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# Heat and Mass Transfer on Three Dimensional Vertical Channel Flow in the Presence of Radiation

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**ABSTRACT**

An analysis is made on the three dimensional free convection and mass transfer flow through a vertical channel in the presence of radiation. Approximate solutions have been obtained for the velocity, temperature and concentration fields using perturbation technique. It is found that the primary velocity decreases with the increase of both radiation parameter and Schmidt number but increases with the increase of thermal Grashoff number as well as mass Grashoff number. The temperature distribution decreases with the increase of both radiation parameter and Reynolds number. The Concentration field also decrease with the increase of both Schmidt number as well as Reynolds number. The shear stress and mass flux in terms of Sherwood number which are of physical interest are presented in the form of tables.

***Keywords:*** Three-dimensional, injection, periodic suction, mass transfer.

## 1. Introduction

Free convective flow with heat and mass transfer has been a subject of interest of many reseachers due to its day-to day application in science and technology. Such phenomenon are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth, etc. Guria and Jana [1] studied the unsteady three dimensional flow and heat transfer along a porous vertical plate subjected to a periodic suction velocity distribution. Guria and Jana [2] also have studied the effect of periodic suction on three dimensional vertical channel flow. Due to the periodic suction the flow becomes three dimensional In the above studies the radiation effect is ignored. It has important application in space vehicle re-entry problems. Many processes in engineering areas occur at high temperatures and it is important for the design of pertinent equipment. Nuclear power plants, gas turbines, and the various propulsion devices for aircraft missiles, satellites and space vehicles are example of such engineering areas. At high temperature radiation effect can be quite significant. The heating of rooms and buildings by the use of radiators is a familiar example of heat transfer by free convection. Heat losses from hot pipes, ovens etc surrounded by cooler air, are at least in part due to free convection. The effect of radiation on the flow past a vertical plate was discussed by Takhar et al. [3]. Guria et al. [4] investigated the effect of radiation on three dimensional flow in a vertical channel subjected to a periodic suction. Guria et al. [5] also studied investigated the effect of radiation on steady three dimensional flow past a vertical porous plate in the presence of magnetic field.

Sing and Thakar [6] discussed the effect of periodic suction on three dimensional mixed convection flow and mass transfer. Ahmed [7] also studied the effects of heat and mass transfer on the steady three dimensional flow of a viscous incompressible fluid along a moving vertical plate. Ahmed and Liu [8] studied the effects of heat and mass transfer on three dimensional flow past a vertical porous plate with uniform free stream velocity. Reddy and Reddy [9] studied radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate with viscous dissipation. However, the interaction of radiation with mass transfer in three dimension verical channel flow has received little attention. Recently, Guria [10] has studied the heat and mass transfer in three dimensional flow past a vertical porous plate in the presence of radiation. The main object of this paper is to study the three dimensional heat and mass transfer flow throgh the vertical channel in the presence of radiation.

**2. Formulation of the problem and its solution**

Consider the steady flow of viscous, incompressible fluid between vertical parallel porous plates separated by a distance $d$. Here the $x^\star$ - axis is chosen along the direction of the flow, $y^\star$ - axis is perpendicular to the wall of the channel and $z^\star$ - axis normal to the $x^\star y^\star$ - plane [see Fig.1]. The temperature at the plates $y^* = 0$ and $y^* = d$ are $T_w$ and $T_0$ $(T_w > T_0)$ respectively.

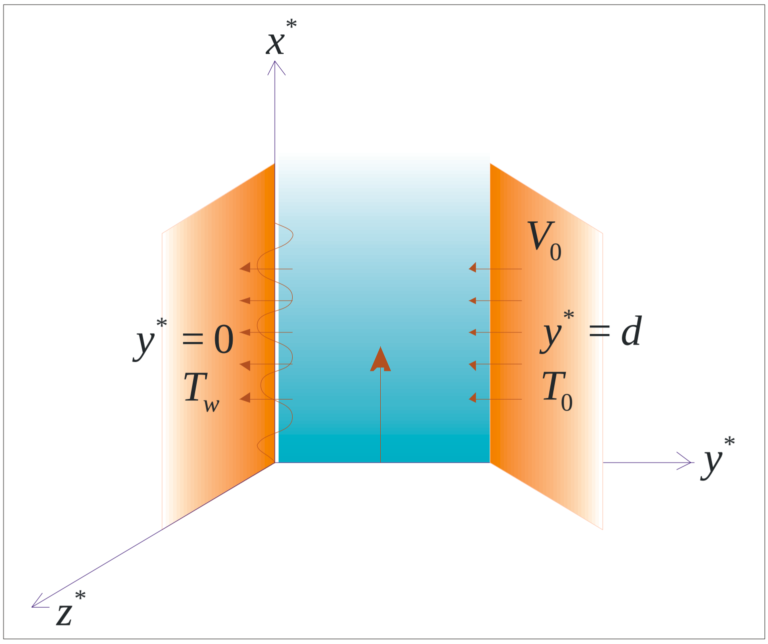


**Figure 1:** Physical model and Co-ordinates system

The plate $y^\star = d$ is subjected to a uniform injection $V_0$ and the plate $y^\star = 0$ to a periodic suction velocity distribution of the form

$$v^{\star} = -V_0[1+\varepsilon\cos(\frac{\pi z^{\star}}{d})], \quad (1)$$

where $\varepsilon(\ll 1)$ is the amplitude of the suction velocity.

