t \\ W_c\theta_1 p\lambda t + W_c\theta_1 p(X - \lambda t) + V_c\theta_2 p(t - X), & \text{if } \lambda t < X < t \\ W_c\theta_1 pX + W_c\theta_2 p(\lambda t - X) + V_c\theta_2 p(t - \lambda t), & \text{if } X \leq \lambda t \end{cases}$$

We obtain the expected value of C_d as

$$E[C_d] = W_c\theta_2 p\lambda t + V_c\theta_2 pt(1 - \lambda) - (\theta_2 - \theta_1)p\left[W_c\int_0^{\lambda t}\overline{G}(x)dx + V_c\int_{\lambda t}^{t}\overline{G}(x)dx\right]$$

Inspection errors

During inspection, an inspector may commit two types of inspection errors (Type I and Type II). Due to Type I error, some non-defective items are classified as defective i.e., $(1-\theta_1)m_1$ in *in-control* state and $(1-\theta_2)m_1$ in *out-of-control* state. Again due to Type II error, some defective items are classified as non-defective i.e., $\theta_1 m_2$ in *in-control* state and $\theta_2 m_2$ in *out-of-control* state. Therefore, the inspector rejects some non-defective items and accepts some defective items due to these two types of inspection errors. Three cases occur during inspection: **Case I** when $X \geq t$, **Case II** when $\lambda t < X < t$ and **Case III** when $X \leq \lambda t$.

Total miscalculation cost due to two types of inspection errors within the time interval $[\lambda t, t]$ is

$$C_i = \begin{cases} C_r p(t-\lambda t)(1-\theta_1)m_1 + C_a\theta_1 p(t-\lambda t)(1-m_2), & \text{if } X \geq t \\ p(X-\lambda t)[C_r(1-\theta_1)m_1 + C_a\theta_1(1-m_2)] & \\ +p(t-X)[C_r(1-\theta_2)m_1 + C_a\theta_2(1-m_2)], & \text{if } \lambda t < X < t \\ p(t-\lambda t)[C_a(1-\theta_2)m_1 + C_r\theta_2(1-m_2)] & \text{if } X \leq \lambda t \end{cases}$$

The expected misclassification cost, due to two types of inspection errors within the time interval $[\lambda t, t]$ is as

$$\begin{aligned} E[C_i] &= p(1-\lambda)t\left\{C_a\theta_2(1-E[m_2]) + C_r(1-\theta_2)E[m_1]\right\} \\ &+ p(\theta_1-\theta_2)\left\{C_a(1-E[m_2]) - C_r E[m_1]\right\}\int_{\lambda t}^{t}\overline{G}(x)dx \end{aligned}$$

Here, the production system runs for a time-period t before preventive maintenance occur. Within the period $[0,t]$, the buffer inventory S builds up at a rate $p-d$ i.e., $S=(p-d)t$. After the time period t, preventive maintenance starts and continues upto time τ, which is a random variable that follows a probability density function $f(\tau)$. The number of shortage units per preventive maintenance cycle is

$$Y(\tau) = \begin{cases} 0 & \text{if } \tau \leq \frac{S}{\alpha}, \\ \alpha\left(\tau - \frac{S}{\alpha}\right) & \text{if } \tau \geq \frac{S}{\alpha}. \end{cases}$$

The expected number of unit shortages is

$$E[Y(\tau)] = \alpha \int_{S/\alpha}^{\infty} \left(\tau - \frac{S}{\alpha}\right) f(\tau) d\tau$$

Total expected cost per unit item is

$$\begin{aligned} ETC(S,\lambda) &= \frac{K}{S}\left(1-\frac{\alpha}{p}\right) + C_m + \frac{H_c S}{2\alpha} + \frac{M_c}{S}\left(1-\frac{\alpha}{p}\right)\int_0^{\infty} \tau f(\tau) d\tau + \frac{S_c \alpha}{S}\left(1-\frac{\alpha}{p}\right) \\ & \int_{S/\alpha}^{\infty}\left(\tau - \frac{S}{\alpha}\right) f(\tau) d\tau - \frac{(\theta_2-\theta_1)(p-d)}{S}\left[W_c \int_0^{\frac{\lambda S}{p-d}} \overline{G}(x) dx\right] + W_c \theta_2 \lambda \\ - & \frac{(\theta_2-\theta_1)(p-d)}{S}\left\{V_c + C_a(1-E[m_2]) - C_r E[m_1]\right\} \int_{\frac{\lambda S}{p-d}}^{\frac{S}{p-d}} \overline{G}(x) dx \\ + & (1-\lambda)\left\{I_c + V_c\theta_2 + C_a\theta_2(1-E[m_2]) + C_r(1-\theta_2)E[m_1]\right\} \end{aligned}$$

Our objective is to find the optimal value of S and λ such that the corresponding expected total cost per unit item $TC(S,\lambda)$ is minimum.

Optimal inspection policy λ^* is obtained. Here, it is proved that the optimal buffer inventory S^*, which minimizes $C(S,\lambda)$, is exists and it is unique.

