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An Innovative Approach to Obtain an Initial Basic Feasible Solution for the Transportation Problems

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ABSTRACT

Operating cost related to transportation is known as transportation cost. The cost of transporting products plays a vital role to maximize profit. In the present competitive global market, industries must have a very good planning of their transportation system to minimize the transportation cost. Modeling of this transporting path is a managerial decision which is concerned with finding optimal distribution plan for a single homogeneous commodity with minimal transportation cost. This modeling is known as transportation problem which can be formulated as a linear programming problem. In the solution procedure of a transportation problem, an initial basic feasible solution is always required to obtain the optimal solution. The well-known solution procedure for finding an initial basic feasible solution of transportation problems are North West Corner Method, Least Cost Method, Vogel's Approximation Method, and Extremum Difference Method. In this study, we propose a new method to obtain an initial basic feasible solution for the transportation problems. Proposed method is illustrated with numerical examples. A comparative study is also carried out to justify the performance of the proposed method. It is observed that the performance of proposed method is suitable for solving transportation problems.

Keywords: Transportation Problem, Transportation Cost, Initial Basic Feasible Solution, Optimal Solution.

1. Introduction

Main target of a business is to earn profit. Transportation or caring cost has significant impact on the economic activities of an industry and this cost is inversely proportional to the profit. Industries should select the routes to receive and deliver their essential items with minimum transportation cost thereby maximize profit. Transportation problem is basically one of the most important and earliest applications of linear programming, which is based on supply and demand of commodities transported from several sources to the different destinations. Transportation problems receive this name because of its wide application in determining optimal transporting route.

Hitchcock [2] in 1941 formally introduced the transportation problem by presenting a paper entitled 'The Distribution of a Product from Several Sources to Numerous Localities'. This presentation is considered as the origin, and first important contribution to the solution of transportation problems. Then Koopman independently began to spearhead research on the potentialities of linear programs for the study of the problems in economics. Continuation of the improvement of transportation problems, Koopmans [3] in 1947, presented his historic paper titled 'Optimum Utilization of the Transportation Systems', which was based on his war time experience. These two contributions are the basis of development of transportation methods which involve a number of shipping sources and a number of destinations. Because of this and work done earlier by Hitchcock, the classical transportation problem is often referred as Hitchcock-Koopman's transportation problem.

Dantzig [4] in 1951 first introduced the logical solution procedure for the transportation problem. It was again developed by Charnes et al. [5] in 1953, and referred as North West Corner Method (NWCM) in which the north-west-corner cost cell is considered at every stage of allocation. And then the next developed method is Least Cost Method (LCM) [5] consists in allocating as much as possible in the lowest cost cell of the Transportation Table in making allocation in every stage. Reinfeld and Vogel [6] in 1958 developed a method known as Vogel's Approximation Method (VAM). These well reputed methods are discussed in all the Operation Research books. Among these existing methods, VAM is the most efficient solution procedures for obtaining an initial basic feasible solution for the transportation problems as it facilitates a very good initial basic feasible solution.