The velocity and temperature fields are independent of $x^*$ since the channel is infinite long along $x^*$-direction. The flow itself will be three dimensional due to cross flow. Let $u^{\star}, v^{\star}, w^{\star}$ be the velocity components in the directions $x^{\star}-, y^{\star}-, z^{\star}-$ axes respectively. The problem is governed by the following equations

$$\frac{\partial v^{\star}}{\partial y^{\star}} + \frac{\partial w^{\star}}{\partial z^{\star}} = 0, \quad (2)$$

$$v^{\star}\frac{\partial u^{\star}}{\partial y^{\star}} + w^{\star}\frac{\partial u^{\star}}{\partial z^{\star}} = \nu(\frac{\partial^2 u^{\star}}{\partial y^{\star 2}} + \frac{\partial^2 u^{\star}}{\partial z^{\star 2}}) + g\beta(T^* - T_0) + g\beta(C^* - C_0), \quad (3)$$

$$v^{\star}\frac{\partial v^{\star}}{\partial y^{\star}} + w^{\star}\frac{\partial v^{\star}}{\partial z^{\star}} = -\frac{1}{\rho}\frac{\partial p^{\star}}{\partial y^{\star}} + \nu(\frac{\partial^2 v^{\star}}{\partial y^{\star 2}} + \frac{\partial^2 v^{\star}}{\partial z^{\star 2}}), \quad (4)$$

$$v^{\star}\frac{\partial w^{\star}}{\partial y^{\star}} + w^{\star}\frac{\partial w^{\star}}{\partial z^{\star}} = -\frac{1}{\rho}\frac{\partial p^{\star}}{\partial z^{\star}} + \nu(\frac{\partial^2 w^{\star}}{\partial y^{\star 2}} + \frac{\partial^2 w^{\star}}{\partial z^{\star 2}}), \quad (5)$$

$$v^{\star}\frac{\partial T^*}{\partial y^{\star}} + w^{\star}\frac{\partial T^*}{\partial z^{\star}} = \frac{1}{\rho C_p}(\frac{\partial^2 T^*}{\partial y^{\star 2}} + \frac{\partial^2 T^*}{\partial z^{\star 2}}) - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*}, \quad (6)$$

$$v^{\star}\frac{\partial C^*}{\partial y^{\star}} + w^{\star}\frac{\partial C^*}{\partial z^{\star}} = D(\frac{\partial^2 C^*}{\partial y^{\star 2}} + \frac{\partial^2 C^*}{\partial z^{\star 2}}), \quad (7)$$

where $\nu$ is the kinematic coefficient of viscosity, $\rho$ is the density, $p^{\star}$ is the fluid pressure, $g$ is the acceleration due to gravity, $\beta$ is the thermal expansion and $C_p$ is the specific heat at constant pressure. $K^*$ is the permeability of the medium.

The equation of conservation of radiative heat transfer per unit volume for all wavelength is

$$\nabla.q_r^* = \int_0^{\infty} K_{\lambda}(T^*)(4e_{\lambda h}(T^*) - G_{\lambda})d\lambda,$$

where $e_{\lambda h}$ is the Plank's function and the incident radiation $G_{\lambda}$ is defined as

$$G_{\lambda} = \frac{1}{\pi}\int_{\Omega=4\pi} e_{\lambda}(\Omega)d\Omega,$$

$\nabla.q_r^*$ is the radiative flux divergence and $\Omega$ is the solid angle. Now, for an optically thin fluid exchanging radiation with an isothermal flat plate at temperature $T_0$ and according to the above definition for the radiative flux divergence and Kirchhoffs law, the incident radiation is given by $G_{\lambda} = 4e_{\lambda h}(T_0)$ then,

$$\nabla.q_r^* = 4\int_0^{\infty} K_{\lambda}(T^*)(e_{\lambda h}(T^*) - e_{\lambda h}(T_0))d\lambda,$$

Expanding $K_\lambda(T^*)$ and $e_{\lambda h}(T_0)$ in a Taylor series around $T_0$, for small $(T^*-T_0)$, we can rewrite the radiative flux divergence as

$$\nabla.q_r^* = 4(T^*-T_0)\int_0^\infty K_{\lambda_0}(\frac{\partial e_{\lambda h}}{\partial T})_0 d\lambda,$$

where $K_{\lambda_0} = K_{\lambda(T_0)}$.

Hence an optical thin limit for a non-gray gas near equilibrium, the following relation holds

$$\nabla.q_r^* = 4(T^*-T_0)I,$$

and hence

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^*-T_0)I,$$

where

$$I = \int_0^\infty K_{\lambda_0}(\frac{\partial e_{\lambda h}}{\partial T})_0 d\lambda.$$

The boundary conditions of the problem are

$$u^\star = 0, v^\star = -V_0[1+\varepsilon\cos(\frac{\pi}{d}z^\star)], w^\star = 0, T^* = T_w, C^* = C_w \quad \text{at} \quad y^\star = 0,$$

$$u^\star = 0, v^\star = -V_0, w^\star = 0, T^* = T_0, C^* = C_\infty, p^\star = p_\infty \quad \text{at} \quad y^\star = d. \tag{8}$$

Introducing the non dimensional variables

$$y = \frac{y^\star}{d}, z = \frac{z^\star}{d}, p = \frac{p^\star}{\rho V_0^2}, u = \frac{u^\star}{V_0}, v = \frac{v^\star}{V_0}, w = \frac{w^\star}{V_0}, \theta = \frac{(T^*-T_0)}{(T_w-T_0)}, \tag{9}$$

equations (2)-(7) become

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{10}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{Re}(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) + Gr\theta + GmC, \tag{11}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re}(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}), \tag{12}$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re}(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}), \tag{13}$$

$$v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \frac{1}{RePr}(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}) - F\theta, \tag{14}$$

$$v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \frac{1}{SRe}(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}), \tag{15}$$

where $Re = V_0 d/\nu$, the Reynolds number, $Pr = \nu/\rho$, the Prandtl number and $Gr = dg\beta(T_w-T_0)/V_0^2$, the Grashof number, $Gm = dg\beta(C_w-C_0)/V_0^2$, the mass

