PG/IIIS/MTM-301/18

M.Sc. 3rd Semester Examination, 2018

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

(Partial Differential Equations and Generalized Functions)

PAPER – MTM-301

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **two** from the rest

The figures in the right-hand margin indicate marks

1. Answer any *two* of the following questions : 4×2

(*a*) Define distribution on an open subset X of $\mathbb{R}^n$ with an example. Using the differentiation of distribution, show that $H' = \delta$.

(*b*) Solve the following : $p^2 + q^2 = u^2$, $u(x, 0) = 1$.

(*c*) Give an example of a second order partial differential equation (PDE) which is not well-posed. Justify your answer.

2. (*a*) Reduce the following equation to a canonical form and hence solve it :

$$3u_{xx} + 10u_{xy} + 3u_{yy} = 0. \qquad 8$$

(*b*) Find the solution of

$$(D^2D' + D'^2 - 2)z = e^{2y}\cos 3x + e^{x}\sin 2y$$

where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$. 5

(*c*) Solve : $2(z + px + qy) = p^2y$. 3

3. (*a*) Show that the following Cauchy problem

$$u_{tt} - c^2u_{xx} = F(x, t), -\infty < x < \infty, t > 0$$
$$u(x, 0) = f(x), u_t(x, 0) = g(x), -\infty < x < \infty$$

is well-posed for $-\infty < x < \infty, 0 \le t \le \tau$ where $\tau > 0$ is fixed; $F, f_x \in C(\mathbb{R}^2), f \in C^2(\mathbb{R})$ and $g \in C^1(\mathbb{R})$. 6

(*Continued*)

(*b*) A pressure wave generated as a result of an explosion satisfies the equation

$$P_{tt} - 16P_{xx} = 0, -\infty < x < \infty, t > 0,$$

where $P(x, t)$ is the pressure at the point x at time t. Suppose a building is located at the point $x_0 = 10$. The engineer who designed the building determined that it will sustain a pressure up to $P = 5$. Find the time t_0 when the pressure at the building is maximal. Will the building collapse ? 8

(*c*) Find the parallelogram identity for the wave equation when the wave speed $c \neq 1$. 2

4. (*a*) Establish the Poisson's formula for the Dirichlet Problem of the Laplace equation in a disk. 5

(*b*) State and prove the mean value principle for the harmonic function. 5

(*c*) (*i*) What is the Laplace transform of Dirac Delta function ?

(*ii*) Find the adjoint of the following PDE :

$$u_{xx} + 4u_{xy} + u_x = 0.$$

(*iii*) Define dirac delta function. 6

[*Internal Assessment* : 10 Marks]
