2018

M.Sc.

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-205

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*General Theory of Continuum Mechanics*)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *two* questions : 4×2

(a) The strain tensor at a point is given by

$$\left(E_{ij}\right)=\begin{pmatrix}5 & 3 & 0\\ 3 & 4 & -1\\ 0 & -1 & 2\end{pmatrix}$$

(Turn Over)

Determine the extension of the line element in the direction of $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$. what is the change of angle between two perpendicular line elements in the direction of $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ and $\left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$? 4

(b) Show that the equation of continuity between Eulerian and Lagrangian forms are equivalent. 4

(c) Define image. Find the image of a source with respect to a straight line. 4

2. What is the concept of stress vector ? Prove that the stress vector at a point on any arbitrary plane surface is a linear function of three stress vectors acting on any three mutually perpendicular planes through that point. 2+6

3. (a) State and prove the Cauchy's first equation of motion. When the continuum is in static equilibrium ? Deduce the equation of equilibrium. 5

(b) *In absence of body forces, do the following stress components*

$$T_{11} = \alpha\left[x_2^2 + \beta(x_1^2 - x_2^2)\right],\ T_{22} = \alpha\left[x_1^2 + \beta(x_2^2 - x_1^2)\right],$$

$$T_{33} = \alpha\beta(x_1^2 + x_2^2), T_{12} = -2\alpha\beta x_1 x_2 \text{ and } T_{23} = T_{31} = 0?$$ 3

4. Derive the equation of energy for a perfect fluid. 8

5. An infinite mass of fluid acted on by a force $\mu r^{-\frac{3}{2}}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere r = c in it, show that the cavity will be filled up after an interval of time

$$\left(\frac{2}{5\mu}\right)^{1/2} c^{5/4}.$$ 8

6. If the equations characterizing the deformation are

$$x_1 = X_1 + \in X_2$$

$$x_2 = X_2 - \in X_1 + \in X_3$$

$$x_3 = X_3 - \in X_2$$

determine the Lagrangian and Eulerian finite strain tensors.

8

7. Define strain deformation. For the deformation defined by the equations

$$X_1 = \frac{1}{2}(x_1^2 + x_2^2),\ X_2 = \tan^{-1}\left(\frac{x_2}{x_1}\right),\ X_3 = x_3,\ x_1 \neq 0,$$

find the deformation gradient tensors in spatial and materials forms. Hence show that the deformation is isochoric. 2+6

[Internal Assessment —10 Marks]
