2017

M.Sc. Part-I Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—V

Full Marks : 50

Time : 2 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

(*Mechanics of Continuous Media*)

Answer *Q. No. 6* and any *three* questions from the rest.

3×16+2

1. (a) Define an isotropic linearly elastic body. Hence, derive the constitutive equation of such a body. 2+6

(b) Define stream line. Test whether the motion specified by $\vec{\upsilon} = \frac{\lambda\left(x_1\vec{j} - x_2\vec{i}\right)}{x_1^2 + x_2 2}$, λ is a constant, is a possible motion for an incompressible fluid. If so, determine the equation of stream lines. Also, let the motion be of potential kind, then determine the velocity potential. 2+6

2. (a) Discuss the gemometrical interpretation of small strain tensors. 8

(b) Find the condition for a given surface $f(x, y, z, t) = 0$ to be a boundary surface of a fluid motion. 5

(c) Show that the equation of continuity in Eulerian and Lagrangian forms are equivalent. 3

3. (a) State and prove Kelvin's minimum energy theorem. 8

(b) Find the change in volume of a continuum body due to strain deformation. 4

(c) A displacement field is given by $x_1 = X_1 - 2X_2 + 3X_3$, $x_2 = 2X_1 + X_2 - 4X_3$ and $x_3 = -3X_1 + 4X_2 + X_3$. Determine the Eulerian finite strain tensors. 4

4. (a) Define a source and sink with its strength. Two sources, each of strength m, are placed at the points $(-a, 0)$ and $(a, 0)$ and a sink of strength $2m$ is placed at the origin. Show that the stream lines of the flow field are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 \lambda xy)$, where λ is a parameter. Also find the fluid speed at any point. 2+6

(b) Prove that the extreme values of normal stress at a point in a continuum body are principal stresses. 8

5. (a) State and prove the Cauchy's first law of motion. Deduce the equation of equilibrium. 8

(b) What are the concepts of stress and stress invariants ? 3

(c) The principal stress at a point in a continuum body are $T_1 = 1$, $T_2 = -1$ and $T_3 = 3$. If stress at a point is given by $\left(T_{ij}\right) = \begin{pmatrix} T_{11} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & T_{33} \end{pmatrix}$ then find the values of T_{11} and T_{33}. 5

6. Find the complex potential of a source. 2

Or

Define stress quadric. 2
