2017

M.Sc. Part-II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—IX (OR/OM)

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Special Paper : OR

Answer Q. No. 11 and any *six* from the rest.

1. (a) Solve the quadratic programming problem using Wolfe's method

Maximize $z = 2x_1 + 3x_2 - 2x_1^2$

subject to $x_1 + 4x_2 \le 4,$ 7

$x_1 + x_2 \le 2,$

and $x_1, x_2 \ge 0.$

(Turn Over)

(b) State and prove Slater's theorem of alternative. 5

(c) State and prove weak duality theorem in connection with dulity in non-linear programming.

2. (a) State and prove Wolfe's duality theorem. 5

(b) What do you mean by quadratic programming problem ? Derive the Kuhn-Tucker conditions for quadratic programming problem. Under what conditions, the above Kuhn-Tucker condition will be necessary and sufficient. 2+5+1

(c) What is differential convex function ? Give the geometrical interpretation of it. 3

3. (a) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. θ is concave if and only if

$\theta(x^2) - \theta(x^1) \leq \theta(x^1)(x^2 - x^1)$ for which $x^1, x^2 \in \Gamma$

Give the geometrical interpretation of the above result. 6+2

(b) Write the relationship among the solutions of local minimization problem (LMP), the minimization problem, (MP), the Fritz-John stationary problem (FJP), the Fritz-John saddle point problems (FJSP), the Kuhn-Tucker stationary point problem (KTP) and the Kuhn-Tucker saddle point problem (KTSP). 5

(c) What is decomposition principle of Dantzig and Wolfe ? What is the advantages of decomposition principle ? 3

4. (a) Define the following terms :

(i) The (primal) quadratic minimization problem (QMP)

(ii) The quadratic dual (maximization) problem (QDP). 3

(b) Solve the following problem by Beale's method

Maximize $Z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

subject to $x_1 + 2x_2 \leq 0$ 8

$x_1, x_2 \geq 0$

(c) State and proof Kuhn-Tucker saddle point necessary optimality theorem. 5

5. (a) State and Prove Tucker's lemma on non-linear programming. 8

(b) Use the artificial constraint method to find the initial basic solution of the following problem and then apply the dual simple algorithm to solve it.

Maximize $Z = x_1 - 3x_2 - 2x_3$

subject to $x_2 - 2x_3 \geq 2$ 8

$$x_1 - 4x_2 - 6x_3 = 8$$
$$2x_2 + x_3 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

6. (a) Describe cutting plane method for multi-variable non-linear optimization with constratints.

(b) Solve the following IPP using branch and bound method :

Maximize $z = 2x_1 + 3x_2$

subject to the constraints

$$6x_1 + 5x_2 \leq 25$$
$$x_1 + 3x_2 \leq 10$$

$x_1, x_2 \geq 0$ and integers. 8

7. (a) Use revised simplex method to solve the following LPP : 8

Maximize $z = 5x_1 + 4x_2$

subject to $6x_1 + 4x_2 \leq 23$

$$x_1 + 2x_2 \leq 6$$
$$-x_1 + x_2 \leq 1$$
$$x_2 \leq 2 \quad \text{and} \quad x_1, x_2 \geq 0$$

(b) Using steepest descent method

Minimize $f(x_1, x_2) = 2x_1 - x_2 + 8x_1^2 + 4x_1x_2 + x_2^2$

starting from the point $(0,1)$. 8

8. (a) Using Golden section method minimize the function

$$f(x) = \begin{cases} (x^2 - 6x + 13)/4, & x \leq 4 \\ x - 2, & x > 4 \end{cases}$$

in the interval [3, 5] upto six experiments. 8

(b) Define goal programming problem. A small furnishing company manufactures tables and chairs. Each chair requires 4 man-hours of labour while each table requires 5 man-hours of labour. If only 80 man-hours are availables each week and both table and chair fetch a profit of Rs. 100 each. Formulate the linear goal programming problem such that management's objectives are (i) the owner of the company would neither hire additional labour nor utilize overtime (ii) the owner has a target to earn a profit of Rs. 2000 per week (iii) he also would like to supply10 chairs, if possible, per week to a sister concern. 8