Grashof number, $F = 4Id/\rho C_p V_0$, the radiation parameter, $S = \nu/D$, the Schmidt number. Using (9), the boundary conditions (8) become

$$u = 0, v = -[1+\varepsilon\cos(\pi z)], w = 0, \theta = 1, C = 1, \quad at \quad y = 0,$$
$$u = 0, v = -1, w = 0, \theta = 0, C = 0, \ p = \frac{p_\infty}{\rho V^2} \quad at \quad y = 1. \tag{16}$$

**3. Solution of the problem**

In order to solve the differential equations (10)-(15), we assume the solution of the following form

$$\begin{aligned} u(y,z) &= u_0(y) + \varepsilon u_1(y,z) + \varepsilon^2 u_2(y,z) + \cdots, \\ v(y,z) &= v_0(y) + \varepsilon v_1(y,z) + \varepsilon^2 v_2(y,z) + \cdots, \\ w(y,z) &= w_0(y) + \varepsilon w_1(y,z) + \varepsilon^2 w_2(y,z) + \cdots, \\ p(y,z) &= p_0(y) + \varepsilon p_1(y,z) + \varepsilon^2 p_2(y,z) + \cdots, \\ \theta(y,z) &= \theta_0(y) + \varepsilon\theta_1(y,z) + \varepsilon^2\theta_2(y,z) + \cdots. \\ C(y,z) &= C_0(y) + \varepsilon C_1(y,z) + \varepsilon^2 C_2(y,z) + \cdots. \end{aligned} \tag{17}$$

On substituting (17) in equations (10)-(15) and equating the terms independent of $\varepsilon$, we get the following system of differential equations

$$v_0' = 0, \tag{18}$$
$$u_0'' - Rev_0u_0' = -ReGr\theta_0 - ReGmC_0, \tag{19}$$
$$\theta_0'' - RePrv_0\theta_0' - FRePr\theta_0 = 0, \tag{20}$$
$$C_0'' - SRev_0C_0' = 0, \tag{21}$$

where primes denotes differentiation with respect to $y$ and the corresponding boundary conditions become

$$u_0 = 0, v_0 = -1, \theta_0 = 1, C_0 = 1 \quad at \quad y = 0$$
and
$$u_0 = 0, v_0 = -1, \theta_0 = 0, C_0 = 0 \quad at \quad y = 1. \tag{22}$$

The solutions of the equations (18) to (21), subject to the boundary conditions (22) are

$$v_0(y) = -1, \qquad C_0(y) = \frac{1}{(e^{-S\,Re} - 1)}\left[e^{-S\,Re} - e^{-S\,Re\,y}\right], \tag{23}$$
$$\theta_0(y) = \frac{1}{(e^{-\lambda_1} - e^{-\lambda_2})}[e^{-\lambda_1}e^{-\lambda_2 y} - e^{-\lambda_2}e^{-\lambda_1 y}], \tag{24}$$
$$u_0(y) = [A_1 + A_2e^{-Rey} + A_3y + A_4e^{-SRey} + A_5e^{-\lambda_1 y} + A_6e^{-\lambda_2 y}] \tag{25}$$

where

$$\lambda_1 = \frac{1}{2}\{RePr + \sqrt{Re^2Pr^2 + 4FRePr}\},$$

$$\lambda_2 = \frac{1}{2}\{RePr - \sqrt{Re^2Pr^2 + 4FRePr}\},$$

$$A_1 = \frac{-1}{(1-e^{-Re})}[A_3 + A_4(e^{-SRe} - e^{-Re}) + A_5(e^{-\lambda_1} - e^{-Re}) + A_6(e^{-\lambda_2} - e^{-Re})],$$

$$A_1 = \frac{1}{(1-e^{-Re})}[A_3 + A_4(e^{-SRe} - 1) + A_5(e^{-\lambda_1} - 1) + A_6(e^{-\lambda_2} - 1)],$$

$$A_3 = \frac{-Gme^{-SRe}}{(e^{-SRe} - 1)} \tag{26}$$

$$A_4 = \frac{Gm}{(e^{-SRe} - 1)SRe(S-1)}$$

$$A_5 = \frac{ReGre^{-\lambda_2}}{(e^{-\lambda_1} - e^{-\lambda_2})\lambda_1(\lambda_1 - Re)},$$

$$A_6 = \frac{-ReGre^{-\lambda_1}}{(e^{-\lambda_1} - e^{-\lambda_2})\lambda_2(\lambda_2 - Re)},$$

On substituting (17) in equations (10)-(15) and equating the coefficient of $\varepsilon$, we get the following system of differential equations

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{27}$$

$$v_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y} = \frac{1}{Re}(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}) + Gr\theta_1 + GmC_1, \tag{28}$$

$$v_0\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{Re}(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}), \tag{29}$$

$$v_0\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{Re}(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}), \tag{30}$$

$$v_0\frac{\partial \theta_1}{\partial y} + v_1\frac{\partial \theta_0}{\partial y} = \frac{1}{RePr}(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}) - F\theta_1. \tag{31}$$

$$v_0\frac{\partial C_1}{\partial y} + v_1\frac{\partial C_0}{\partial y} = \frac{1}{SRe}(\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2}). \tag{32}$$