Initial basic feasible solution leads to obtain the optimal solution of transportation problems. There are many procedures available in the literature for finding an initial basic feasible solution for the transportation problems. These models were formulated to minimize transportation cost; Klein [7] in 1967 developed a primal method for minimal cash flows with applications to the assignment and transportation problems. Mackinnon & James [8] in 1975 developed an algorithm for the generalized transportation problem. Shimshaket. al. [9] in 1981 propose a modification (SVAM) which ignores any penalty that involves a dummy row/column. Like, if there is dummy columns in the cost –matrix, the penalties are ignored not only for the dummy column, but also for all the rows since the calculation of row-penalties involves the dummy column. Goyal [10] in 1984 suggests another modification for solving unbalanced transportation problems, where the cost of transporting goods to or from a dummy point is set equal to the highest transportation cost in the problem, rather than to zero. This method is known as GVAM. Ramakrishnan [11] in 1988 again brought out an improvement to GVAM for solving transportation problems. Kirca and Satir [12] in 1990, developed a heuristic to obtain an efficient initial basic feasible solutions, called Total Opportunity Cost Method (TOCM). Balakrishnan [13] in 1990, proposed a modified version of VAM only for solving the unbalanced transportation problems. Adlakha and Kowalski [14] in 2003 presented a simple heuristic algorithm for the solution of small fixed-charge transportation problems. Mathirajan and Meenakshi [15] in 2004 analyzed some variants of VAM and extended TOCM using the VAM procedure. They coupled VAM with TOCM and achieved very efficient initial solutions. Kasana and Kumar [16] in 2005 proposed Extremum Difference Method (EDM) where they define the penalty as the differences of the highest and lowest unit transportation cost in each row and column and allocate as like as the VAM procedure. Lukač et al. [17] in 2008 proposed a model for solving the production-transportation problem in the petroleum industry. Kulkarni, and Datar [18] in 2010 proposed a new heuristic method of obtaining an initial basic feasible solution for unbalanced transportation problems. In the solution procedure of the proposed method an unbalanced transportation problems is converted to a modified unbalanced transportation problem. Kulkarni, and Datar claimed that their proposed method reduces number of iterations to reach optimality. Koruko˘glu and Balli [19] in 2011 proposed an improved version of the well-known VAM by taking the total opportunity cost into account. They claimed through computational experiments that this improved VAM provided more efficient initial feasible solution to a large scale transportation problem. Uddin, et al. [20] in 2011, developed a network model to obtain the minimum transportation cost. Khan [21] in 2011 presented a method by defining pointer cost as the difference of highest and next to highest cost in each row and column of a transportation table and allocate to the minimum cost cell corresponding to the highest three pointer cost. This method is known as “Highest Cost Difference Method (HCDM)”. Florez et al. [22] in 2011 proposed a new model for planning Multi-Modal transportation problems. Again, Singh et al. [23] in 2012 modified the solution procedure of VAM using total opportunity cost and allocation costs. Deshmukh [23] in 2012 proposed a new method called an innovative method (NMD) to obtain a better initial feasible solution to the transportation problem. Sudhakar et al. [25] in 2012 proposed a new approach called “Zero Suffix Method (ZSM)” for obtaining a minimal total cost solution to the transportation problem. Islam. et al. [26] in 2012, applied EDM on TOCM, and allocate to the minimum cost cell corresponding to the highest distribution indicator and again HCDM on TOCM for obtaining an initial basic feasible solution. Kamrul Hasan [27] in 2012 evaluated the performance of the methods for finding the optimal solution directly. He concluded that the performance of this type of methods does not guarantee that the obtaining solution be optimal always. Sharma et al. [28] in 2012 proposed an approach for solving transportation problem with various methods of linear programming problem. Hlayel and Alia [29] in 2012 proposed an approach for solving transportation problems using the best candidates’ method. Md. Babu et al. [30] in 2013, proposed a method for solving transportation problems, where first allocation was made in the lowest cost cell which appears along lowest demand/supply. They named the method "Lowest Allocation Method (LAM)". Rashid, et al [31] in 2013, proposed a new procedure for finding the initial basic feasible solution only for the balanced transportation problems. Girmay and Sharma [32] in 2013 proposed a heuristic approach to solve balanced and unbalanced transportation problems. Soomro et al. [33] in 2014 carried out a comparative study of initial basic feasible solution methods for transportation problems using mathematical theory and modeling. Das et al. [34] in 2014 brought out a logical development in the solution procedure of VAM. Ahmed, et al. [35] in 2014 developed an algorithm for finding an initial basic feasible solution for both the balanced and unbalanced TP. Khan et al. [36] in 2015, used TOCM and define the pointer cost as the sum of all entries in the respective row or column of the TOCM and then allocate to the minimum cost cell corresponding to the highest pointer cost. Khan et al. [37] again in 2015 used TOCM and determine the distribution indicator for each cell of the TOCM by subtracting corresponding row and column highest element of every cell from the respective element. Finally the maximum possible allocation is allotted to the cell having the smallest distribution indicator. Kadhim et al. [38] in 2015 proposed an approach for solving transportation problem using modified Kruskal's Algorithm., Hoque [39] in 2015, proposed a better efficient heuristic solution technique denoted by JHM (Juman & Hoque Method) to obtain a better initial basic feasible solution for the transportation problems. Ahmedet al. [40] in 2016, applied the first allocation on lowest odd valued cost cell and proposed to form an allocation table to allocate rest of the allocation for finding an initial basic feasible solution for the transportation problems. This method is called Allocation Table Method (ATM). Ahmed et al. [41] again in 2016 proposed a computationally easier solution procedure to solve the transportation problems. In this method continuous allocation system is followed for which this method is known as "Incessant Allocation Method (IAM)".

Moreover, very recently Uddin, et al. [42] in 2016 proposed a method to solve the transportation problems named as Improved Least Cost Method (iLCM). This improvement is basically done by bringing changes in the existing solution procedure of the classical LCM. This method is also yielding a better initial basic feasible solution.

In this research, we studied solution procedure for the transportation problems thoroughly and found that the performances of the existing methods are inconsistent for finding an initial basic feasible solution. We also found that NWCM is basically a position based method, and most of all other methods are basing on cost cells, for which the solution obtained by NWCM varies with the mathematical formulation of the transportation problems. Again not a single method can be considered as the best method for solving transportation problems. We also observe that VAM and EDM are generally yielding a good initial basic feasible solution for transportation problems.

The solution procedure of VAM and EDM starts with the calculation of penalties for all rows and columns. For each row and column of the transportation table, the difference between the second minimum and the first minimum unit transportation costs is known as the penalty for VAM, where as the difference between the highest and lowest unit transportation cost is calculated for EDM. Then it allocates as many units as possible to the lowest cost cell in the row or column having maximum penalty. Then the allocated row/column is deleted, penalties are revised and procedure repeated successively until all units are supplied. But this is a time consuming solution procedure.

Finally we observed that a better initial basic feasible solution makes the optimal solution procedure easier. Considering this, in our study we aimed to propose a method for obtaining a better initial basic feasible solution rather than the optimal solution for the transportation problems.