Convexity of $ETC(S,\lambda)$ at the optimal S^* and λ^*, is also proved by using Hessian matrix. It has shown that (S^*,λ^*) is the global minimum solution. A numerical example along with graphical representations are provided to illustrate the proposed model. Sensitivity analysis of the optimal solution with respect to major parameters of the system has been carried out and the implications are discussed.

6.3 Conclusions

This model focused on a deteriorating production process with preventive maintenance and inspection errors. Further, it was considered that during long-run production, the production system moves to an *out-of-control* state after some time. The probability of defective items being produced in *in-control* state is smaller than in the *out-of-control* state. On-line inspection and a human-based inspection were considered to detect the defective items. During inspection, two types of inspection errors were carried out, where human based inspection is not error-free. Inspected items are immediately salvaged at a cost in a parallel system before they were released for sale. Non-inspected items were sold at the market with post-sale warranty. During the maintenance period, there was a buffer inventory, which fulfilled the demand during maintenance. The optimal buffer inventory and optimal non-inspected fraction batch were obtained to minimize the total expected cost per unit product. A solution methodology was provided to find the optimal solution. Finally, numerical example and sensitivity analysis of the optimal solution with respect to key parameters were studied to illustrate the model and some managerial insights were provided. This model can be extended by considering the production rate and demand for products as random variables. During the production run time, machine breakdown can be considered in future.

7 Chapter 6

Optimal production run time and inspection errors in an imperfect production system with warranty

Abstract

This model considers an imperfect production system to obtain the optimal production run time and inspection policy. Contrary to the existing literature this model considerers that product inspection performs at any arbitrary time of the production cycle and after the inspection, all defective products produced until the end of the production run are fully reworked. Due to some misclassification during inspection, from the inspector's side two types of inspection errors as Type I and Type II are considered to make the model more realistic rather than existing models. Defective items, found by the inspector, are salvaged at some cost before being shipped. Non-inspected defective items are passed to customers with free minimal repair warranty. The model gives three special cases, where it is found that this model converges over the exiting literature. Some numerical examples along with graphical representations are provided to illustrate the proposed model with comparison with the existing models. Sensitivity analysis of the optimal solution with respect to key parameters of the model has been carried out and the implications are discussed.

7.1 Formulation of the model

This section contains problem definition, and mathematical model.

Problem definition

An imperfect production system for a single-type of item is considered. Production starts from *in-control* state and after a period of operation, the production system may shift to *out-of-control* state until the end of the production-run. At each state, θ_1 and θ_2 represent the percentage of the number of defective items during *in-control* state and *out-of-control* state, respectively with $\theta_1 < \theta_2$. The elapsed time until the production system shifts to the *out-of-control* state is denoted by X, which follows an exponential distribution with $g(x)$ as probability density function, $G(x)$ as distribution function, and $\overline{G}(= 1 - G(x))$ as survival function. The failure rate function of the random variable X is defined as $\phi(x) = g(x)/G(x)$. After completion of a lot, the system is inspected with fixed cost η to obtain the information about the state of the system. If the system is in *out-of-control* state, after completion of the production cycle, the production system brought back to the *in-control* state with an additional restoration cost r. To detect the defective items produced in a produced lot, a product inspection policy is carried out at a fixed cost I_c. The inspection time is considered as negligible. Product inspection policy starts from (pu_1t)th item to (pu_2t)th item, and the defective items from those inspected will be salvaged at some fixed cost C_s before being shipped. After completion of inspection, all produced products during production time u_2t to the end of production are reworked without inspections. During inspection, due to misclassification an inspector classified some non-defective items as defective with a fixed rate m_1 and classified some defective items as non-defective with a fixed rate m_2. The non-inspected defective items are taken as salvageable and those items are sent to the market with post sale (warranty) cost C_w with the assumption $I_c + C_s < C_w$.

Mathematical model

The inventory level starts with $p - d$ rate and depletes with a rate $-d$, where production rate $(p) >$ demand rate $(d) > 0$. The total produced items are pt during cycle time t and the time duration of a production cycle is pt/d. The production cost per product is C_p and the inventory holding cost per unit per unit time is C_h. Thus, the maximum inventory is $(p - d)t$ and the holding cost is $\frac{1}{2}C_h(pt/d)(p - d)t$. Hence, the holding cost per item is $\frac{C_h(p-d)t}{2d}$. Setup cost for each production-run is S_0. Theretofore, setup cost per item is $\frac{S_0}{pt}$ and hence the system inspection cost per item is $\frac{\eta}{pt}$. If the system moves to *out-of-control* state, then r is the fixed cost to transfer the system back to the *in-control*

state. Therefore, the restoration cost per unit item is $\frac{rG(t)}{pt}$.
Expected number of defective items in the time interval $[u_1t, u_2t]$ is obtained as

$$E[N_2(t)] - E[N_1(t)] = \theta_2 p(u_2 - u_1)t - (\theta_2 - \theta_1)p \int_{u_1t}^{u_2t} \overline{G}(x)dx$$

