2016

M.Sc. Part–I Examination

PHYSICS

PAPER—I

Full Marks : 75

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Use separate Answerscripts for Gr. A & Gr. B.

Group—A

[Marks : 30]

1. Answer any *four* of the following :   4×2

   (a) Prove that if F (q, p, t) and G (q, p, t) are two integrals of motion, then [F, G] is also integral of motion.

(b) If U be a generating function depends only on Q_α, P_α, t. Prove that

$$P_\alpha = -\frac{\partial U}{\partial Q_\alpha} ; \quad q_\alpha = -\frac{\partial U}{\partial p_\alpha}, \quad K = H + \frac{\partial U}{\partial t} .$$

(c) Write down Hamilton's canonical equation in terms of Poisson's Bracket.

(d) Consider a 3-D circular plate with Z-axis perpendicular to its plane and origin through the centre of plate. Show that the inertia tenvor has the form

$$\begin{vmatrix} I_{xx} & I_{xy} & 0 \\ I_{yx} & I_{yy} & 0 \\ 0 & 0 & I_{xx} + I_{yy} \end{vmatrix}$$

(e) Deduce the Lagrange's equation of a system, it the Lagrangian of this system has the form $L = -\sqrt{(1 - \dot{q}^2)}$.

(f) Derive the Hamiltonian corresponding to the Lagrangian.

$$L = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_1\dot{q}_2 + \dot{q}_2^1\right) - v(q) .$$

(g) Consider any arbitrary function F of co-ordinate (q_k) and canonical momenta (p_k) and canonical momenta (p_k) and time (t). Show that :

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F, H]$$

where $[F, H]$ is the Poisson Bracket of F and H.

2. Answer any *two* of the following :   2×3

(a) Find out the natural frequencies of a linear non symmetric triatomic modecule (AB$_2$), vibrates only in a horizontal plane. What will happen if the mass of A atom is very much greater than mass of B atom ?

(b) A linear transformation of a generalized co-ordinate q and corresponding momentum p to Q and P is given by Q = q + p

and $P = q + \alpha p$ is canonical.

Find the value of α.

(Continued)

(Turn Over)

(c) Derive Lagrange's equation of motion from Hamilton's principle.

3. Answer any *two* of the followings : 2×8

(a) (i) Establish Euler's dynamical equations of rotational motion.

(ii) Discuss the torque free motion of a symmetrical top and explain the precessional motion.
Prove that angular velocity, kinetic energy and angular momentum are constant in this motion. 3+2+3

(b) The Lagrangian (L) for a charged particle in an electromagnetic field is given as :

$$L = \frac{1}{2} \sum_k mv_k^2 + q \sum_k v_k A_k - q\varphi$$

Derive the equation of motion. Find the expression of Hamiltonian for this. 4+4

(c) (i) If L is the Lagrangian of a system of n degrees of freedom satisfying Hamilton's variation variational principle, show that,

$$L' = L + \frac{dF}{dt} (q_1, q_2 \dots q_n, t)$$

will also satisfy Hamilton's variational principle, where F is any arbitrary well behaved function.

(ii) Prove that if a generalised coordinate is cyclic in the Lagrangian it should be cyclic in the Hamiltonian also.

(iii) Explain the physical significance of Hamilton's principle function. 3+3+2

Group—B

[*Marks : 45*]

Answer Q. No. 1 and any *three* from the rest.

1. Answer any *three* from the following : 3×3

(a) Find the packing fraction of simple cubic structure. What is meant by Zinc Blende structure ?

$$1\tfrac{1}{2} + 1\tfrac{1}{2}$$

(b) What is the matrix representation of 3 fold rotational symmetry ? What is meant by Screw axis ?

$$1\frac{1}{2}+1\frac{1}{2}$$

(c) Derive the condition $G^2 = 2kG$ on the basis of Ewald's Sphere. G is the reciprocal lattice vector and k is the radius of sphere. 3

(d) Find the condition for systematic absence in a BCC structure ? What is the application of neutron diffraction ? 2+1

(e) Find the Fermi energy of electron in a metal at absolute Zero. 3

(f) Find an expression of effective mass of electron in a solid ? 3

(g) What is Meissner's effect ? 3

2. What is meant by Single Crystal and polycrystal ? Find an expression of structure factor assuming scattering X-rays from a crystal. What is meant by Brillouin Zone ?

3+7+2

3. Derive the dispersion relation assuming diatomic lattice vibration in a solid. What is TO Phonon ? In an assembly if 10^{23} SHO each has frequency of 10^{13} Hz, calculate the mean energy of the system (ignoring zero point energy) at 20 K.

8+1+3

4. (a) Prove that average energy of electron in a metal increases with increase of temperature.

(b) Evaluate the temperature that there is one percent probability that a state with an energy 0·5 electron volt above the Fermi energy will be occupied by an electron.

8+4

5. (a) Describe the essential features of Kronig Penny model.

(b) Clearly distinguish metal, insulator and semiconduc-
tor on the basis of band theory. 9+3

6. (a) Distinguish type-1 and type-II super conductor.

(b) Find the expression of local field in a dielectric
according to Lorentz.

(c) What is Hund's Rule? 4+6+2

———