The corresponding boundary conditions become

$$u_1 = 0, v_1 = -\cos(\pi z), w_1 = 0, \theta_1 = 0, C_1 = 0 \quad \text{at} \quad y = 0,$$
$$u_1 = 0, v_1 = 0, w_1 = 0, \theta_1 = 0, C_1 = 0 \quad \text{at} \quad y = 1. \tag{33}$$

These are the linear partial differential equations describing the three dimensional flow. To solve the equations (28)-(32), we assume velocity components and pressure in the following form

$$u_1(y,z) = u_{11}(y)\cos(\pi z),$$

$$\begin{aligned}
v_1(y,z) &= v_{11}(y)\cos(\pi z),\\
w_1(y,z) &= -\frac{1}{\pi}v_{11}^{'}(y)sin(\pi z),\\
p_1(y,z) &= p_{11}(y)\cos(\pi z),\\
\theta_1(y,z) &= \theta_{11}(y)\cos(\pi z),\\
C_1(y,z) &= C_{11}(y)\cos(\pi z),
\end{aligned} \tag{34}$$

$v_1$ and $w_1$ are so chosen that the continuity equation (27) is satisfied automatically. Substituting (34) in (28)-(32) and comparing the coefficients of harmonic terms, we obtain the following set of differential equations

$$v_{11}^{''} + Rev_{11}^{'} - \pi^2 v_{11} = Rep_{11}^{'}, \tag{35}$$

$$v_{11}^{'''} + Rev_{11}^{''} - \pi^2 v_{11}^{'} = Re\pi^2 p_{11}, \tag{36}$$

$$\theta_{11}^{''} + RePr\theta_{11}^{'} - (FRePr + \pi^2)\theta_{11} = RePrv_{11}\theta_0^{'}, \tag{37}$$

$$C_{11}^{''} + SReC_{11}^{'} - \pi^2 C_{11} = SRev_{11}C_0^{'}, \tag{38}$$

$$u_{11}^{''} + Reu_{11}^{'} - \pi^2 u_{11} = Rev_{11}u_0^{'} - Re(Gr\theta_{11} + GmC_{11}). \tag{39}$$

The corresponding boundary conditions are

$$\begin{aligned}
&u_{11} = 0, v_{11} = -1, v_{11}^{'} = 0, \theta_{11} = 0, C_{11} = 0 \quad \text{at} \quad y = 0,\\
&u_{11} = 0, v_{11} = 0, v_{11}^{'} = 0, \theta_{11} = 0, C_{11} = 0 \quad \text{at} \quad y = 1.
\end{aligned} \tag{40}$$

Solutions of the equations (35)-(39) subject to (40) and on using (34) yield

$$v_1(y,z) = [B_1 e^{-m_1 y} + B_2 e^{-m_2 y} + B_3 e^{\pi y} + B_4 e^{-\pi y}]\cos(\pi z), \tag{41}$$

$$w_1(y,z) = \frac{1}{\pi}[B_1 m_1 e^{-m_1 y} + B_2 m_2 e^{-m_2 y} - B_3 \pi e^{\pi y} + B_4 \pi e^{-\pi y}]\sin(\pi z), \tag{42}$$

$$p_1(y,z) = [A_7 e^{\pi y} + A_8 e^{-\pi y}]\cos(\pi z), \tag{43}$$

$$\begin{aligned}
\theta_1(y,z) = [&G_1 e^{-\mu_1 y} + G_2 e^{-\mu_1 y} + G_3 e^{-(m_1+\lambda_1)y} + G_4 e^{-(m_2+\lambda_1)y}\\
&+ G_5 e^{(\pi-\lambda_1)y} + G_6 e^{-(\pi+\lambda_1)y} + G_7 e^{-(m_1+\lambda_2)y} + G_8 e^{-(m_2+\lambda_2)y}\\
&+ G_9 e^{(\pi-\lambda_2)y} + G_{10} e^{-(\pi+\lambda_2)y}]\cos(\pi z),
\end{aligned} \tag{44}$$

$$\begin{aligned}
C_1(y,z) = [&D_1 e^{-\alpha_1 y} + D_2 e^{-\alpha_2 y} + D_3 e^{-(m_1+SRe)y} + D_4 e^{-(m_2+SRe)y}\\
&+ D_5 e^{(\pi-SRe)y} + D_6 e^{-(\pi+SRe)y}]\cos(\pi z)
\end{aligned} \tag{45}$$

$$\begin{aligned}
u_1(y,z) = [&E_1 e^{-m_1 y} + E_2 e^{-m_2 y} + E_3 e^{-\mu_1 y} + E_4 e^{-\mu_2 y} + E_5 e^{-(m_1+\lambda_1)y} + E_6 e^{-(m_2+\lambda_1)y}\\
&+ E_7 e^{(\pi-\lambda_1)y} + E_8 e^{-(\pi+\lambda_1)y} + E_9 e^{-(m_1+\lambda_2)y} + E_{10} e^{-(m_2+\lambda_2)y}\\
&+ E_{11} e^{(\pi-\lambda_2)y} + E_{12} e^{-(\pi+\lambda_2)y} + E_{13} e^{-\alpha_1 y} + E_{14} e^{-\alpha_2 y}\\
&+ E_{15} e^{-(m_1+SRe)y} + E_{16} e^{-(m_2+SRe)y} + E_{17} e^{(\pi-SRe)y} + E_{18} e^{-(\pi+SRe)y}\\
&+ E_{19} e^{-(m_1+Re)y} + E_{20} e^{-(m_2+Re)y} + E_{21} e^{(\pi-Re)y} + E_{22} e^{-(\pi+Re)y}
\end{aligned}$$

