M.Sc. 3rd Semester Examination, 2018

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(*Transforms and Integral Equations*)

PAPER —MTM-302

Full Marks : 50

Time : 2 hours

Answer Q.No. 1 and any three from the rest

The figures in the right-hand margin indicate marks

1. Answer any *five* questions : 2 × 5

(*a*) If * is the convolution operator concerning on Laplace transform, then show the operator * is commutative.

(*b*) For each of the following functions, determine

(Turn Over)

which has a Laplace transform. If it exists, find it; if does not satisfy, why ?

(*i*) $e^{\frac{1}{t}}$

(*ii*) e^{t^2}

(*c*) Define Fourier transform and state the conditions of existence of the transform.

(*d*) Define eigen value and eigen vector in terms of an integral equation.

(*e*) Define singular integral equation with an example.

(*f*) Verify the initial value theorem in connection with Laplace transform for the function $(4 + t)^2$.

(*g*) Define the wavelet function and analyze the parameters involving in it.

(*h*) State Bromwich's integral formula concerning on inverse Laplace transform.

2. (*a*) Form an integral equation corresponding to the differential equation

$$\frac{d^2y}{dx^2} - \sin(x)\frac{dy}{dx} + e^x y = x,$$

with the conditions $y(0) = 1, y'(0) = -1$ and find its kernel. 5

(*b*) Find the Laplace transform of the triangular wave function $f(t)$ which is defined as follows :

$$f(t) = \begin{cases} t, & \text{if } 0 \le t < c \\ 2c - t, & \text{if } c \le t < 2c \end{cases}$$

where $f(t + 2c) = f(t)$. 3

(*c*) Find the exponential Fourier transform of

$$f(t) = \begin{cases} 1 - |t|, & |t| < 1 \\ 0, & |t| > 1 \end{cases}.$$ 2

3. (*a*) Solve the fillowing integral equation

$$y(x) = f(x) + \lambda \int_{-1}^{1} (x + t) y(t) dt$$

and find the eigen values. 5

(*b*) Evaluate

$$L\left\{\int_0^t \frac{\sin u}{u} du\right\}$$

by the help of initial value theorem. 5

4. (*a*) Solve the following boundary value problem in the half plane $y > 0$, described by PDE :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \ -\infty < x < \infty, \ y > 0,$$

with boundary conditions $u(x, 0) = f(x)$, $-\infty < x < \infty$. u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 7

(*b*) Find the value of $\sin(t) * t^2$ where * denotes the convolution operator on Laplace transform. 3

5. (*a*) State and prove convolution theorem on Laplace transform. 5

(*b*) Solve the integral equation

$$\varphi(x) = \int_0^x \frac{1}{(x-t)^\alpha} y(t)dt, \; 0 < \alpha < 1.$$ 5

6. (*a*) Use the Laplace transformation technique to solve the differential equation :

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 2t - e^{-t}, t > 0$$

which satisfies $x(0) = \frac{1}{2}, \; \frac{dx}{dt} = 0,$ at $t > 0$. 5

(*b*) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform. 5

[*Internal Assessment* : 10 Marks]