2018

M.Sc.

2nd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-201

Subject Code—21

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(*Fluid Mechanics*)

Answer Q. No. *1* and any *four* from the rest.

1. Answer any *four* questions : 4×2

(a) Write the main three branches of studying of fluid dynamics with a short description on computation study.

(b) What are the differences between laminar and turbulent flows ?

(c) Write the continuity equation of incompressible flow in spherical and cylindrical polar coordinates systems.

(d) What are the source of forces for the Newton's second law applied to a infinitesimally small fluid element ?

(e) Discuss the boundary conditions for both tangential and normal components of velocity of a fluid element.

(f) If a velocity potential exists for the velocity field of the form :

$u = a(x^2 - y^2)$, $v = -2axy$, $w = 0$, then find the velocity potential.

2. (a) Derive the continuity equation for the model of an infinitesimally small element fixed in space.

(b) Write all the four forms of the continuity equation: Integral-Conservation, Integral-Nonconservation, Differential-Conservation and Differential-Nonconservation. 5+3

3. (a) What are the observations of Ludwig Prandtl for the formulation of boundary layer theory ?

   (b) Based on the above observation, derive the set of governing equations for the boundary layer flow along a flat plate. 2+6

4. (a) Draw infinitesimally small moving fluid element and show the forces in the x-direction for derivation of the x-component of the Navier-Stokes equation.

   (b) Write the x-component of the Navier-Stokes equations in nonconservation form. Convert this equation to its conservation form.

   (c) With the help of Stokes hypothesis, convert the equation resulted from part-(b) for incompressible flow case into its simplified form. 3+2+3

5. (a) What do you mean by analytical/exact solution of Navier-Stokes Equation ?

   (b) With the necessary assumptions, find the exact solution for the case of Couette flow.

   (c) Show the velocity profile graphically for the above Couette flow. 2+5+1

6. (a) Find the necessary and sufficient conditions that vortex lines may be at right angle to the streamlines.

(b) Let a uniform flow at speed U in the x-direction with complex potential $w(z) = Uz$. If an impermeable circular obstacle at $|z| = a$ say, is inserted then calculate the disturbed flow. 4+4

7. (a) Let us suppose an infinite number of vortices each of strength k placed at points $z = 0, \pm a, \pm 2a, \pm 3a, \ldots\ldots$ Find the complex potential at any point z.

(b) Show that for the above situation the vortex at the origin is at rest. 5+3

[Internal Assessment —10 Marks]