9. (a) The optimal result of the LPP

Maximize $z = 2x_1 + 2x_2$

subject to $5x_1 + 3x_2 \le 8$

$x_1 + 2x_2 \le 4$

and $x_1, x_2 \ge 0$

is given in the following table

C_B	X_B	b	y_1	y_2	y_3	y_4
2	x_1	4/7	1	0	2/7	−3/7
2	x_2	12/7	0	1	−1/7	5/7
$z_j - c_j$			0	0	2/7	4/7

Find the optional results after addition of the following constraints :

(i) $2x_1 + x_2 \le 3$

(ii) $x_1 + x_2 \le 2$ 2+6

(b) Define unimodel manimization and minimization function. Write down the steps of Golden section search method of a unimodal maximum function of single variable. 2+6

10. (a) What do you mean by "post optimality analysis"? Derive the ranges of descrete changes of requirement vector (b) of the LPP

Maximum $z = Cx$

Subject to $Ax = b$, $x_2 \ge 2$

So that the feasibility of the optimal basic feasible soultion remains unchange. 8

(b) Solve the following IPP using Gomory's cutting plane method

Maximize $z = x_1 + x_2$

subject to $3x_1 + 2x_2 \le 5$ 8

$x_2 \le 2$

$x_1, x_2 \ge 0$ and are integers

11. Answer any *one* : 4×1

(a) Write a short note in constraint Qualification in connection with non-linear Programming. 4

(b) If $f(x)$ is a quadratic function, then show that the minimum point can be obtained in a single step by Newton's method. 4

Special Paper : OM

Answer Q. No. *1* and any *six* from the rest.

1. Answer any *two* questions : 2×2

(a) Write the substantial derivative of temperature in cartesian coordinate and state the physical meaning of each term.

(b) What are the source of forces for the Navier-Stokes equation? Write down all of them.

(c) Define the Rossby number and discuss its physical interpretation for the case of higher value.

2. (a) What is the physical principal considered for energy equation?

(b) Draw an infinitesimal small fluid element and show the energy fluxes along x- direction associated with the element.

(c) Derive the energy equation in non-conservation form. 2+6+8

3. Derive the Navier-Stokes equation in non-conservation form and then reduce it to its conservation form. 12+4

4. (a) Write the substantial derivative of temperature in isobaric coordinate.

(b) Convert the continuity, momentum and thermodynamic equations from cartesian to isobaric coordinate system.

(c) State the physical interpretation of each terms of *y*-momentum equation derived in part-(b) 2+12+2

5. (a) Write the set of governing equations for the case of linear waves (in shallow- water) in the absence of rotation.

(b) For the above case, find the expression for horizontal velocity in terms of initial surface elevation $F(x)$. 4+12

6. (a) Assuming that the mass exchange process across the free ocean surface $F(r,t)=0$ amount to a flux b of pure water in unit time per unit area, obtain the boundary conditions at the free ocean surface.

(b) Assuming that sea-water is a two component mixture of salt and pure water, show that the principal of conservation of mass leads to the pair of equations

$$\frac{D\rho}{Dt} + \rho\, div\, \vec{q} \quad \text{and} \quad \frac{\rho Ds}{Dt} = -div\, \vec{I_S}$$

where symbols have their usual meaning.

7. Assuming two-dimensional model of ocean currents, obtain the solution for a problem of boundary layer flow

and show that a weak back appears close to the external edge of the boundary western shore. 16

8. Derive the following relations :

(i) $C_v = C_p + \tau \dfrac{\left(\dfrac{\partial \tau}{\partial T}\right)^2}{\left(\dfrac{\partial \tau}{\partial p}\right)}$

(ii) $K_n = K_T - \Gamma\alpha$

(b) Explain the Brunt-*Väisälä* frequency. Express it in term of *C*. Where symbols have their usual meaning. (4+4)+(5+3)

9. Derive the governing equations of motion of seawater uner β-plane approximations stated by you. 16

10. Derive verticity equation in the form of

$$\frac{D\vec{\xi}_a}{Dt} = \left(\vec{\xi}_a \cdot \vec{\nabla}\right)\vec{\upsilon} - \left(\vec{\nabla} \cdot \vec{\upsilon}\right)\vec{\xi}_a - \vec{\nabla}\alpha \times \vec{\nabla} p + \gamma \nabla^2 \vec{\xi}_a$$

Write the physical interpretation of each term of the above equation. 12+4