2. Transportation problem

Transportation problem is one of the subclasses of linear programming problems, a typical transportation problem must have the following elements:

i. Source(s) ii. Destination(s) iii. Weighted edge(s)

and, the objective of transportation problem is to determine the schedule for transporting goods from various source to a number of destinations in a way that minimizes the shipping cost while satisfying supply and demand constraints [1].

To describe the transportation problem, following notations are to be used:

- m Total number of sources/origins
- n Total number of destinations
- S_i Amount of supply at source i
- d_j Amount of demand at destination j
- c_{ij} Unit transportation cost from source i to destination j
- x_{ij} Amount to be shipped from source i to destination j

Network representation of the transportation problems using the above mentioned notations is shown in the Figure1.

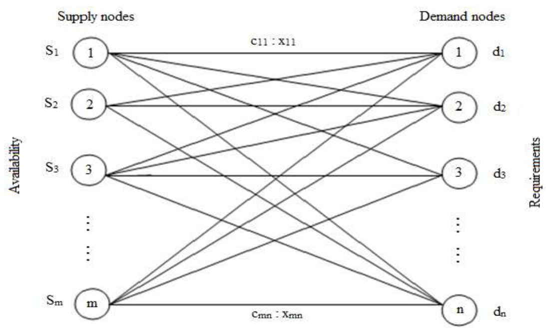


Figure 1. Network Diagram for Transportation Problem

The objective of the transportation model is to determine the unknowns' x_{ij} that will minimize the total transportation cost while satisfying both the supply (capacity) and demand (requirement) restrictions. Basing on this objective transportation problem can mathematically be formulated as:

Minimize: $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$ (Objective function)

subject to : $\sum_{j=1}^{n} x_{ij} \le S_i$; $i = 1, 2, \ldots\ldots, m$ (Capacity constraints)

$\sum_{i=1}^{m} x_{ij} \ge d_j$; $j = 1, 2, \ldots\ldots, n$ (Requirement constraints)

$x_{ij} \ge 0$, for all i and j (Non-negative condition)

Balanced and unbalanced are the types of transportation problems. If the sum of the supplies of all the sources equals the sum of the demands of all the destinations, the problem is termed as a balanced transportation problem. If this sum is not equal the problem is termed as unbalanced transportation problem.

After the mathematical formulation of transportation problem following steps are followed to obtain the optimal solution:

- Step 1: Verify the transportation problem either it is balanced or unbalanced. If the problem is unbalanced first balance it.
- Step 2: Determine the initial basic feasible solution.
- Step 3: Verify the optimality condition of the initial basic feasible solution. If the solution is not optimal improve it for obtaining optimal solution.

A feasible solution, of a transportation problem with m sources and n destinations, which contains no more than (m+n-1) non-negative allocations, is called basic feasible solution to the Transportation Problem. And a feasible solution (may not be basic feasible) is said to be optimal if it optimizes the transportation cost.

3. Proposed method

In this study we mainly introduce an allocation procedure in which complete the allocation making the allocation(s) in the smallest cost cell(s) along each row or column until it exhausted. Each and every step of iterations, we follow this allocation procedure either along a row & column or vice-versa. We apply this allocation procedure after finding the difference of unit transportation costs to obtain an initial basic feasible solution for the transportation problems. Finally, we apply this concept on EDM, and propose a new method to obtain an initial basic feasible solution for the transportation problems. Computational procedure of the proposed method for solving the balanced and the unbalanced transportation problems are described separately.

3.1. Proposed method for balanced transportation problem

Step by step solution procedure for balanced transportation problem is described below:

- Step-1: Mathematically formulate the transportation table.

- Step-2: Determine the maximum row cost difference (MRCD) for each row, which is the difference between the highest and lowest unit transportation costs along the row, and put it on the right of the corresponding rows of the cost matrix. Similarly, find the maximum column cost difference (MCCD) for each of the columns and write them below the corresponding columns of the cost matrix.

- Step-3: Select the highest of MRCDs (HoMRCD) and the highest of MCCDs (HoMCCD) separately. In case of tie for selecting HoMRCDs/HoMCCDs choose the HoMRCD/HoMCCD along which the smallest cost cell appears. In case of ties for lowest cost cells, select the cell where maximum allocation can be made. If a tie occurs again, select the cell for which sum of demand and supply is minimum in the original transportation table. Again if all these are same, select the topmost and then the extreme left corner cell.

- Step-4: To complete the allocation, after selecting the HoMRCD and HoMCCD, following cases may arise:
  Case-I: If the HoMRCD>HoMCCD, at first complete the row allocation along the selected HoMRCD making the allocation(s) in the smallest cost cell(s) continuously. For any allocation other than the final one, made along the row/column satisfies both the row and column, in such case find the smallest cost cell which is along the column/row and assign a zero in that cell. And then, complete the column allocation(s) along the selected HoMCCD following the same procedure if not exhausted during the row allocation procedure.
  Case-II: Again if the selected HoMRCD<HoMCCD, at first complete the column allocation along the selected HoMCCD and then row allocation according to the procedure described in Case-I.
  Case-III: If the selected HoMRCD = HoMCCD, at first complete the row/column allocation along which smallest cost cell appears and then complete the column/row allocation following the procedure described in above cases. Again if the smallest cost cell is common for both the HoMRCD and HoMCCD, in such case at first complete either the row/column allocation, and then follow the procedure described before to complete allocation along column/row.

