2018

M.Sc.

1st Semester Examination

PHYSICS

PAPER—PHS-101

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Unit—101.1

[Marks : 20]

Answer Q. No. *1*, *2* and any *one* from the rest.

1. Answer any *two* questions : 2×2

(a) Discuss the nature of singularities of the function

$$f(z) = \frac{z - 1 - i}{z^2 - (4 + 3i)z + (1 + 5i)},$$

where $z \in c$.

(b) An unitery matrix $\begin{pmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{pmatrix}$ is given, where a, b, c, α and β are real. Find the inverse of it.

(Turn Over)

(c) Develop the Laurent expansion of $\frac{1}{(z-1)(z-2)}$ in the region $|z| < 1$.

(d) If P be a $n \times n$ diagonalizable matrix which satisfies the equations

$$P^2 = P, \qquad T_r(P) = n - 1.$$

Find Det. (P).

2. Answer any *two* questions : 2×4

(a) Suppose a 2×2 matrix X (not necessarily Hermitian, nor unitary) is written as $X = a_0 + \vec{\sigma}.\vec{a}$ where a_0 and $a_{1,2,3}$ are numbers. $\vec{\sigma}$ is the Pauli Spin matrices. How a_0 and a_k $(k = 1, 2, 3)$ are related to $T_r(X)$ and $T_r(\sigma_k X)$?

(b) Prove that $\int_0^\infty e^{-\alpha x} J_0(x) dx = \frac{1}{\sqrt{1+\alpha^2}}$.

(c) Prove that $H_n(0) = (-1)^{n/2} \frac{n!}{(n/2)!}$ where n is even

$= 0$ when n is odd.

(d) Show that $f(z) = \ln z$ has branch points.

3. (a) If $\hat{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Find $\cos\left(\frac{\pi \hat{A}}{6}\right)$.

(b) Evaluate $\frac{1}{2\pi i} \oint_C \frac{e^{4z} - 1}{\cos h(z) - 2\sin h(z)} dz$

around the unit circle C traversed in the anti-clockwise direction. 4+4

4. (a) Prove that $\sum_n (2n+1) P_n(x) = 2\delta(x-1)$.

(b) Evaluate $\int_0^\infty \frac{\ln x}{\left(x^2+1\right)} dx$ by residue theorem. 4+4

Unit—101.2

[Marks : 20]

1. Answer any *two* questions : 2×2

(a) Obtain the relation between Hamilton's principal function and Hamilton's characteristic function.

(b) Show that the following transformation is canonical

$$Q = \sqrt{2q}\, e^{-\alpha} \cos p \text{ and } P = \sqrt{2q}\, e^{\alpha} \sin p.$$

(c) A particle of mass m moves in a potential $V(x) = \frac{1}{2} m w^2 x^2 + \frac{1}{2} m \mu v^2$ where x is the position coordinate, v is the velocity, w and μ are constants. Find the canonically conjugate momentum of the particle.

(d) Consider a mass m moving in one dimension under a force with the potential $U(x) = K(2x^3 - 5x^2 + 4x)$, where the constant $K > 0$. Show that the point $x = 1$ corresponds to a stable equilibrium position for the particle.

2. Answer any *two* questions : 2×4

(a) Prove that if $F(q, p, t)$ and $G(q, p, t)$ are two integrals of motion, then $[F, G]$ is also integral of motion.

(b) Prove that the shortest distance between two points in a plane is a straight line.

(c) Find out the equation of motion of a simple pendulum using Hamilton's principle.

(d) Explain stable, unstable and neutral equilibrium on the basis of potential function.

3. Answer any *one* question : 1×8

(a) A particle slides from rest at one point on a friction less wire in a verticle plane to another point under the influence of earth's gravitational field. If the particle travels in shortest time, show that the path followed by it is a cycloid. Derive Euler-Lagrange equation. 4+4

(b) What is action-angle variable ? Find out the frequency of a linear harmonic oscillator using action-angle variable method. 3+5

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