$$+E_{23}ye^{-m_1y}+E_{24}ye^{-m_2y}+E_{25}e^{\pi y}+E_{26}e^{-\pi y}]\cos(\pi z), \qquad (46)$$

where

$$m_1=\frac{1}{2}\{Re+\sqrt{Re^2+4\pi^2)}\},$$
$$m_2=\frac{1}{2}\{Re-\sqrt{Re^2+4\pi^2}\},$$
$$\mu_1=\frac{1}{2}\{RePr+\sqrt{Re^2Pr^2+4(FRePr+\pi^2)}\},$$
$$\mu_2=\frac{1}{2}\{RePr-\sqrt{Re^2Pr^2+4(FRePr+\pi^2)}\},$$
$$\alpha_1=\frac{1}{2}\{SRe+\sqrt{S^2Re^2+4\pi^2)}\},$$
$$\alpha_2=\frac{1}{2}\{SRe-\sqrt{S^2Re^2+4\pi^2}\},$$
$$B_1=[\pi r_2(e^{\pi}-e^{-\pi})+r_4(e^{\pi}+e^{-\pi})]/2(r_1r_4-r_2r_3),$$
$$B_2=-[\pi r_1(e^{\pi}-e^{-\pi})+r_3(e^{\pi}+e^{-\pi})]/2(r_1r_4-r_2r_3),$$
$$B_3=-\frac{1}{2\pi}[\pi+A_5(\pi-m_5)+A_6(\pi-m_6)],$$
$$B_4=-\frac{1}{2\pi}[\pi+A_5(\pi+m_5)+A_6(\pi+m_6)],$$
$$r_1=e^{-m_5}-\frac{1}{2\pi}[e^{\pi}(\pi-m_5)+e^{-\pi}(\pi+m_5)],$$
$$r_2=e^{-m_6}-\frac{1}{2\pi}[e^{\pi}(\pi-m_6)+e^{-\pi}(\pi+m_6)],$$
$$r_3=m_5e^{-m_5}+\frac{1}{2}[e^{\pi}(\pi-m_5)-e^{-\pi}(\pi+m_5)],$$
$$r_4=m_6e^{-m_6}+\frac{1}{2}[e^{\pi}(\pi-m_6)-e^{-\pi}(\pi+m_6)]. \qquad (47)$$

The other constants are not given here to save space.

**4. Results and discussion**

The velocity, temperature and concentration field for different values of the non dimensional parameters are plotted in the diagram.The value of dimensionless parameter $Gr$ is taken as positive. The positive value corresponds to an extremely cooled plate by the free convection currents. The value of Prandtl number is taken equal to and this value corresponds to the air. The values of Grashof numbers are taken to be large from the physical point of view. The large Grashof number values correspond to free convection problem. The Schmidt number (S) are taken for helium (S=0.3), water vapor $(S=0.60)$, oxygen $(S=0.66)$ and ammonia $(S=0.78)$. The effect of Grashoff number, mass

Grashoff number, radiation parameter and Schmidt number on main flow velocity is shown in Figs.2-5. It is observed from Fig.2 that greater cooling of surface (an increase in Gr)results in an increase in the velocity. It is due to the fact that in the values of thermal Grashof number has the tendency to increase the thermal buoyancy effect. This gives rise to an increase in the induced flow. The primary velocity also increases with increase in mass Grashoff number. It is seen from Figs.4 and 5 that the primary velocity decreases with increase in radiation parameter as well as Schmidt number for cooling of the plate $(Gr > 0)$. Knowing the velocity field it is interesting to know the shear stress at the plate. The shear stress at the plate $y^* = 0$ due to the primary flow is given by

$$\tau_x^* = \mu(\frac{\partial u^*}{\partial y^*})_{y^*=0} = \frac{\mu V_0}{d}(\frac{\partial u}{\partial y})_{y=0} \tag{48}$$

In non-dimensional form the shear stress at the plate $y = 0$ can be written as

$$\begin{aligned}\tau_x &= \frac{\tau_x^* d}{\mu V_0} = (\frac{\partial u}{\partial y})_{y=0} \\ &= u_0^{'}(0) + \varepsilon u_1^{'}(0). \\ &= -\sum_{i=1}^{4} A_i m_i + \varepsilon[-Am_5 - Bm_6 - C\lambda_1 - D\lambda_2 - \sum_{i=1}^{4} C_i(m_i + m_5) \\ &- \sum_{i=1}^{4} D_i(m_i + m_6) + \sum_{i=1}^{4} E_i(\pi - m_i) - \sum_{i=1}^{4} E_i(\pi - m_i)]\cos(\pi z).\end{aligned} \tag{49}$$

The shear stress due to the primary flow in terms of $\tau_x$ is shown in Table.1 for different values of Reynolds number and Schmidt number for cooling of the plate. It is seen that $\tau_x$ increases with increase in Reynolds number but it decreases with increase in Schmidt number for cooling of the plate.

| | $\tau_x$ | | | |
|---|---|---|---|---|
| $Re \setminus S$ | 0.3 | 0.6 | 0.66 | 0.78 |
| 2 | 3.08 | 2.58 | 2.43 | 2.00 |
| 3 | 4.34 | 3.31 | 3.04 | 2.27 |
| 4 | 5.49 | 3.84 | 3.44 | 2.35 |
| 5 | 6.50 | 4.15 | 3.62 | 2.27 |

**Table 1:** Shear stress component due to primary flow for for $Gr = 5.0, Pr = 0.71,$ $\varepsilon = 0.25,\ z = 0.0$.

The temperature $\theta$ is plotted for different values of radiation parameter and Reynolds number in Figs.6 and 7 for $Re = 5.0$, $Gr = 5.0$, $\varepsilon = 0.05$, $z = 0.0$ for cooling of the plate. It is found that the temperature $\theta$ decreases with increase in radiation parameter as well as Reynolds number. In Figs. 8 and 9 we have presented the concentration field for several values of Schmidt number and Reynolds number. It is found that the concentration field decrease with increase in both Schmidt number and Reynolds number.