- Step-5:Now ignore the row or column those are exhausted during the allocation(s) made in Step-4, for further consideration.

- Step-6: Then repeat Step-2 to Step-5 until all the rim requirements are satisfied.

- Step-7: Finally calculate the total transportation cost which is the sum of the product of cost and corresponding allocated value.

3.2. Proposed method for unbalanced transportation problem

Unbalanced transportation problems can easily be solved by introducing a dummy source or a dummy destination. If the total supply is more than the total demand, a dummy destination (dummy column) with demand equal to the surplus supply is added. If the total demand is more than the total supply, a dummy source (dummy row) with supply equal to the surplus demand is added. In this case unit transportation cost for the dummy column and dummy row are assigned zero values, as no shipment is actually made in case of a dummy source and dummy destination. To solve the unbalanced transportation problems, follow the proposed procedure described for solving the balanced transportation problems considering the following cases:

Case-I: Balance the unbalanced transportation problem. Determine the MRCD and MCCD ignoring the dummy row or column. And then, select the smallest cost along the selected HoMRCD or HoMCCD without considering the zero cost cells assigned in the dummy sources/destinations.

Case-II: To complete the allocation along the row or the column, first allocation is not to be made in the dummy cost cell (if appears). Again, if any basic cell is there which was made before during other row/column allocation, in such case the first allocation is also not to be made in the dummy cost cell. For rest of the allocations second allocation is to be made in the dummy cost cell (if present).

Case-III: If HoMRCD=HoMCCD, also the smallest cost cell is a common cell for both the HoMRCD and HoMCCD. In this case, at first complete the allocation along which the dummy source/destination is not appeared.

4. Numerical Examples

To justify the efficiency of the proposed method we have solved a good number of problems from various literature and books, which are listed in Table1.

Balanced Transportation Problems	**Unbalanced Transportation Problems**
Example-1: [Ref No. 39] $[c_{ij}]_{3x3}$=[6 8 10; 7 11 11; 4 5 12] $[s_i]_{3x1}$=[150, 175, 275] $[d_j]_{1x3}$=[200, 100, 300]	**Example- 1:** [Ref No. 32] $[c_{ij}]_{3x3}$=[4 8 8; 16 24 16; 8 16 24] $[s_i]_{3x1}$=[76, 82, 77] $[d_j]_{1x3}$=[72, 102, 41]
Example-2: [Ref No. 40] $[c_{ij}]_{3x3}$=[15 7 25; 8 12 14; 17 19 21] $[s_i]_{3x1}$=[12, 17, 7] $[d_j]_{1x3}$=[12, 10, 14]	**Example-2:** [Ref No. 41] $[c_{ij}]_{3x4}$=[10 8 4 3; 12 14 20 2; 6 9 23 25] $[s_i]_{3x1}$=[500, 400, 300] $[d_j]_{1x4}$=[250, 350, 600, 150]
Example--3: [Ref No. 36] $[c_{ij}]_{3x4}$=[3 6 8 4; 6 1 2 5; 7 8 3 9] $[s_i]_{3x1}$=[20, 28, 17] $[d_j]_{1x4}$=[15, 19, 13, 18] ;	**Example-3:** [Ref No. 41] $[c_{ij}]_{4x4}$=[12 10 6 13; 19 8 16 25; 17 15 15 20; 23 22 26 12] $[s_i]_{4x1}$=[150, 200, 600, 225] $[d_j]_{1x4}$=[300, 500, 75, 100]

