2017

M.Sc. Part–II Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—VII

Full Marks : 100

Time : 4 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

[Marks : 25]

Answer Q. No. *1* and any *two* from the rest.

1. What is the difference between conduction current and diplacement current ? 1

2. Find the expression for the potential and field due to an electric dipole. Find the mutual potential energy of two dipoles in a plane. 6+6

3. (a) Prove the following Maxwell's equation 6

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

(The symbols have their usual meanings)

(b) Show that electromagnetic waves propagate in free space with velocity of light in vacuum. 6

4. (a) Find the expressions of the electric and magnetic field in terms of electromagnetic potentials.

(b) Prove that a plane electromagnetic wave can not propagate in a conducting medium without attenuation. 6+6

Group—B

(Fuzzy Sets and its application in O.R.)

[Marks : 25]

Answer Q. No. 5 and any *three* from Q. No. 6 to Q. No. 10.

5. Answer any *one* question : 1×2

(a) Determine the α-cut of the triabgular fuzzy number

$\tilde{A} = (a, b, c)$.

(b) Give an example of a trapezoidal fuzzy number.

6. (a) (i) Using subtracting rule of two fuzzy numbers prove the subtraction rule of two interval numbers. 4

(ii) Also show that the law of contradiction donot hold for fuzzy sets. 2

(b) Evaluate the following :

5(9, 11, 13, 15) − 6(0, 3, 5) + 9 2

7. (a) Let $\tilde{A}$ be the fuzzy set with membership function

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 & \text{for } x < 1 \\ x-1 & \text{for } 1 \le x \le 2 \\ (4-x)/2 & \text{for } 2 < x < 3 \\ (x-2)/2 & \text{for } 3 \le x \le 4 \\ 1 & \text{for } 4 < x < 5 \\ (13-2x)/3 & \text{for } 5 \le x \le 6 \\ (7-x)/3 & \text{for } 6 < x < 7 \\ 0 & \text{for } x \ge 7 \end{cases}$$

Show that it is normal but not convex. Determine the membership function of $\underset{\sim}{A}^C$. 4

(b) Let us consider to triangular fuzzy numbers A and B with membership functions given by

$$\mu_A(x) = \begin{cases} 0, & x \le -1 \\ (x+1)/2, & -1 < x \le 1 \\ (3-x)/2, & 1 < x \le 3 \\ 0 & x > 3 \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x \le 1 \\ (x-1)/2, & 1 < x \le 3 \\ (5-x)/2, & 3 < x \le 5 \\ 0 & x > 5. \end{cases}$$

Find $A + B$. 4

8. (a) Discuss Werner's approach to find an equivalent crisp LPP for a fuzzy LPP with fuzzy resources.

(b) Discuss the method of solving "Non Symmetric Fuzzy LPP". 6+2

9. (a) What are the reasons to consider fuzzy LPP ?

(b) Describe the Zimmermann's method to convert fuzzy LPP into crisp LPP. 2+6

10. (a) Explain the difference between random uncertainity and fuzzy uncertainity with example. What are the causes of uncertainity ? 4

(b) State Zadeh's Extension principle. Define addition of triangular fuzzy numbers. Is it possible to add a crisp number with a triangular fuzzy number ? 2+1+1

Group—C

[Marks : 30]

11. Answer any *two* questions : 2×3

(a) Define the Reynolds number and discuss its physical significance. 3

(b) Prove that lines of constant ϕ (equi-potential lines) are othogonal to lines of constant ψ (streamlines) at all points where they intersect. 3

(c) Is the function $\psi = 4x - 3y$, the stream function for a two-dimensional potential flow ? 3

12. Answer any *three* questions : 3×8

(a) Determine the velocity potential and stream function when an elliptic cylinder moves in an infinite mass of liquid with velocity U parallel to the axial plane through the major axis of a cross-section. 8

(b) Derive the vorticity equations in vector form and deduce the above equation for 2D flow. 6+2

(c) (i) Write the Navier-Stokes equation in component form and state the physical significance of each term. 4

(ii) Derive the equation of continuity for compressible viscous fluid flow. 4

(d) State and prove Blasius theorem in a steady two-dimensional irrotational motion of an incompressible fluid under no external forces. 8

(e) Explain the phenomenon of the separation of boundary layer flow and discuss the role of pressure gradiuent in the boundary layer formed adjacent top the solid body. 8

Group—D

(Magnetic hydrodynamics)

[Marks : 20]

Answer any *two* questions : 2×10

13. (a) Write Maxwell's electromagnetic field equations.

(b) State and Prove Ferraro's law of isorotation. 2+(2+6)

14. A viscous incompressible electrically conducting fluid of uniform density flowing steadily between a horizontal conducting plane $z = 0$ (lower at rest) and a non-conducting plane $z = L$ (upper), but the upper one moves horizontally with uniform velocity. Suppose a uniform magnetic field acts vertically upwards and there is no pressure gradient in the liquid. Find the velocity of the fluid and the magnetic field. 10

15. (a) Deduce magnetic induction equation. Give the definition of magnetic Reynolds number and interpret it physically. 3+2

(b) Write short notes on

(i) Hartmann number

(ii) Alfve'n number 2×2½

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