2018

M.Sc.

4th Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-404 (OR/OM)

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

MTM-404 (OR)

(Non-linear Optimization)

Answer Q. No. *1* and any *three* from the rest.

1. Answer any *five* questions : 5×2

(a) Let X° be an open set in R^n, let θ and g be defined on X°. Find the conditions under which a solution $(\bar{x}, \bar{r}_0, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely.

(b) Define posynomial and polynomial in connection with geometric programming with an example.

(c) What is the "Theorem of Alternatives" in connection with non-linear programming ?

(d) Define bi-matrix game with an example.

(e) State Dorn's duality theorem in connection with duality in quadratic programming.

(f) What is the necessity of constraint qualification related with non-linear programming ?

(g) Write the basic difference(s) between Beale's and Wolfe's method for solving quadratic programming problem.

(h) Write the advantages of geometric programming.

2. (a) Solve the quadratic programming problem using Wolfe's method

Maximize $z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

subject to $x_1 + 2x_2 \leq 10,$

$x_1 + x_2 \leq 9,$

$x_1, x_2 \geq 0.$

(b) State and prove Weak duality theorem in connection with duality in non-linear programming. 7+3

3. (a) Minimize the following using geometric programming

$f(x) = 16x_1x_2x_3 + 4x_1x_2^{-1} + 2x_2x_3^{-2} + 8x_1^{-3}x_2$

$x_1, x_2, x_3 > 0.$

(b) State and prove Motzkin's theorem of alternative. 6+4

4. (a) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem.

(i) Complete optimal solution (ii) Pareto optimal solution

(iii) Local Pareto optimal solution (iv) Weak Pareto optimal solution.

(b) Give the geometrical interpretations of differentiable convex function and concave function. 2+4+4

5. (a) State and prove Fritz-John saddle-point necessary optimality theorem.

(b) Use the chance constrained programming to find an equivalent deterministic problem to following stochastic programming problem, when c_j's are random variables

$$\text{Minimize } F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to } \sum_{j=1}^{n} a_{ij}\, x_j \le b_i$$

$$x_i \ge 0, i, j = 1, 2, \ldots, n$$

6+4

6. (a) Prove that a pair { y*, z* } constitutes a mixed strategy Nash equilibrium solution to a bimatrix game (A, B) if and only if, there exists a pair {p*, q*} such that {y*, z*, p*, q*} is a solution of the following bilinear programming problem:

Minimize $\left[y'Az + y'Bz + p + q\right]$

subject to $Az \ge -pl_m$

$B'y \ge -ql_n$

$y \ge 0, z \ge 0, y'l_m = 1, z'l_n = 1.$

(b) Define the following :

(i) Kuhn-tucker stationary point problem;

(ii) Fritz-john stationary point problem. 6+4

[Internal Assessment : 10 Marks]

MTM-404 (OM)

(Dynamical Oceanology-II)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions : 4×2

(a) Calculate the circulation within a small fluid element with area $\delta x \delta y$.

(b) Calculate the inertial radius for inertial waves for a typical velocity 20 cm/s with a standard Coriolis parameter's value, and Rossby radius of deformation for a deep ocean with height = 3000 m and a shallow ocean with height = 100m.

(c) Define the vertical velocity in isoberic coordinates and discuss the physical significance for its positive and negative values.

(d) Derive the depth-integrated continuity equation.

(e) Write the thermodynamic equation in isobaric coordinate systems. Also explain physical interpretation of each term.

(f) Derive the ageostrophic continuity equation.

2. Derive the Klein-Gordon equation with necessary assumptions. 8

3. For the initial surface elevation

$$\eta_0 = \begin{cases} h \text{ for } y > 0 \\ 0 \text{ for } y < 0 \end{cases}$$

and with the assumption that the motion is independent of x-coordinate, find the stationary surface elevation and the geostrophic velocity distribution. 8

4. (a) Write the depth-averaged 2D shallow water equations and then derive the equation for relative vorticity.

(b) For a Couette flow over a plane wall with zero incident, find the relative vorticity. 5+3

5. (a) Write the kinematic and dynamic boundary conditions on both the surface (free and bottom) of shallow water theory and hence derive

$$\eta_t = -\frac{\partial}{\partial x}\int_{-H}^{\eta} u\,dz - \frac{\partial}{\partial y}\int_{-H}^{\eta} v\,dz.$$

(b) Also derive the hydrostatic equation for this shallow water theory. 6+2

6. (a) Write the x- and y-momentum equations in rectangular coordinate systems and then convert these equations to their isobaric coordinate systems.

(b) What are the physical interpretation of each terms of both the equations ?

7. (a) Write the governing equations for equatorial Kelvin waves and then find the surface elevation and velocity of this wave.

(b) Derive the equation for geostrophic relative vorticity and discuss the physical meaning of each term of this equation. 4+4

[Internal Assessment : 10 Marks]