Example-4: [Ref No. 39] $[c_{ij}]_{3x4}$=[4 6 8 8; 6 8 6 7; 5 7 6 8] $[s_i]_{3x1}$=[40, 60, 50] $[d_j]_{1x4}$=[20, 30, 50, 50]	**Example-4:** [Ref No. 41] $[c_{ij}]_{3x5}$=[5 8 6 6 3; 4 7 7 6 5; 8 4 6 6 4] $[s_i]_{3x1}$=[800, 500, 900] $[d_j]_{1x5}$=[400, 400, 500, 400, 800]
Example-5: [Ref No. 41] $[c_{ij}]_{3x5}$=[5 7 10 5 3; 8 6 9 12 14; 10 9 8 10 15] $[s_i]_{3x1}$=[5, 10, 10] $[d_j]_{1x5}$=[3, 3, 10, 5, 4]	**Example-5:** [Ref No. 18] $[c_{ij}]_{4x3}$=[3 4 6; 7 3 8; 6 4 5; 7 5 2] $[s_i]_{4x1}$=[100, 80, 90, 120] $[d_j]_{1x3}$=[100, 110, 60]
Example-6: [Ref No. 24] $[c_{ij}]_{3x5}$=[4 1 2 4 4; 2 3 2 2 3; 3 5 2 4 4] $[s_i]_{3x1}$=[60, 35, 40] $[d_j]_{1x5}$=[22, 45, 20, 18, 30]	**Example-6:** [Ref No. 43] $[c_{ij}]_{3x3}$=[4 5 6; 3 1 5; 2 4 4] $[s_i]_{3x1}$=[12, 11, 7] $[d_j]_{1x3}$=[6, 5, 8]
Example-7: [Ref No. 30] $[c_{ij}]_{4x4}$=[7 5 9 11; 4 3 8 6; 3 8 10 5; 2 6 7 3] $[s_i]_{4x1}$=[30, 25, 20, 15] $[d_j]_{1x4}$=[30, 30, 20, 10]	**Example-7:** [Ref No. 43] $[c_{ij}]_{3x4}$=[25 17 25 14; 15 10 18 24; 16 20 8 13] $[s_i]_{3x1}$=[300, 500, 600] $[d_j]_{1x4}$=[300, 300, 500, 500]
Example-8: [Ref No. 39] $[c_{ij}]_{4x5}$=[25 14 34 46 45; 10 47 14 20 41; 22 42 38 21 46; 36 20 41 38 44] $[s_i]_{4x1}$=[27, 35, 37, 45] $[d_j]_{1x5}$=[22, 27, 28, 33, 34]	**Example-8:** [Ref No. 43] $[c_{ij}]_{3x5}$=[5 4 8 6 5; 4 5 4 3 2; 3 6 5 8 4] $[s_i]_{3x1}$=[600, 400, 1000] $[d_j]_{1x5}$=[450, 400, 200, 250, 300]
Example-9: [Ref No. 21] $[c_{ij}]_{4x6}$=[7 10 7 4 7 8; 5 1 5 5 3 3; 4 3 7 9 1 9; 4 6 9 0 0 8] $[s_i]_{4x1}$=[5, 6, 2, 9] $[d_j]_{1x6}$=[4, 4, 6, 2, 4, 2]	**Example-9:** [Ref No. 44] $[c_{ij}]_{4x5}$=[10 2 16 14 10; 6 18 12 13 16; 8 4 14 12 10; 14 22 20 8 18] $[s_i]_{4x1}$=[300, 500, 825, 375] $[d_j]_{1x5}$=[350, 400, 250, 150, 400]
Example-10: [Ref No. 42] $[c_{ij}]_{5x5}$=[73 40 9 79 20; 62 93 96 8 13; 96 65 80 50 65; 57 58 29 12 87; 56 23 87 18 12] $[s_i]_{5x1}$=[8, 7, 9, 3, 5] $[d_j]_{1x5}$=[6, 8, 10, 4, 4]	**Example -10:** [Ref No. 45] $[c_{ij}]_{3x4}$=[42 48 38 37; 40 49 52 51; 39 38 40 43] $[s_i]_{3x1}$=[160, 150, 190] $[d_j]_{1x4}$=[80, 90, 110, 160]

Table1: Numerical examples of transportation problem

5. Initial basic solution and its explanation

A feasible solution to a transportation problem is a set of non-negative allocation, x_{ij} that satisfies the row and column restrictions. In this section, solution and its explanation of few numbers of numerical problems are given below to make the proposed method very clear and well understandable.

5.1. Solution of the balanced transportation problem (Example-2)

Mathematical formulation, solution and its explanation of Example-2 of the balanced transportation problem is illustrated below in Table 2:

Sources	**Destinations**			**Supply**	**MRCD**	
	1	**2**	**3**			
1	2 15	10 7	25	**12**	18	-
2	10 8	12	7 14	**17**	6	6
3	17	19	7 21	**7**	4	4
Demand	**12**	**10**	**14**			
MCCD	9	12	11			
	9	-	7			

Table 2: Initial Basic Feasible Solution using Proposed Algorithm

Explanation:

- Here the HoMRCD and HoMCCD are respectively 18 and 12. As HoMRCD>HoMCCD, so according to the proposed algorithm at first, complete the row allocation along the HoMRCD by allocating 10 in the cell (1, 2) and 2 in the cells (1, 1). And then complete the column allocation along the HoMCCD. But during the procedure of row allocationthis column is exhausted. For these allocations row-1 and column-2 are exhausted.

- Now in the reduced matrix HoMRCD and HoMCCD are respectively 6 and 9. It is clear that, HoMRCD<HoMCCD, so complete the column allocation along the HoMCCD by allocating 10 in the cell (2, 1). Now complete the row allocation along the HoMRCD by allocating 7 in the cell (2, 3). Row-2 and column-1 are exhausted for these allocations.

- Finally, complete the solution by allocation 7 in the cell (3, 3).

- **Total transportation cost is**, 2x15+10x7+10x8+7x14+7x21=**425.**

5.2. Solution of the balanced transportation problem Example 9

Mathematical formulation, solution and its explanation of Example-9 of the balanced transportation problem is illustrated below in Table 3:

Explanation:

- Here the HoMRCD is 9 appears along row-4. Again HoMCCD is 9 appears along column-2 and column-4. According to the proposed algorithm HoMCCD along column-4 is to be selected as the smallest cost cell zero appears along this column.