The non-dimensional mass flux at the plate $y = 0$ in terms of Sherwood number $Sh$ is

$$Sh = (\frac{\partial C}{\partial y})_{y=0}$$
$$= -C_0'(0) - \varepsilon C_1'(0),$$
$$= -C_0'(0) - \varepsilon C_{11}'(0)\cos(\pi z),$$

| | $Sh$ | | | |
|---|---|---|---|---|
| $Re \setminus S$ | 0.3 | 0.6 | 0.66 | 0.78 |
| 2 | 1.29 | 1.62 | 1.69 | 1.83 |
| 3 | 1.45 | 1.98 | 2.10 | 2.33 |
| 4 | 1.62 | 2.38 | 2.54 | 2.87 |
| 5 | 1.80 | 2.79 | 3.01 | 3.44 |

**Table 2:** Sherwood number for $Gr = 5.0$, $Pr = 0.71$, $\varepsilon = 0.25$, $z = 0.0$.

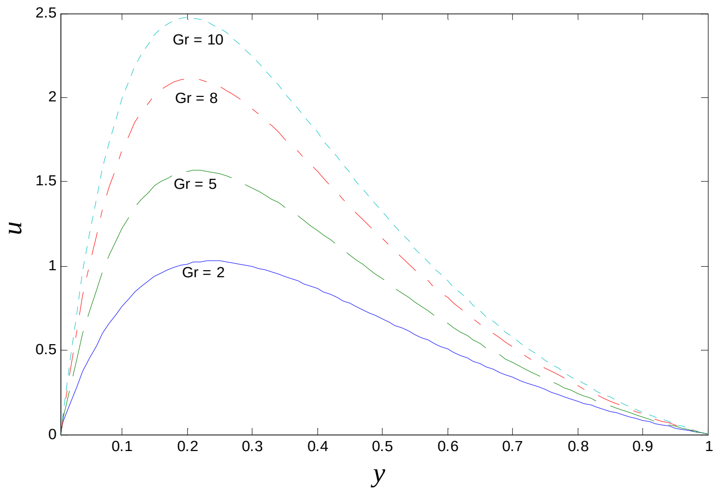


**Figure 2:** Primary velocity $u$ for $S = 0.3, Gm = 5, F = 2, Pr = 0.71, Re = 4, \varepsilon = 0.25$.

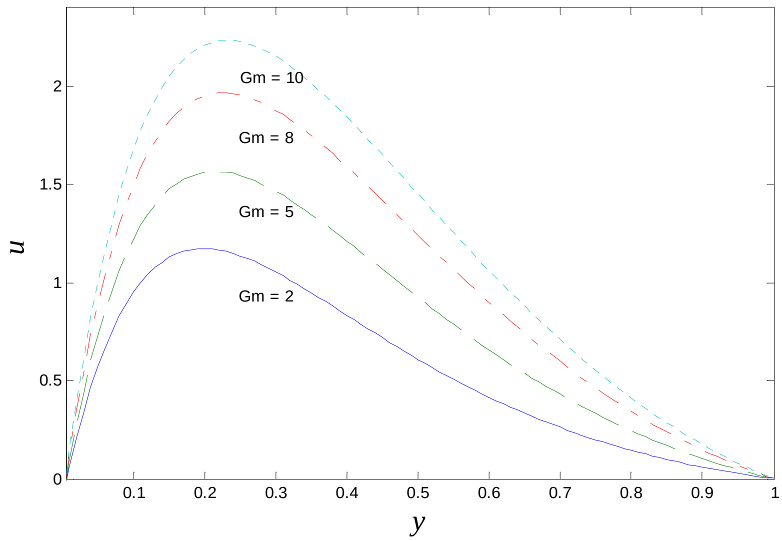


