M.Sc. 3rd Semester Examination, 2018

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER – MTM-303 (OR & OM)

Full Marks : 50

Time : 2 hours

Answer Q. No. 1 and any **four** from the rest

The figures in the right-hand margin indicate marks

Paper – MTM-303 (OR)

1. Answer any *four* questions : 2 × 4

   (*a*) Describe the basic principle of dynamic programming method.

   (*b*) Write the Kuhn-Tucker necessary conditions for the problem :

   $$\text{Maximize } Z = f(x_1, x_2, \ldots\ldots, x_n)$$
   $$\text{Subject to } g(x_1, x_2, \ldots\ldots, x_n) \le C$$

(Turn Over)

(*c*) $f: R^n \to R^n$. Prove that $f(x)$ is convex if $f(x_2) - f(x_1) \geq [\nabla f(x_1)]^T(x_2 - x_1)$ for any two points $x_1, x_2 \in R^n$ where symbols are usual meaning.

(*d*) Write, some reasons for maintaining the inventories.

(*e*) Explain, replenishment and de-coupling inventories.

(*f*) What do you mean by the term "Post Optimality Analysis".

2. Describe dynamic programming method to solve the following problem :

Maximize $Z = f_1(y_1) + f_2(y_2) + \cdots + f_n(y_n)$
subject to $y_1 y_2 \cdots y_n \geq p, p > 0, y_j > 0$

for all j.

Using this method find the values of y_1, y_2, y_3 for the following problem :

Maximize $Z = y_1 + y_2 + y_3$
s.t. $y_1 y_2 y_3 \geq 27, y_1, y_2, y_3 \geq 0.$ 8

3. (*a*) Use Kuhn-Tucker conditions, solve the problem :

$$\text{Maximize } Z = 3x_1^2 + 14x_1x_2 - 8x_2^2$$
$$\text{Subject to } 3x_1 + 6x_2 \leq 72$$
$$x_1, x_2 \geq 0 \text{ and are integers.}$$ 3

(*b*) Write, the formulation of Beale's method for solving quadratic programming problem. 5

4. State the assumptions and notations for purchasing inventory model with no shortage. Then, derive the expression for economic lot size and find out the minimum cost per unit time. Hence, calculate the economic order quantity and minimum average cost for the problem : An engineering factory consumes 5000 units of a component per year. The ordering, receiving and handling costs are Rs. 300 per order while the trucking cost is Rs. 1200 per order, interest cost Rs. 0·06 per unit per year, deterioration and obsolence Rs. 0·004 per unit per year and storage cost Rs. 1000 per year for 5000 units. 2 + 3 + 3

5. Solve the problem using Gomory cutting plane method :

$$\begin{aligned} \text{Maximize } & Z = 7x_1 + 9x_2 \\ \text{Subject to } & -x_1 + 3x_2 \leq 6 \\ & 7x_1 + x_2 \leq 35 \\ & x_1, x_2 \geq 0 \text{ and are integers.} \end{aligned}$$ 8

6. Solve the following LPP by revised simplex method :

$$\begin{aligned} \text{Minimize } & Z = x_1 + x_2 \\ \text{subject to } & 2x_1 + x_2 \geq 4 \\ & x_1 + 7x_2 \geq 7 \\ & x_1, x_2 \geq 0. \end{aligned}$$ 8

7. What are the types of parameter changes that affect the optimal solution of LPP. Derive the range of the components of requirement vector (b) of the following LPP

$$\begin{aligned} \text{Maximize } & Z = CX \\ \text{subject to } & AX = b \\ & X \geq 0 \end{aligned}$$

such that after discrete changes of these components, the optimal basic feasible solution table does not altered. 2 + 6

[*Internal Assessment* : 10 Marks]

Paper – MTM-303 (OM)

(*Dynamical Oceanology*)

1. Answer any *four* questions : 2 × 4

(*a*) Define solvent and solute with example. Give one example for each of Gaseous solution, liquid solution and solid solution.

(*b*) State the basic laws of physics which are taken as axiomatic in developing the study of the dynamics of the ocean.

(*c*) Write all the equations of motion in oceanography in terms of eddy viscosities.

(*d*) Define relative and specific humidity.

(*e*) Show that the potential temperature of an air parcel is invariant.

(*f*) Derive the relationship $\vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = -\Omega^2 \vec{R}$, where $\vec{r}$ is the distance to the center of the earth, $\vec{R}$ is a vector perpendicular to the axis of rotation with magnitude to the distance to the axis of rotation of the earth and $\vec{\Omega}$ be the angular velocity of the earth.

2. Consider an ocean with horizontal and vertical length scales 500 KM and 500 M, respectively and horizontal speed of order 0·2 m/s.

(*a*) Calculate the vertical speed of the above situation. 2

(*b*) With the necessary assumptions, calculate values of eddy viscosities involved in the *y*-momentum equation. 2

(*c*) With the above values from parts-(*a*)-(*b*) and with a typical time scale 10 days, scale the *y*-momentum equation in oceanography involving eddy viscosities. 4

3. (*a*) Derive the equation of continuity of volume for compressible fluid. 2

(*b*) What types of frictionless flow are there in the ocean, and then define them. 2

(*c*) With necessary assumptions for the Inertial flow, reduce the horizontal equation of motion in to simplified form and show that the motion is described by a circle. 4

4. Derive the Reynolds equation for the z-component of velocity. 8

5. (*a*) Derive the expression of the pressure gradient force in the atmosphere. 4

(*b*) Derive an expression for the density ρ of an air parcel at pressure p if it is adiabatically expands from a level where pressure and density are p_s and ρ_s respectively. 4

6. (*a*) Deduce the equation of state for moist air in the atmosphere in the following form

$$p\alpha = R_d T\left[1-(1-\epsilon)\frac{e}{p}\right]. \qquad 5$$

(*b*) Derive the geostrophic wind equation in the atmosphere. 2

(*c*) What is isentropic process ? 1

7. (*a*) Obtain the atmospheric energy equation and interpret each term. 6

(*b*) Derive the relation between pressure and height in the atmosphere, when the environment lapse rate of temperature is constant. 2

[*Internal Assessment* : 10 Marks]