- Now it is found that the selected HoMRCD = HoMCCD = 9. Again zero is the smallest cost cell appears in the cell (4, 4) and (4, 5) for HoMRCD. Also, zero is the smallest cost cell appears in the cell (4, 4) for HoMCCD. In these smallest cost cells, 4 is the maximum allocation can be made in the cell (4, 5) which appears along the highest MRCD.

Sources	**Destinations**						**Supply**	**MRCD**		
	1	**2**	**3**	**4**	**5**	**6**				
1	7	10	5 7	4	7	8	**5**	6	3	1
2	5	4 1	0 5	5	3	2 3	**6**	4	4	2
3	1 4	3	1 7	9	1	9	**2**	8	6	-
4	3 4	6	9	2 0	4 0	8	**9**	9	-	-
Demand	**4**	**4**	**6**	**2**	**4**	**2**				
MCCD	3	9	4	9	7	6				
	3	9	2	-	-	6				
	-	-	2	-	-	5				

Table 3: Initial basic feasible solution using proposed algorithm

- According to the proposed algorithm, at first complete the row allocation along the HoMRCD by allocating 4, 2 and 3 in the cells (4, 5); (4, 4) and (4, 1) respectively. And then complete the column allocation along the HoMCCD, but during the procedure of row allocationthis column is exhausted. For the allocations in this stage row-4, column-4, and column-5 are exhausted.

- Now in the reduced matrix HoMRCD and HoMCCD are respectively 6 and 9. It is clear that, HoMRCD<HoMCCD, so complete the column allocation along the HoMCCD by allocating 4 in the cell (2, 2). Now complete the row allocation along the HoMRCD by allocating 1 in the cell (3, 1), and again 1 in the cell (3, 3). Row-3, column-1, and column-2 are exhausted for these allocations.

- Again, in the reduced matrix HoMRCD and HoMCCD are respectively 2 and 5. As HoMRCD<HoMCCD, so complete the column allocation along the HOMCCD by allocating 2 in the cell (2, 6). But this allocation satisfies both the row and column, for this reason according to the algorithm allocate 0 (zero) in the cell (2, 3) to avoid the degeneracy. Now complete the row allocation HoMRCD, but his row is exhausted during the column allocation.

- Finally, allocating 5 in the cell (1, 3), complete the allocations.

- **Total transportation cost is**, 5x7+4x1+0x5+2x3+1x4+1x7+3x4+2x0+4x0=**68**

5.3. Solution of the unbalanced transportation problem Example10

Mathematical formulation, solution and its explanation of Example10 of the unbalanced transportation problem is illustrated below:

Sources	**Destinations**					**Supply**	**MRCD**
	1	**2**	**3**	**4**	**5**		
1	42	48	38	160 / 37	0 / 0	**160**	11
2	80 / 40	10 / 49	52	51	60 / 0	**150**	12
3	39	80 / 38	110 / 40	43	0	**190**	5
Demand	**80**	**90**	**110**	**160**	**60**		
MCCD	3	11	14	14	-		

Table 4: Initial Basic Feasible Solution using Proposed Algorithm

Explanation:

- In this problem, HoMRCD<HoMCCD as the HoMRCD is 12 and HoMCCD is 14. To complete the column allocation along the HoMCCD, column-4 is selected for allocation between column-3 and column-4. Because the smallest cost cell 37 among the cells along these two columns appears along column-4. Now 160 is allocated in the cell (1, 4), and both the row-1 and column-4 are exhausted for this allocation, so we assigns a zero in the cell (1, 5).

- Now complete the allocation along the HoMRCD, first allocation 80 is in the cell (2, 1); second allocation 60 is in the dummy cell (2, 5) and then next to the second allocation 10 in the cell (2, 2) made.

- Finally complete the allocation by putting 80 in the cell (3, 2) and 110 in the cell (3, 3).

- **Total transportation cost is**,
160x37+0x0+80x40+10x49+60x0+80x38+110x40=**17050**

6. Comparative study

Initial basic feasible solution obtained by various methods and the optimal solution of the numerical examples are tabulated in the Table 5:

7. Evaluation of the performance of NWCM

It is already mentioned that in the solution procedure of NWCM, top left corner cell or north-west corner is always considered at every stage of allocation. For this allocation procedure, NWCM is called a positioned based method. Performance of this method is justified by solving Example 1. The tabular form of Example 1 is shown in the Table 6.

