2017

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VIII

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

Answer Q. No. *1* is compulsory and any *three* from the rest.

1. Answer any *two* of the following : 2×4

(a) (i) State Fredholm Alternative concerning on integral equation. 2

(ii) Define finite Hankel transform of order n of a function $f(r)$, $0 \le r \le a$ and state its inversion formula. 2

(b) (i) Define Fourier sine and cosine transform of a function $f(x)$. 1+1

(ii) If $F_c(\alpha)$ is the Fourier cosine transform of $f(x)$, then show that the Fourier cosine transform of $f(x/a)$ is $F_c(a\alpha)$, whose a is a constant. 2

(c) (i) Find the Laplace transform of $f(x) = [x]$, where $[x]$ represents the greatest integer less than or equal to x. 2

(ii) When a function $f(x)$ is said to be of exponential order $O(e^{ax})$ for $x > 0$? If $f(x)$ is a exponential order, what extra condition is needed for the existence of its Laplace transform ? 2

2. (a) Solve the following boundary value problem in the half plane $y > 0$, described by PDE : $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, $-\infty < x < \infty, y > 0$, with boundary conditions $u(x,0) = f(x)$, $-\infty < x < \infty$. u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. 8

(b) State and prove Bromwich's integral formula on Laploace transform. 6

3. (a) Find the first order Hankel transform of $f(r)=\frac{1}{r}e^{-ar}$. 4

(b) Give an example to show that the integral of a good function is not necessarily a good function. 3

(c) Using Green's function method, solve the following differential equation $\frac{d^2y}{dx^2}+y=x^2$; $y(0) = y(\pi/2) = 0$. 7

4. (a) Form an integral equation corresponding to the following differential equation

$$\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}+e^{-x}y(x)=x^3-5x$$

with the given initial conditions, $y(0) = -3$ and $y'(0) = 4$ 7

(b) Define the following terms: Fairly good function, Regular Sequence, Generalized function. 3

(c) Show that zero order Hankel transform of e^{-ar} is

$$\frac{a}{(a^2+\alpha^2)^{3/2}}, a > 0$$ 4

5. (a) If a and b are real constants, solve the following integral equation $ax + bx^2 = \int_0^x \frac{y(t)}{(x-t)^{1/2}} dt.$ 6

(b) If a real valued function $f(t)$ of real variable which is piecewise continuous in any finite interval of t and is exponential ofer $O(e^{vt})$ as $t \to \infty$, when $t \geq 0$ then prove that the integral $\int_0^\infty f(t)e^{-pt} dt$, converges in the domain Real $(p) > v$. 5

(c) Find Fourier transform of $e^{-\frac{x^2}{2}}$ and find the function whose Fourier transform is $e^{-\frac{\alpha^2}{2}}$. 3

6. (a) Find the characteristics values and characteristics functions of the Sturm-Liouville problem

$$(x^3 y')' + \lambda xy = 0;\ y(1) = 0,\ y(e) = 0.$$

(b) With the help of a resolvent Kernel, solve the Volterra integral equation

$$y(x) = x3^x - \int_0^x 3^{x-t} y(t) dt$$ 6

(c) State and prove Parseval's theorem on Hankel Transform. 2

Group—B (OR)

(Elements of Optimization and Operations Research)

[Marks : 50]

Answer Q. No. 7 and any *three* from the rest.

7. Define lead time and demand in connection with inventory model. 2

Or

State the necessary and sufficient conditions for optimum point of a multi-variable optimization problem. 2

8. (a) Discuss the steps of revised simplex method to determine an optimal solution of a LPP. What are the advantages of it? 6+2

(b) Solve the following IPP using branch and bound method : 8

Maximize $Z = 3x_1 + 4x_2$

subject to constraints $4x_1 + 6x_2 \leq 15$

$x_1, x_2 \geq 0$ and integers.

