# 2017

## M.Sc. Part–I Examination

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

### PAPER—IV

*Full Marks* : 100

*Time* : 4 Hours

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

**Write the answer to questions of each group in Separate answer booklet.**

### Group—A
### (*Principles of Mechanics*)

[Marks : 50]

Answer *Q. No. 1* and any *three* questions from the rest.

**1.** Answer any *one* question : 2

(a) State and prove conservation law of angular momentum.

(b) State basic postulates of special theory of relativity.

2. (a) Deduce Lagrange's equations of motion for unconnected conservative system. 8

(b) A Lagrangian for a particular physical system can be written as

$$L = \frac{m}{2}(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2) - \frac{k}{2}(ax^2 + 2bxy + cy^2)$$

where $a$, $b$ and $c$ are arbitrary constants staisfying $b^2 - ac \neq 0$. What are the equations of motion? Examine particularly the two cases $a = 0 = c$ and $b = 0$, $c = -a$. 8

3. (a) What is the effect of the Coriolis force on a particle falling freely under the action of gravity? 6

(b) State and prove Jacobi's identity in connection with Poisson bracket. 6

(c) Show that the transformation $q = \sqrt{2P}\sin Q$, $p = \sqrt{2P}\cos Q$ is canonical. 4

4. (a) Show that the path followed by a particle in sliding from one point to another in absence of friction in the shortest times is a cycloid. 5

(b) Show that the following functional $J = \int_{x_0}^{x_1} \frac{1+y^2}{y'2} dx$ will be extremum if $y = \sinh(c_1 x + c_2)$ where $c_1$, $c_2$ are arbitrary constants. 5





(c) What do you mean by generating function related to canonical transformation? Show that if canonical transformation is known that one can determined generating function. 6

5. (a) Solve the Harmonic oscillation problem by Hamilton-Jacobi method. 6

(b) Deduce Lorentz transformation equation. 10

6. (a) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian. 4

(b) Solve the following system of equations in connction with rotation of rigid body about a fixed point. 8

$$A\dot{w}_x - (B - C)w_y w_z = 0$$

$$B\dot{w}_y - (C - A)w_x w_z = 0$$

$$C\dot{w}_z - (A - B)w_x w_z = 0$$

In case of $A = B$. The symbols have their usual meanings.

(c) Prove that $[q_i, p_j] = \delta_{ij}$, where $q_i$ and $p_j$ are called generated momentum. 4

## Group—B

### (*Partial Differential Equation*)

[Marks : 50]

Answer Q. No. 1 and any *three* from the rest.

**1.** Define a semi-linear and a quasi-linear partial differential equation. 2

*Or*

Define domain of dependence and region of influence for the Cauchy problem of homogeneous wave equation. 2

**2.** (a) Show that the equations 5

$$xp - yq = x,\ x^2p + q = xz$$

are compatible and find their solution.

(b) Find a complete integral of the PDE $2(z + xp + yq) = yp^2$ by Charpit's Method. 5

(c) Find the integral surface of the differential equation $(y + zq)^2 = z^2(1 + p^2 + q^2)$ circumscribed about the surface $x^2 - z^2 = 2y$. 6

**3.** (a) Show that the equation $\frac{\partial^2 y}{\partial t^2} + 2k\frac{\partial y}{\partial t} = c^2\frac{\partial^2 y}{\partial x^2}$ possesses solutions of the form

$$\sum_{r=0}^{\infty} c_r e^{-kt}\cos(\alpha_r x + \epsilon_r)\cos(w_r t + \delta_r)$$

where $c_r, \alpha_r, \epsilon_r, \delta_r$ are constants and $w_r^2 = \alpha_r^2 c^2 - k^2$. 6





(b) Prove that for the equation

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{2}{(x + y)} \cdot \left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = 0$$

the Green's function is given by

$$w(x, y, \xi, \eta) = \frac{(x + y)\{2xy + (\xi - \eta)(x - y) + 2\xi\eta\}}{(\xi + \eta)^3}.$$

Hence find the solution of the differential equation which satisfies the conditions

$$z = 0, \frac{\partial z}{\partial x} = 3x^2 \text{ on } y = x.$$ 10

**4.** (a) Reduce the equation

$$x(xy - 1)r - (x^2y^2 - 1)s + y(xy - 1)t + (x - 1)p + (y - 1)q = 0$$

to Canonical form and hence solve it. 10

(b) Derive the D'Alembert's formula for the Cauchy problem for the one-dimensional homogeneous wave equation. 6

**5.** (a) Show that $z(x, y) = 4e^{-3x}\cos 3y$ is a solution to the boundary value problem

$$\nabla^2 z = 0, z(x, \pi/2) = 0, z(x, 0) = 4e^{-3x}.$$ 8

(b) State and prove Mean-value theorem for harmonic function. Also, Show that if a harmonic function vanishes at all points on the boundary, then it is identically zero everywhere. 5+3

6. (a) Find the solution of the one-dimensional diffusion equation $k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$ satisfying the following boundary conditions ;

(i) $u$ is bounded as $t \to \infty$

(ii) $u_x(0,t)=0, u_x(a,t)=0$ for all $t$.

(iii) $u(x,0)=x(a-x), 0<x<a$. 8

(b) Solve the initial value problem described by the non-homogeneous wave equation 8

$$u_{tt}-c^2u_{xx}=f(x,t)$$

subject to the initial conditions

$$u(x,0)=\phi_1(x), u_t(x,0)=\phi_1(x), x\leq 0<\infty$$

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