Balanced Transportation Problems						**Ex. No.**	**Unbalanced Transportation Problems**					
NWCM	LCM	VAM	EDM	Proposed	Opt.		NWCM	LCM	VAM	EDM	Proposed	Opt.
5925	4550	5125	4550	4550	452	**1**	2968	2968	2424	2424	2424	2424
545	433	425	439	425	425	**2**	18800	8800	8350	1015	7750	7750
273	231	204	218	200	200	**3**	14725	1462	1322	1307	12475	1247
980	960	960	960	930	920	**4**	13100	9800	9200	1010	9200	9200
234	191	187	218	183	183	**5**	1010	1210	880	950	930	840
363	305	290	290	290	290	**6**	90	57	57	57	57	57
540	435	470	415	410	410	**7**	20400	1640	1640	1640	15800	1550
4782	3572	3663	3572	3572	345	**8**	8150	6450	6000	6000	6050	5600
95	70	68	70	68	68	**9**	19700	1375	1225	1225	11800	1150
1994	1123	1104	1102	1102	110	**10**	20530	1878	1706	2053	17050	1705

Table 5: A comparative study of different solutions

Sources	Destinations			Supply
	P	Q	R	
A	6	8	10	150
B	7	11	11	175
C	4	5	12	275
Demand	200	100	300	

Table 6: Tabular form of Example 1

Again the same problem (Example-1) can be rearranged, and the rearranged tabular formulation is shown in Table 7:

Sources	Destinations			Supply
	Q	R	P	
A	8	10	6	150
B	11	11	7	175
C	5	12	4	275
Demand	100	300	2000	

Table 7: Rearranged Tabular form of Example 1

Solution of Example-1 obtained by various methods for both the mathematical formulations is shown in Table 8:

	NWCM	LCM	VAM	EDM	**Proposed**
Table6	5925	4550	5125	4550	4550
Table 7	4925	4550	5125	4550	4550

Table 8: Initial Basic Feasible Solution

In the Table 8, it is observed that all the methods are yielding the same result for both the formulation of Example1, except the NWCM. Result varies in case of NWCM for the mathematical formulation of the problem, and also for the position based solution procedure of NWCM.

8. Evaluation of the obtained result

In this research we studied the initial basic feasible solution obtained by proposed method and few other reputed methods. Again we have shown the comparisons of the obtained result in Table 9, Table 10, Chart 1, Chart 2 and Chart 3 in various forms to evaluate in depth performance of the initial basic feasible solution. These comparisons are shown in following.

Balanced Transportation Problems				**Ex. No**	**Unbalanced Transportation Problems**			
NWCM	**LCM**	**VAM**	**EDM**		**NWCM**	**LCM**	**VAM**	**EDM**
+	+	+	=	**1**	+	+	=	=
+	+	=	+	**2**	+	+	+	+
+	+	+	+	**3**	+	+	+	+
+	+	+	+	**4**	+	+	=	+
+	+	+	+	**5**	+	+	**-**	+
+	+	=	=	**6**	+	=	=	=
+	+	+	+	**7**	+	+	+	+
+	=	+	=	**8**	+	+	**-**	**-**
+	+	=	+	**9**	+	+	+	+
+	+	+	=	**10**	+	+	+	+

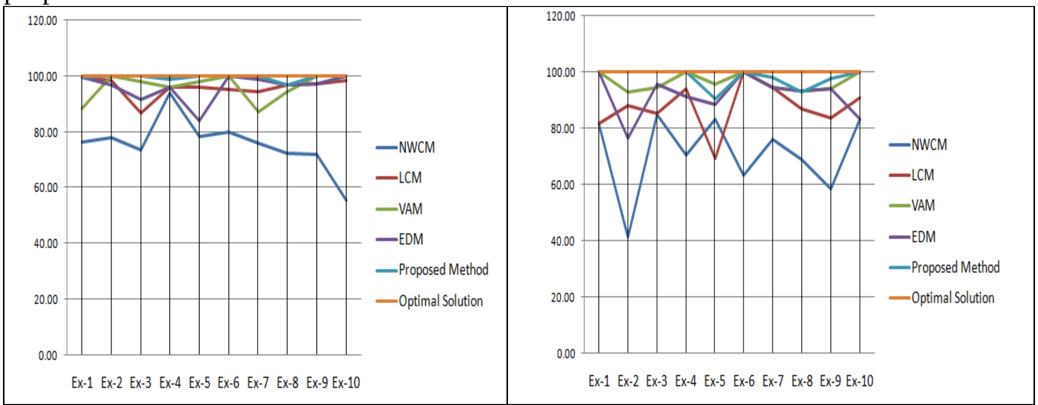
Table 9: Performance of the proposed Method

Table 9 is constructed basically to observe the performance of the proposed method over other classical methods. To represent this performance of the proposed method, we have used three signs “ +, **-** , and =”. These signs represent the better, poor and same/equal performance of the proposed method. Here it is observed that the proposed method performs either better or equal in comparison to other methods in most of the cases. And poor performance of proposed method is very less.

