$$\begin{aligned} \text{Maximize } & z = 2x_1 + 2x_2 \\ \text{subject to } & 5x_1 + 3x_2 \le 8 \\ & x_1 + 2x_2 \le 4 \\ & x_1, x_2 \ge 0 \\ & x_1, x_2 \text{ are integers.} \end{aligned}$$ 8

5. A manufacturing firm produces two types of products : A and B. The unit profit from product A is Rs. 100 and that of product B is Rs. 50. The goal of the firm is to earn a total profit of exactly Rs. 700 in the next week and also wants to achieve a sales volume for products A and B atleast 5 and 4 respectively. Then formulate this problem as a goal programming model and solve its graphically. 4 + 4

6. Solve the LPP

$$\begin{aligned} \text{Maximize } & z = 2x_1 - x_2 - x_3 \\ \text{subject to } & 4x_1 + x_2 - x_3 \ge 6 \\ & x_1 - x_2 + 3x_3 \le 10 \\ & x_1 + x_2 \le 4 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

using simple method. Determine the ranges for discrete changes of the components of b when changed one at a time, so as to maintain the optimality of the current optimum solution. 8

7. Using Fibonacci method maximize the function

$$f(x) = \begin{cases} x-1 & , \ 1 \le x \le 3 \\ 2(4-x), & 3 < x \le 4 \end{cases}$$

in the integral [1, 4] taking $n = 5$. 8

[*Internal Assessment* — 10 marks]

MTM-305 (OM)

(*Dynamical Oceanology*)

Answer **Q. No. 1** and any **four** from the rest

1. Answer any *four* questions : 2×4

(*a*) Suppose that we had a hand-basin with two taps, a waste pipe at the bottom and an overflow pipe near that top. With the taps full on (and a good flow of water from them) it is possible that the water level would rise in the basin. It may reach a final stage when the basin fills completely and water starts to drip over the edge. Express the continuity of volume of water in the basin.

(*b*) Combine the Coriolis terms acting on the horizontal plane into a single term and then calculate its magnitude at different latitude $\phi = 45^\circ$ and 30° for a current speed of $0{\cdot}15\ ms^{-1}$.

(*c*) Define concentration of a solution quantitatively in four possible ways.

(*d*) Describe shallow, intermediate and deep water waves.

(*e*) What is the reason for wave breaking ? State the condition of wave breaking for shallow and deep water waves.

(*f*) Explain "wave refraction".

2. (*a*) Derive the pressure term in the equation of motion for oceanography. 3

(*b*) Show contour plot of density as functions of temperature and salinity over ranges appropriate to most of the oceans. 2

(*c*) For an ocean with horizontal and vertical length scales 100 KM and 1000 M respectively, horizontal speed of order 0·1 m/s, time scale of 10 days and with an order of accuracy of 1%, write the simplified three momentum equations of motion for the interior of the ocean a few degree or more away from the equator. Then derive the following :

$$v\frac{\partial f}{\partial y} = f\frac{\partial w}{\partial z}.$$

where symbols have their usual meaning. 3

3. (*a*) With the help of geostrophic method, derive the following expression for calculating relative velocities.

$$V_1 - V_2 = \frac{1}{Lf}[\Delta\varphi_B - \Delta\varphi_A]$$

where symbols have their usual meaning. 5

(*b*) For a solution containing *i* number of components show that the sum of all the mole fractions is unity. 3

4. (*a*) Write all the Reynolds equations of motion in oceanography in component form. 3

(*b*) Considering the x-component of Reynolds equation, define molecular stress as well as the turbulent viscosities with the similarity of molecular stress. Are the Reynolds stress preserved symmetry ? Justify. 3

(*c*) Define the Rossby number and all the Ekman numbers involved in the z-component of Reynolds equation. 2

5. Prove that, the path of the particle describes an ellipse of a progressive wave in the surface of a cannel of finite depth. Also, show that a submarine whose depth is half a wave length would hardly notice the motion of a submarine due to surface waves. 4 + 4

6. Define kinetic and potential energy. Prove that the total energy of progressive wave is $\frac{1}{2}\rho g a^2 \lambda$ where a, λ are the wave amplitude and wave length respectively. 2 + 6

7. Explain, group velocity. Write an expression of group velocity for wave length (λ). Derive the equation for surface tension (T) of capillary waves as

$$\frac{\partial^2\phi}{\partial t^2} - g\frac{\partial\psi}{\partial x} + \frac{T}{\rho}\frac{\partial^3\psi}{\partial x^3} = 0,$$

where symbols have usual meaning. 2 + 1 + 5

[*Internal Assessment* — 10 marks]