**Figure 3:** Primary velocity $u$ for $S = 0.3, Gr = 5, F = 2, Pr = 0.71, Re = 4, \varepsilon = 0.25$.

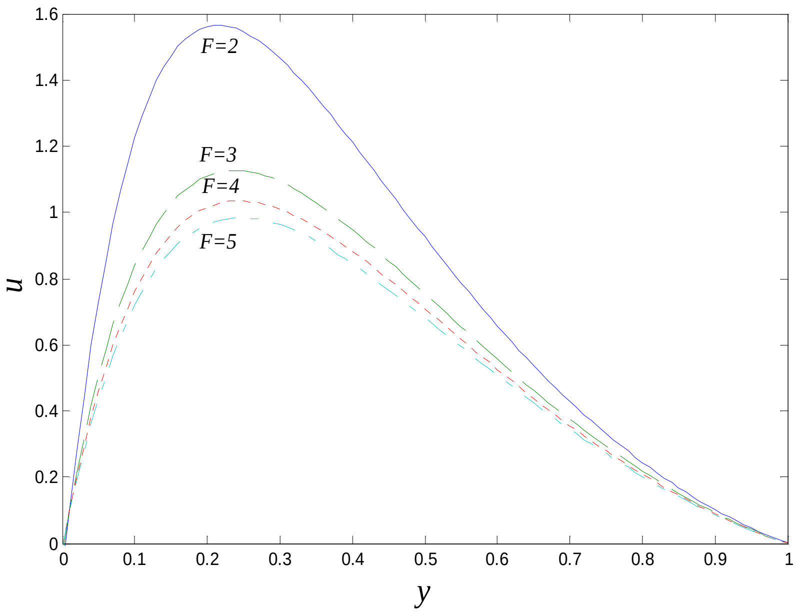


**Figure 4:** Primary velocity $u$ for
$S = 0.3, Gm = 5, Gr = 5, Pr = 0.71, Re = 4, \varepsilon = 0.25$.

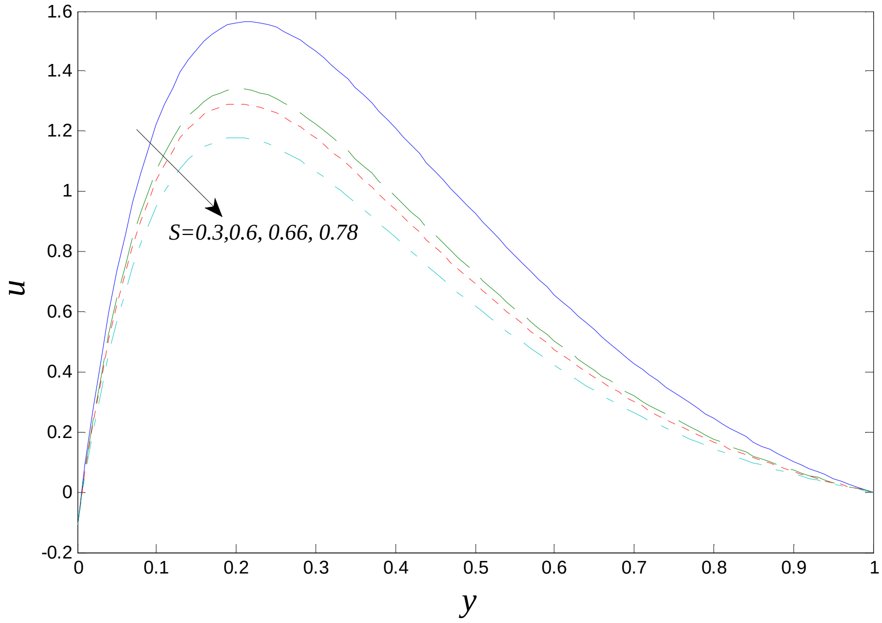


**Figure 5:** Primary velocity $u$ for $F = 2, Gm = 5, Gr = 5, Pr = 0.71, Re = 4, \varepsilon = 0.25$.

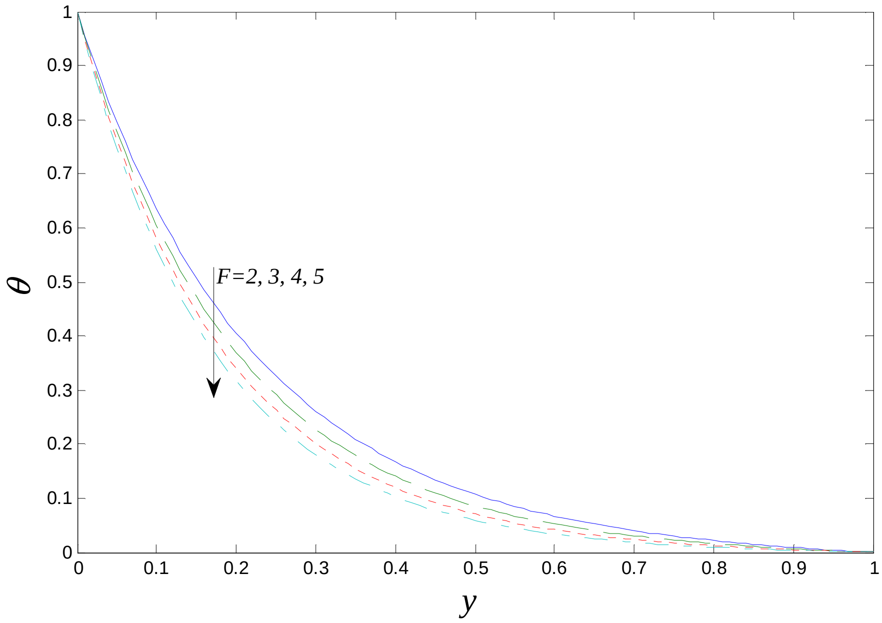


**Figure 6:** Temperature profile $\theta$ for $Gr = 5.0$, $Re = 5.0$, $Pr = 0.71, \varepsilon = 0.25$.

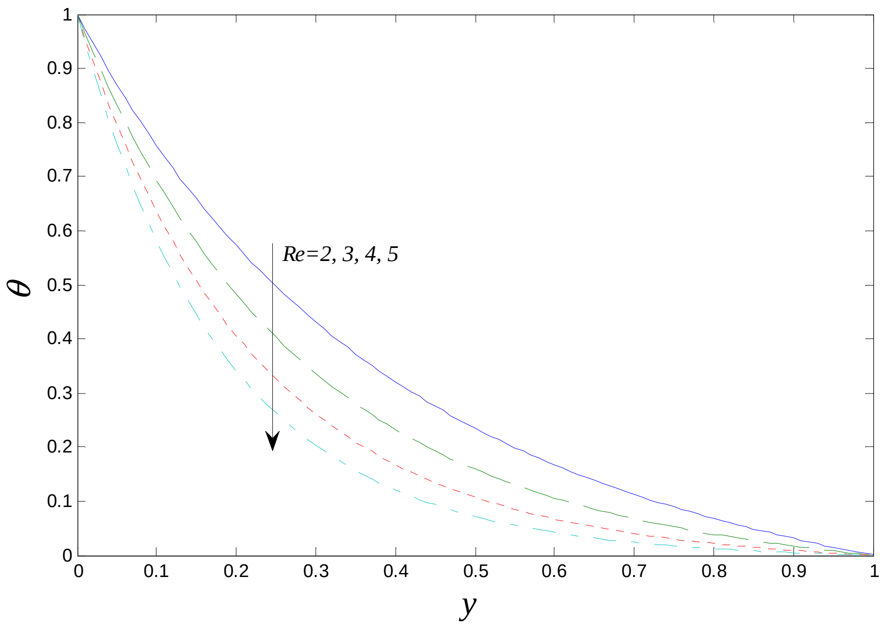


**Figure 7:** Temperature profile $\theta$ for $Gr = 5.0$, $F = 2.0$, $Pr = 0.71, \varepsilon = 0.25$.

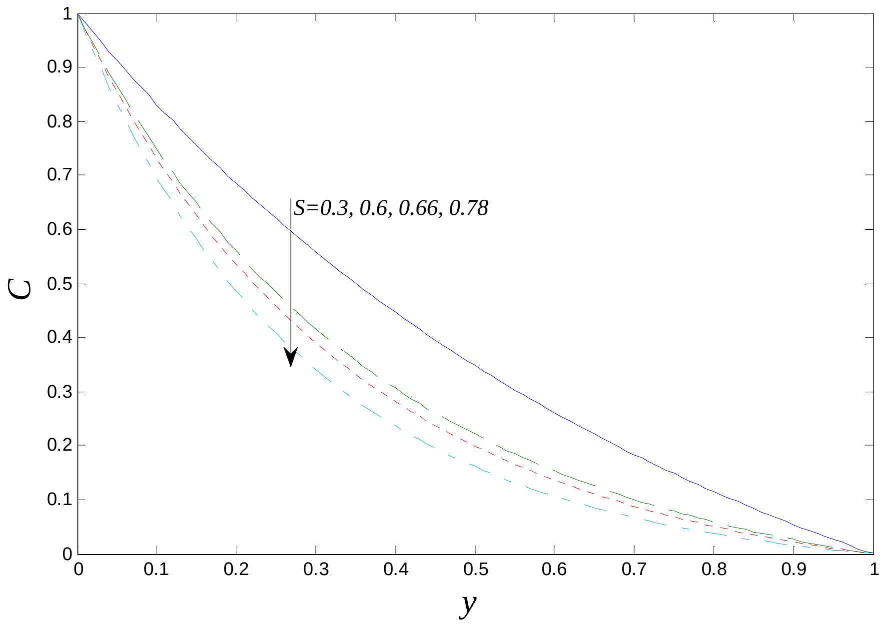


**Figure 8:** Variations of concentration field for $Re = 4$.

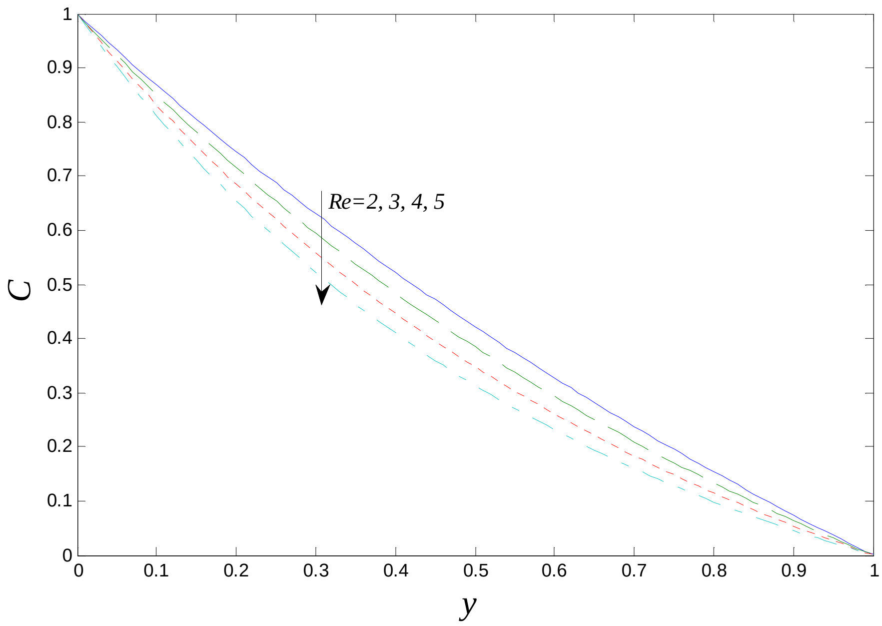


**Figure 9:** Variations of concentration field for $S = 0.3$.

**5. Conclusion**

The steady heat and mass transfer flow of viscous incompressible fluid passing through the vertical channel has been studied in the presence of radiation. It is found that the primary velocity decreases with increase in radiation parameter as well as Schmidt number for cooling of the plate. It is also found that with increase in thermal Grashoff number and mass Grashoff number the primary velocity increases for cooling of the plate. It is observed that the temperature profile decreases with increase in either radiation parameter or Reynolds number for cooling of the plate. The Concentration field also decrease with the increase of both Schmidt number as well as Reynolds number.

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