2017

M.Sc. Part-I Examination

PHYSICS

PAPER—V

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answerscripts for Gr. A & Gr. B.

Group—A

[Marks—35]

Attempt Q. No. *1, 2, 3* and any *two* from the rest.

1. Answer any *five* questions : 5×2

(a) Find the second order forward difference of $f(x) = \cos 2x$.

(Turn Over)

(b) Distinguish between PAM and RAM..

(c) What (i) computed GOTO statement (ii) Assign GOTO statement ?

(d) To point (i) -14.7145 (ii) $-0.589178 \times 10^{-14}$ use suitable Format in FORTRAN77.

(e) What is the syntax of reading complex variables.

2. Answer any *two* questions : 2×3

(a)
$$f(x) = x^2 + \sin 2x \quad \text{if } x < 3$$
$$= 10.3 \quad \text{if } x = 3$$
$$= x^3 - \cos 3x \quad \text{if } x > 3$$

Write a function subprogram to find $f(x)$ in Fortran.

(b) Write a Fortran program to read n values x_1, x_2, ..., -- x_n and evaluate $\sum_{i=1}^{n} f(x_i)$

(c) Write a Fortran program to print the following sequence of numbers.

1 2 2 3 3 3 4 4 4 4 5 5 5 5 5

(d) Write a Fortran program to find the mean and standard deviation of n numbers.

(e) Write a Fortran program to check whether a matrix is upper triangular or not ?

3. Answer any *one* question : 1×4

(a) Apply Runge-Kutta method (fourth order) to find an approximate value of y when $x = 0.2$.

(Given $\frac{dy}{dx} = x + y^2, y(0) = 1$).

(b) Describe least square method to find fitting the parabola $y = a + bx + cx^2$.

(c) Find the largest eigen value of the following matrix

$$A = \begin{pmatrix} 3 & 4 & -1 \\ 1 & 2 & 0 \\ 2 & 5 & 1 \end{pmatrix} \text{ with initial vector } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

(d) Deduce simpson $\frac{1}{3}$ rule of integration.

(e) A particle is moving along a straight line. The displacement x at some time instantce t are given below :

t	0	1	2	3	4	
x	5	8	12	17	26	

Find the velocity and acceleration of the particle at $t = 4$.

(f) Deduce Lagrange's method for interpolation.

Group—B

[Marks—40]

Attempt Q. No. *1*, *2* and any *two* from the rest.

1. Answer any *five* questions : 5×2

(a) Show that $(1+x+iy)^4(7-x-iy)^3$ is an analytic fucntion of the complex variable $z = x + iy$ in the domain $|x|<2$.

(b) Find the value of the residue of $\dfrac{\sin z}{z^6}$.

(c) Find the fourier transform of

$$f(x) = e^{-a|x|}(-\infty < x < \infty); (a > 0)$$

(d) The laplace transfrom of $f(t) = \sin \pi t$ is $F(s) = \dfrac{\pi}{s^2+\pi^2}, s > 0$. Find the laplace transfrom of $t \sin \pi t$.

(e) Show that $\sum_n p_n(x) = \dfrac{1}{\sqrt{2-2x}}$ where $p_n(x)$ is the Lagendre polynomial.

(f) Show that $\sqrt{\dfrac{\pi x}{2}} J_{3/2}(x) = \dfrac{\sin x}{x} - \cos x$.

2. (a) Find the normalized eigen vector a of the matix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, corresponidng to its positive eigen value.

The normalized eigenvectors of the matrix

$B = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ are β_1 and β_2 with the eigen values λ_1 and λ_2 respectively and $\lambda_1 > \lambda_2$. If the eign vector α is expressed as $\alpha = P\beta_1 + Q\beta_2$. Find the constants P and Q. 2+3

(b) Evaluate $\int_0^\infty \frac{\ln x}{(1+x^2)} dx$ by using Cauchy's residue theorem. 5

3. (a) Prove that $H_n(x) = 2^n \exp\left\{-\frac{1}{4}\frac{d^2}{dx^2}\right\} x^n$ where $H_n(x)$ is the Hermite polynomial. 5

(b) Prove that $xL_n^{'(x)} = nL_n^{(x)} - nL_{n-1}^{(x)}$ where $L_n^{(x)}$ is the Laguerre polynomial. 5

4. (a) The character table of c_{3v}, the group of symmetries of an equilateral triangle is given below :

	$(x)^0$	$x^{(1)}$	$x^{(i)}$
$1C_1$	1	1	b
$1C_2$	1	a	c
$2C_3$	1	1	d

where C_1, C_2, C_3 denotes the three clases of c_{3v}, containing 1, 3, 2 elements respectively $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(3)}$ are the characters of three irreducible representives $\Gamma^{(0)}$, $\Gamma^{(1)}$ and $\Gamma^{(2)}$ of c_{3v}. Find a, b, c and d. Find also the reducible representation Γ of c_{3v} with character $\chi = (4,0,1)$ in terms of decomposition $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$. 5

(b) Consider a vector $\vec{p} = \hat{i} + 2\hat{j} + \hat{k}$. The axes are rotated in the clockwise direction about the z-axis by an angle 45°. Find the vector $\vec{p}$.

If $\hat{T}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$. Find the transformation matrix $\hat{T}$. 2+3

5. (a) Find the Fourier transform of $e^{-\frac{r^2}{a^2}}$ where a is a constant and $r^2 = x^2 + y^2 + z^2$.

(b) Show that $\bar{L}^{1}\left[\ln\left(1+\frac{w^2}{s^2}\right)\right] = \frac{2}{t}(1-\cos wt)$. 5+5

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