2018

M.Sc.

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-401

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Functional Analysis*)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions : 4×2

(a) Let F and G be subspaces of a Hilbert space H. Show that $(F+G)^{\perp} = F^{\perp} \cap G^{\perp}$.

(Turn Over)

(b) Let $T : H \to H$ be a bounded linear operator on a Hillert space H. Prove that if T is self-adjoint then $\langle Tx, x \rangle$ is real for all $x \in X$.

(c) Check whether $C^1[0, 1]$ with the supremum norm is a Banach space.

(d) Let X be a Banach space and $f(x) = f(y)$, $\forall f \in X^*$, $x, y \in X$. Then show that $x = y$.

(e) Let X be an inner product space and $A, B \subseteq X$. Then show that $A \subseteq B \Rightarrow B^{\perp} \subseteq A^{\perp}$.

(f) Let $A \in BL(H)$ where H is a Hilbert space.

Show that there exist unique self-adjoint operators B and C such that $A = B + iC$.

2. (a) Let Y be a normed space and Y_0 be a dense subspace of Y. Suppose Z is a Banach space and $T \in B(Y_0, Z)$. Prove that there exists a unique $\tilde{T} \in B(Y, Z)$ such that $\tilde{T}/_{Y_0} = T$.

(b) In a finite dimensional normed linear space X, prove that any norm is equivalent to any other norm. 5+3

3. (a) Suppose $X = C^1[0, 1]$, the set of all functions $f : [0, 1] \to \mathbb{C}$ such that f' exists and is continuous. Let $Y = C[0, 1]$ and let X and Y be equipped with supremum norm. Define $A : X \to Y$ by $Af = f'$. Show that the graph of A is closed.

(b) Let $\{T_n\}$ be a sequence of bounded self adjoint linear operators $T_n : H \to H$ on a Hilbert space H. If $T_n \to T$ then show that T is bounded self adjoint linear operator on H.

4+4

4. (a) Assume that $\{u_\alpha\}_{\alpha \in I}$ is an orthonormal set in the inner product space X and $x \in X$. Let $E_x = \{u_\alpha : < x, u_\alpha > \neq 0\}$.

Then show that E_x is a countable set.

(b) If H is simply an inner product space, then show that $T \in BL(H, Y)$ may not have an adjoint. Here, Y is also an inner product space.

4+4

5. (a) Let $A \in BL(H)$ is self adjoint, where H is a Hilbert space. Then show that

$$\|A\| = \sup\{|< Ax, x >| : x \in H, \|x\| \leq 1\}.$$

(b) Let X and Y be inner product spaces. Then a linear map $F : X \to Y$ satisfies $< F(x), F(y) > = < x, y >$ for all $x, y \in X$ if and only if it satisfies $\|F(x)\| = \|x\|$ for all $x \in X$, where the norms on X and Y are induced by the respective inner products.

4+4

6. (a) Let $P \in BL(H)$ be a non-fero Projection on a Hilbert space H. Show that $\|P\| = 1$ implies that P is an orthogonal Projection.

(b) Let $A \in BL(H,Y)$ where H is a Hilbert space and Y is an inner Product space. Then show that the adjoint T^* of T is the unique mapping of Y into H. ALso, show that $T^* \in BL(Y,H)$. 4+4

[Internal Assessment : 10 Marks]