9. (a) A company producing three items has limited storage space of avaragely 750 items of all types. Determine the optimal production quantities for each item separately, when the following information is given : 8

Product	1	2	3
Holding cost (Rs.)	0.05	0.02	0.04
Set-up cost (Rs.)	50	40	60
Demand rate	100	200	75

(b) Briefly describe Beale's algorithm for solving a quadratic programing problem. 8

10. (a) The optimal result of the LPP

Maximise $z = 2x_1 + 2x_2$

subject to $5x_1 + 3x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

and $x_1, x_2 \geq 0$

is given in the following table :

C_B	K_B	b	y_1	y_2	y_3	y_4
2	x_1	4/7	1	0	2/7	−3/7
2	x_2	12/7	0	1	−1/7	5/7
$z_j - c_j$			0	0	2/7	4/7

Find the optimal results after changes of cost vector (C = [2 2]) to the cost vectors (i) C = [4 2], (ii) C = [2 1] 4+4

(b) Solve the following LPP by the method of dynamic programming : 8

Maximize $Z = 2x_1 + 5x_2$

subject to $2x_1 + x_2 \leq 430$

$2x_2 \leq 460$

and $x_1, x_2 \geq 0$

11. (a) Determine x_1 and x_2 such that maximize the following non-linear optimization problem :

Maximize $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$

subject to the constraints $x_2 \leq 8$ 8

$x_1 + x_2 \leq 10$

and $x_1, x_2 \geq 0$

(b) Write down the expression for optimum oder quantity of a purchasing inventory model with fully backlogged shortages. Hence find the optimum order quantity when the following situations occur :

(i) Shortage cost is negligible.

(ii) inventory carrying cost is negligible.

(iii) inventory carrying cost and shortage cost are equal.

12. (a) Solve the following LPP be revised simplex method

Maximize $z = x_1 + 2x_2$

subject $x_1 + x_2 \le 3,\ x_1 + 2x_2 \le 5$

$2x_1 + x_2 \le 6,\ x_1, x_2 \ge 0$ 8

(b) Apply Wolfe's method to solve the quadratic programming problem :

Maximize $z = 2x_1 + x_2 - x_1^2$ 8

subject to $2x_1 + 3x_2 \le 6$

$2x_1 + x_2 \le 4$, and $x_1, x_2 \ge 0$

Group—B (OM)

(Dynamical Oceanology and Methodology)

[Marks : 50]

Answer Q. No. *12* and any *three* from the rest.

7. (a) Deduce the equation of state for moist air in the atmosphere in the following form $p\alpha = R_d \dfrac{T}{\left[1-(1-\epsilon)\frac{e}{p}\right]}$.

(b) Discuss the different cases of pressure changes in the atmosphere with respect to altitude.

(c) Define homogeneous atmosphere. Show that the height of the homogeneous atmosphere depends entirely on the temperature at the bottom. Also prove that the pressure at the top of the homogeneous iosthermal is equal to $\frac{1}{e}$ times that at the sea level.

5+5+6

8. (a) What is thermal wind in the atmosphere ? Derive the equation of thermal wind.

(b) Derive an expression for the density ρ of an air parcel at pressure p if it is adiabatically expands from a level where pressure and density are p_s and ρ_s respectively.

(c) Derive the expression of the pressure gradient force in the atmosphere. 8+4+4

9. (a) Derive the momentum equation of motion of an air parcel in the atmosphere in the cartesian co-ordinate system.

(b) Derive the hypsometric equation in the atmosphere.

(c) Derive the adiabatic lapse rate of unsaturated moist air. 8+4+4

10. (a) Prove that i) $C_v = C_P + T\dfrac{\left(\dfrac{\partial \tau}{\partial T}\right)^2}{\left(\dfrac{\partial \tau}{\partial P}\right)}$,

ii) $K_\eta = K_T - \Gamma\alpha$.

(Symbols have their usual meanings)

(b) **Derive the boundary conditions at the free ocean surface $F(\vec{r},t)=0$.** (4+4)+8

11. Derive governing equations of motion of sea-water under Boussinesq approximations stated by you. 16

12. What is geodynamic paradox?

Or

Express adiabatic temperature gradient in terms of T, C_p, s variables. 2
