NEW

2015

BCA

1st Semester Examination

DISCRETE MATHEMATICS WITH APPLICATION TO COMPUTER SCIENCE

PAPER—1103

Full Marks : 70

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Answer *Q. No. 1* and any *six* from the remaining.

1. Answer any *five* questions : 5×2

(a) Define directed graph G(V, E) with example.

(b) Let X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} and A = {3, 5, 7, 9} and B = {1, 2, 7, 10}.

Verify $(A \cup B)' = A' \cap B'$.

(c) Define Tautology in Mathematical logic.

(d) Show that the permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 6 & 5 & 2 \end{pmatrix}$$ is odd.

(e) Construct the Truth table $p \wedge (\sim q \vee q)$.

(f) Define an abelian group (G, O).

(g) Construct the truth table of $x.x' + y.x$.

(h) Let A = {2, 5, 9} and B = {6, 7, 11}. Write down the total number of distinct relation from A to B.

2. (a) Prove that for any set A, B, C, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(b) Define equivalence relation. A relation 'ρ' defined on the set Z by $a \rho b$ iff $ab > 0$ for $a, b \in Z$. Examine ρ is (i) reflexive, (ii) symmetric, (iii) transitive.

5+2+3

3. (a) Show that the mapping $f : R \to R$ be defined by $f(x) = ax + b$ where $a, b, x \in R$, $a \neq 0$ is one to one and onto. Define its inverse. 4+1

(b) Find the number of possible ways in which the letters of the word COTTON can be arranged so that the two T's don't come together. 5

4. (a) Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with the initial condition $a_0 = 1$ and $a_1 = -1$. 5

(b) In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics but not Computer Science. Find the number of students who have taken Mathematics and Computer Science and those who have taken Computer Science but not Mathematics. 5

5. (a) Write down the negation of each of the following statements using propositional logic :

(i) If it is raining, then game is cancelled ;

(ii) He swims if and only if the water is warm. 2+2

(b) Rewrite the following argument using quantifiers, variables and predicate symbols. Prove the validity of the argument :

If a number is odd, then its square is odd

K is a particular number that is odd

so K^2 is odd. 4

(c) Prove the equivalence by using laws of propositional algebra $(p \rightarrow q) \rightarrow q \equiv p \vee q$. 2

6. (a) Define subgroup of a group (G, O). Show that

$$a\,0\,b = a + b + 1 \text{ where}$$

$a, b \in Z$, (G, O) is a group. 1+3

(b) Define Cyclic group $(G, \cdot)$. Show that the set of integers with respect to + i.e. (Z, +) is a cyclic group and 1 is its generator. 1+3

(c) Define isomorphism of groups (G,0) and (G,π).

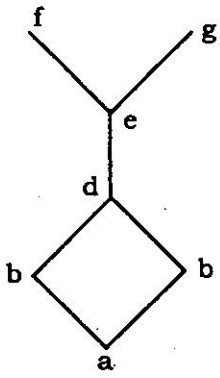
7. (a) Define integral domain and show that ring of integers $(Z, +, \cdot)$ is an integral domain. 1+4

(b) Show that in a non-directed graph, the total number of odd degree vertices is even. 5

8. (a) Define Hasse diagram of a poset (S, R) where S is a non-empty set with the relation R.

Let X = {1, 2, 3, 4, 5, 6} and '/' (divides by) is a partial order relation on X. Draw the Hasse diagram of (X, 1). 4

(b) Test whether the poset given below by the Hasse diagram is a lattice or not : 2



(c) Find the complement of the expression :

$$((A' + C).(B + D'))'$$

by De-Morgan's theorem. 4

9. (a) Draw the graph with help of a adjacency matrix : 5

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

(b) Define lattice (S, C). Show that the poset (P(S), $\subseteq$) is a lattice where P(S) is a power set of S and '$\subseteq$' means subset. 1+4

10. (a) By Boolean Algebra prove that :

(i) $(a + b)' = a'.b'$;

(ii) $(a \cdot b)' = a' + b'$. 4

(b) Find the sum of product of the Boolean expression from the truth table given below and simplify the expression using k-map : 3+3

x	y	z	f
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1