During product screening process, the inspectors make Type I and Type II errors. They classify some non-defective item as defective item i.e., $(1-\theta_1)m_1$ in *in-control* state and $(1-\theta_2)m_1$ in *out-of-control* state. In another side, the inspector classifies some defective items as non-defective i.e., $\theta_1 m_2$ in *in-control* state and $\theta_2 m_2$ in *out-of-control* state. Therefore, the inspector rejects some non-defective items and accepts some defective items, thus, two cases arise: **Case I:** $X < u_1t$ and **Case II:** $X \geq u_1t$. Total defective cost is

$$C_d = \begin{cases} C_R p[(1-\theta_1)X + (1-\theta_2)(u_1t - X)]m_1 & \\ +C_A p[\theta_1 X + \theta_2(u_1t - X)](1-m_2), & \text{if } X < u_1t \\ C_R(1-\theta_1)pu_1tm_1 + C_A\theta_1 pu_1t(1-m_2), & \text{if } X \geq u_1t \end{cases}$$

The expected value of defective cost, $E[C_{d_1}]$ within the time interval $[0, u_1t]$ and expected value of defective cost $E[C_{d_2}]$ in the time interval $[0, u_2t]$ are obtained separately. Then, the expected defective cost within the interval $[u_1t, u_2t]$ is

$$\begin{aligned} E[C_{d_2}] - E[C_{d_1}] &= p(u_2 - u_1)t \left\{C_A\theta_2(1 - E[m_2]) + C_R(1-\theta_2)E[m_1]\right\} \\ &+ p(\theta_1 - \theta_2)\left\{C_A(1 - E[m_2]) - C_R E[m_1]\right\} \int_{u_1t}^{u_2t} \overline{G}(x)dx \end{aligned}$$

The expected total warranty, salvage, defective, and rework cost with lot size pt under the inspection policy (u_1, u_2) is

$$C_w E[N_1(t)] + C_s\{E[N_2(t)] - E[N_1(t)]\} + E[C_{d_2}] - E[C_{d_1}] + C_r pt(1-u_2)$$

Now, the expected total cost per item i.e., $C(t, u_1, u_2)$ is the addition of manufacturing cost, holding cost, setup cost, process inspection cost, restoration cost, product inspection

Table 1: Summary of numerical results

This model	Sarkar and Saren (2016)	Hu and Zong (2009)	Wang (2005)
(t^*, u_1^*, u_2^*)	(t^*, u^*)	(t^*, u_1^*, u_2^*)	(t^*, u^*)
$C(t^*, u_1^*, u_2^*)$	$C(t^*, u^*)$	$C(t^*, u_1^*, u_2^*)$	$C(t^*, u^*)$
(2.04,0.058,0.251)	(1.85,0.064)	(2.05,0.062,0.239)	(1.83,0.069)
\$8.48	\$8.90	\$8.49	\$8.97

cost, warranty cost, salvage cost, defective cost, and rework cost as follows:

$$\begin{aligned}
C(t, u_1, u_2) &= C_p + \frac{C_h(p-d)t}{2d} + \frac{K + rG(t)}{pt} + (I_c + C_s\theta_2)(u_2 - u_1) + C_w\theta_2 u_1 \\
&+ C_r(1 - u_2) + (u_2 - u_1)\Big[C_A(1 - E[m_2])\theta_2 + C_R E[m_1](1 - \theta_2)\Big] \\
&+ \frac{(\theta_1 - \theta_2)}{t}\{C_s + C_A(1 - E[m_2]) - C_R E[m_1]\} \int_{u_1 t}^{u_2 t} \overline{G}(x)dx \\
&+ \frac{C_w(\theta_1 - \theta_2)}{t} \int_0^{u_1 t} \overline{G}(x)dx
\end{aligned}$$

where $K = S_0 + \eta$.

The objective is to obtain the optimum value of t, u_1, and u_2 such that $C(t, u_1, u_2)$ is minimum. The optimization of this model is done by numerical example. This model is the extension of several models.

Some special cases:

Case I If $u_2 = 1$ and $C_r = C_s$, then this model converges to Sarkar and Saren's (2016) model.

Case II If $C_A = C_R = 0$, and $C_s = C_r$, then the model becomes Hu and Zong's (2009) model.

Case III If $u_2 = 1$, $C_A = C_R = 0$, and $C_s = C_r$, then the model becomes Wang's (2005) model.

Numerical comparison of these three models with our model is given in following table. Sensitivity analysis of the optimal solution with respect to major parameters of the system has been carried out and the implications are discussed.

7.2 Conclusions

This model developed an imperfect production process under the presence of Type I and Type II errors. The *out-of-control* probability of the system as well as Type I and Type II inspection errors are considered as random variable with known probability density function. Variable inspection interval was considered. Inspected items are salvaged as some fixed cost before they are released for sale. Non-inspected items are sold at market with post-sale warranty. This model minimized the total cost per item by obtaining the production run time and inspection policy. The numerical comparison between other models has been conducted to display the impact of inspection errors on the optimum solutions. This model can be extended for items having linear increasing demand, price, and advertising-dependent demand or power-demand. This study can be extended further by considering stochastic demand.

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