2016

M.Sc. Part-II Examination

PHYSICS

PAPER—VIII

Full Marks : 75

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate answer-scripts for Group-A and Group-B

Group–A

(Advanced Quantum Mechanics-II)

[Marks : 40]

Answer Q. No. *1, 2, 3* and any *two* from the rest.

1. Answer any *five* bits : 5×2

(a) For any vector $\vec{A}$, show that $\left[\sigma, \vec{A}.\vec{\sigma}\right] = 2i\vec{A} \times \vec{\sigma}$.

(b) If the eigenvalues of J^2 and J_z are given by

$$J^2 \left|\lambda m\right> = \lambda \left|\lambda m\right> \text{ and } J_z \left|\lambda m\right> = m \left|\lambda m\right>,$$

Show that $\lambda \geq m^2$.

(c) Show that for hard sphere potential, total scattering cross-section $\sigma = 2\pi a^2$ (at high energy).

(d) Show that in M.W. region, stimulated emission is predominant.

(e) Show that for proton – neutron system

$\sigma_p . \sigma_n = -3$ for singlet system.

$= 1$ for triplet state.

(f) Write down the equation of continuity for Klein–Gordon equation.

(g) Prove that $C\vec{\alpha}$ is the velocity operator in Dirac equation.

(h) Give the zeroth order wave function for helium atom in the ground state ($1s^2$).

2. Answer any *two* bits : 2×3

(a) Sixteen noninteracting electrons are confined in a potential $V(x) = \infty$ for $x < 0$ and $x > 0$;

$V(x) = 0$ for $0 < x < a$

Find the number of possible configurations.

(b) For a Dirac particle moving in a central potential, show that the orbital angular momentum is not a constant of motion.

(c) An electron is in a state described by the wave function

$$\psi = \frac{1}{\sqrt{4\pi}}\left(\cos\theta + e^{-i\phi}\sin\theta\right)R(r).$$

What are the possible values of L_z ?

3. Answer any *one* bit : 4

(a) Establish the expansion of a plane wave in terms of an infinite number of spherical waves.

(b) Prove that $|jm>$ is the eigenket of the commutator $[J_y, J_+]$.

4. (a) Find the elastic and total cross-section for a black sphere of radius R.

(b) In the analysis of scattering of particles of mass m and energy E from a fixed centre with range a, the phase shift for the l th partial wave is given by

$$\delta_l = \sin^{-1}\left[\frac{(iak)^l}{[(2l+1)!l!]^{1/2}}\right]$$

Show that the total cross-section at a given energy is approximately given by $\sigma = \left(\frac{2\pi\hbar^2}{mE}\right)\exp\left(-\frac{2mEa^2}{\hbar^2}\right)$.

5+5

5. Prove that the quantity $\vec{L} + \frac{1}{2}\hbar\vec{\sigma}$, where L is the orbital angular momentum of a particle and $\sigma^1 = \begin{pmatrix} \sigma^1 & 0 \\ 0 & \sigma^1 \end{pmatrix}$ is a constant of motion for the particle in Dirac's fermaions. Hence give an interpretation for the additional angular momentum $\frac{1}{2}\hbar\sigma^1$.

8+2

6. (a) Two electrons having spin angular momentum vectors $\vec{S}_1$ and $\vec{S}_2$ have an interaction of the type $H = A\left(\vec{S}_1.\vec{S}_2 - 3S_{1z}S_{2z}\right)$, A being constant. Express it in terms of $\vec{S} = \vec{S}_1 + \vec{S}_2$ and obtain its eigenvalues.

(b) Obtain the selection rule for electric dipole transitions of a linear harmonic oscillator.

5+5

Group–B

(Statistical Mechanics)

[Marks : 35]

Answer Q. No. *1* and *two* from the rest.

1. Answer any *five* bits : 5×3

(a) If the density of states between ν and $\nu + \alpha\nu$ is $g(\nu)d\nu = A\nu^2 d\nu$. Find the zero point energy of a Qu. Harmonic oscillator.

(b) If the grand portion function is given by $l\,n\,\xi = F - AH^2V$, where F and A are constants, independent of H. Find the magnetization of the system.

(c) Prove that for pure state density matrix $\hat{\rho}$ is a projection operator.

(d) For non-interacting photons radiation pressure is $\frac{1}{3}u$; where u is the energy density, why ?

(e) Find the canonical partition function of quantum mechanical harmonic oscillator in three dimension.

(f) If $E = \pm\frac{\mu_B H}{2}$ for a spin $\frac{1}{2}$ particle, show that entropy gives rise to concept of negative temperature.

(g) Distinguish between He-I and He-II in the light of two fluid model.

(h) Define 2nd order phase transition in terms of order parameter.

2. (a) Show that for a two dimensional ideal B-E gas, number of particles

$$N = \frac{A 2\pi m K_B T}{h^2} B_1(\alpha)$$

Where $\alpha = -\mu B$; A is the area. Other symbols have usual meanings. Can it undergo B – E condensation ? 5

(b) For spin $\frac{1}{2}$ particle, magnetic specific heat

$C_H = NK_B(\beta\epsilon)^2 \cosh^{-2}(\beta\epsilon)$, where $\epsilon = \mu_B H$. 5

3. (a) Derive an exppression of entropy for BE/FD gases.

(b) Prove that $i\hbar\dot{\hat{\rho}} = [\hat{H}, \hat{\rho}]$, where $\hat{\rho}$ is the density matrix. 5+5

4. (a) What is Landau diamagnetism ? Prove that magnetic moment of the gas M = $\langle N \rangle \mu_{eff}$ L (x), where L (x) is the Langevin function, if the energy E is given by

$$E = \frac{e\hbar B}{m}\left(j + \frac{1}{2}\right) + \frac{p_z^2}{2m}.$$

(b) Prove that in one-dimensional Ising model, spontaneous magnetism does not exist.

5. (a) Prove that entropy of an ideal gas in d-dimension

$$S(E, N, V) = NK_B\left[\frac{d+2}{2} + \ln\left\{\frac{V}{N}\left(\frac{4\pi mE}{dNh^2}\right)^{d/2}\right\}\right].$$

(b) Prove that r.m.s. fluctuation in energy in canonical ensemble is proportional to $\frac{1}{\sqrt{N}}$. 5+5