Balanced Transportation Problems					Example No.	Unbalanced Transportation Problems				
NWCM	**LCM**	**VAM**	**EDM**	**Proposed Method**		**NWCM**	**LCM**	**VAM**	**EDM**	**Proposed Method**
76.37	99.45	88.29	99.45	99.45	**1**	81.67	81.67	100	100	100
77.98	98.15	100	96.81	100	**2**	41.22	88.07	92.81	76.35	100
73.26	86.58	98.04	91.74	100	**3**	84.72	85.30	94.33	95.41	100
93.88	95.83	95.83	95.83	98.92	**4**	70.23	93.88	100	91.09	100
78.21	95.81	97.86	83.94	100	**5**	83.17	69.42	95.45	88.42	90.32
79.89	95.08	100	100	100	**6**	63.33	100	100	100	100
75.93	94.25	87.23	98.80	100	**7**	75.98	94.51	94.51	94.51	98.10
72.31	96.81	94.40	96.81	96.81	**8**	68.71	86.82	93.33	93.33	92.56
71.58	97.14	100	97.14	100	**9**	58.38	83.64	93.88	93.88	97.46
55.27	98.13	99.82	100	100	**10**	83.05	90.79	99.94	83.05	100

Table 10: Percentage of near optimality

Table 10 is constructed to observe the near optimality status of various methods for finding an initial basic feasible solution. If the percentage is 100, indicates that the obtained result is numerically equal to the optimal solution. The simulation results of Table-9, shows proposed method(IEDM) is directly yielding optimal result for balanced transportation problems in 70% cases. Again this performance is Nil for NWCM, LCM, 30% for VAM, and 20% for EDM. In case of unbalanced transpiration problems, this performance is zero, 10%, 30%, 20%, and 60% respectively for NWCM, LCM, VAM, EDM and proposed method. This data analysis ensures the better performance of the proposed method.



Balanced Transportation Problems	**Unbalanced Transportation Problems**

Chart 1: Percentage of near optimality

Chart 1 is inserted in our study to observe the performance of near optimality graphically. From the graph it is clearly observed that the line indicates the performance of proposed method is closely running with optimal line. This also ensures the better performance of the proposed method.

In this study we have also calculated the average percentage of near optimality to see the overall performance of various methods which is shown in the Table 11.

Balanced Transportation Problems					Unbalanced Transportation Problems				
NWCM	LCM	VAM	EDM	Proposed	NWCM	LCM	VAM	EDM	Proposed
75.47	95.72	96.15	96.05	99.52	71.05	87.41	96.43	91.60	97.84

Table11: Average percentage of near optimality

This performance is also shown in Chart 2 for balanced transportation problems and in Chart 3 for unbalanced transportation problems.

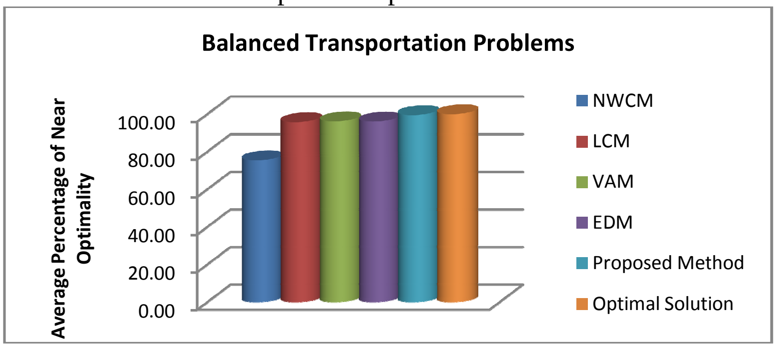


Chart 2: General performance of various methods for Balanced Transportation Problem

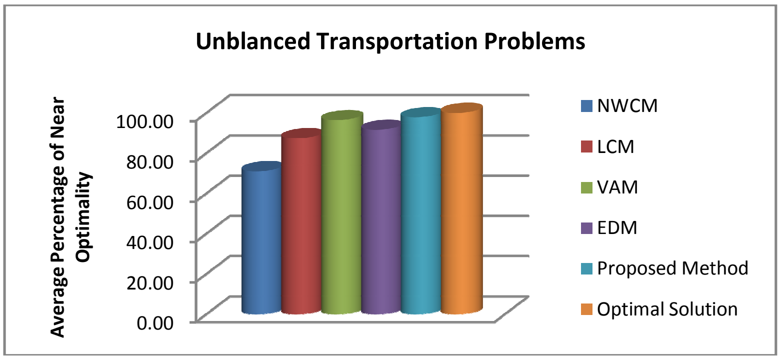


Chart 3: General performance of various methods for Unbalanced Transportation Problem

From the above charts it is also observed that the overall performance of the proposed method is comparatively better than all other methods.

9. Conclusion

Transportation models are the least cost means to transport a product manufactured at different plants or factories to a number of different warehouses. This is an important factor in logistic management. In this research we brought an improvement to the existing solution procedure of EDM with the help of our newly introduced allocation procedure for obtaining an initial basic feasible solution for the transportation problems. Proposed method will ultimately help the management of an industry to determine their supply route. Again the same concept may apply on other methods where it is necessary to find the penalty cost or cost cell difference. Further study may be carried out to evaluate the performance of the same operation on other methods. Efficiency of the newly introduced method is justified in the result and evaluation part of this study, where it is observed the overall performance of the proposed method is better in comparison to other well discussed traditional methods. It is also found that the newly introduced method is computationally easier and requires less time to obtain an initial basic feasible solution for the transportation problems. Again, better initial solution takes less number of iterations to obtain optimal solution. Considering all these, we claim that our proposed method may be used to obtain an initial basic feasible solution for the transportation problems and may be included in all the operation research study.

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