2018

M.Sc.

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-203

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Unit-I

(Abstract Algebra)

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions : 2×2

(a) Show that a group of order $p^n q$ where p and q are primes with $p > q$, contains unique subgroup of index q.

(b) Show that the alternating group A_4 is not isomorphic to the Dihedral group D_{12} of all symmetries of a regular hexagon.

(c) Is quotient ring of an integral domain always an integral domain ? Justify your answer.

2. (a) Verify the class equation for the Permutation group S_3.

(b) If G is a group of order 30, show that either Sylow – 5 – subgroup of G is unique or there are six Sylow–5–subgroups; one Sylow–3–subgroup and three Sylow–2–subgroups. 4+4

3. (a) State and prove the second Isomorphism theorem of groups. 4+4

(b) Show that every group is Isomorphic to a subgroup of the group of all Permutations. 4+4

4. (a) Let R be a commutative ring with unity. An element of R is said to be nilpotent if $a^n = 0$ for some $n \in \mathbb{N}$. Show that the set of all nilpotent elements of R forms an ideal of R.

(b) Define Principal ideal domain with an example. Give an example of a Principal ideal domain which is not an Euclidean demain.

(c) Let R be the set of all real valued functions on [0, 1]. Define f + g, fg by (f + g) (r) = f(r) + g(r),

fg (r) = f(r) g(r), $r \in [0,1]$.

Let $I = \left\{ f \in R / f\left(\frac{1}{7}\right) = 0 \right\}$.

Show that I is a maximal ideal of R.

[Internal Assessment : 5 Marks]

Unit-II

(Linear Algebra)

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. Answer any *two* questions : 2×2

(a) Define complete lattice with an example. Also give an example which does not form the complete lattice.

(b) What is Jordan matrix ? Give an example.

(c) Let $V = M_2(R)$.

Define $T : V(R) \to R$ by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + b + c + d - 1$.

Is T a linear transformation ? Justify your answer.

2. (a) Prove that every chain is a lattice. Examine that the poset $D_{12} = \{2, 3, 4, 6\}$ under divisibility 12 forms a lattice or not. Show that in a poset a < a for no a and a < b, b <c implies a < c.

(b) Let $T : R^4 \to R^2$ be a linear transformation defined by

$$T\left((x, y, z, w)^t\right) = \begin{bmatrix} x - y + 2z \\ x + z + w \end{bmatrix}.$$

Find : (i) Nullity (T)

(ii) Rank (T)..

(iii) Verify that, Nullity (T) + Rank (T) = 4.

1+2+2+3

3. (a) Determine all possible Jordan canonical forms for a linear operator T whose characteristic polynomial is $(t\text{-}2)^5$ and minimal polynomial is $(t\text{-}2)^2$.

(b) Let T be a linear operator on a finite dimensional vector space V. Then prove that T is diagonalizable if the minimal polynomial of T is of the form $P(t) = (t - \lambda_1)(t - \lambda_2) \ldots\ldots (t - \lambda_k)$ where $\lambda_1, \lambda_2 \ldots\ldots \lambda_k$ are the distinct eigen values of T.

(c) A linear transformation $T : E^n \to E^1$ is non null i.e., $T(x) \neq 0, \forall x \in E^n$, where E^n is the n dimensional Euclidean space. Find the dimension of null space of T.

3+3+2

4. (a) Let $T : R^3 \to R$ be linear. Show that there exist scalars a, b and c such that $T(x, y, z) = ax + by + cz$, for all $(x, y, z) \in R^3$. Can you generalise this result for $T : F^n \to F$? Justify your answer.

(b) Let V and W be vector spaces of same dimension, and let $T : V \to W$ be linear. Then prove that the following are equivalent :

(i) T is one-to-one.

(ii) T is onto.

(iii) Rank (T) = Dim (V)

(c) Is there a linear transformation $T : R^3 \to R^2$ such that $T(1, 0, 3) = (1, 1)$ and $T(-2, 0, -6) = (2, 1)$? Justify your answer. 3+3+2

[Internal Assessment —5 Marks]
