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Harmonic Module of Electron and its Space-Time Geometry

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ABSTRACT
A theoretical framework that may connect electromagnetism and gravitation has been developed. In this new scenario a harmonic module of electron has been proposed and has been extended to general relativistic approach and space-time geometry.

Keywords: photon-photon collision, module of electron

1. Introduction

The experimental result in **[1]** claim that two photons of same frequency can combine to form one photon having double the frequency. In **[1]** it has been shown that two infrared photons with same directional wave vector passing through frequency doubling crystal form one green photon. Again, photons may be of two types **[2-4]-** right rotating and left rotating. Therefore, helicity is +1 and -1. The process $\gamma + \gamma = e + \overline{e}$, pair production in photon-photon collision has been shown in [**5,6]** where, γ has electromagnetic wave nature and it has no mass at rest. But e and $\overline{e}$ have rest masses i.e. they are stable at rest. In this work trial would be made to find out the process of combination of two photons which can satisfy the experimental results [1,5,6] mentioned below.

Case-I: frequency of oscillation of the the composed system will be double that of interacting photons of same frequency and

Case-II: pair production in photon-photon collision creates the electron as shown in the relation $\gamma + \gamma = e + \overline{e}$. Therefore, under some conditions composed system will be stable at rest.

2. System of electron

Photon has dual nature: particle like and like an electromagnetic wave. Propagation of photon means flow of energy through space time. Again, each photon possesses rotational motion, each electromagnetic wave is transverse in nature so, every photon is a system composed of three velocities: linear, rotational and that of oscillation. Hence, oscillation is in all directions on a plane and the wave becomes unpolarized. In absence of rotational motion it possesses one dimensional oscillation i.e. the wave becomes linearly polarized. Both cases are shown in figure 1.

Case-II implies that when two photons interact with each other or be superimposed on each other then, under some boundary conditions, one electron is formed. This is possible under the condition that the composed system will be stable and independent of linear motion. Therefore, we may assume, in the composed system i.e. in the system of electron, rotational and oscillatory motions of each propagating photon has been composed. Also, for simplification let, these two photons or two unpolarised waves are interacted perpendicularly.

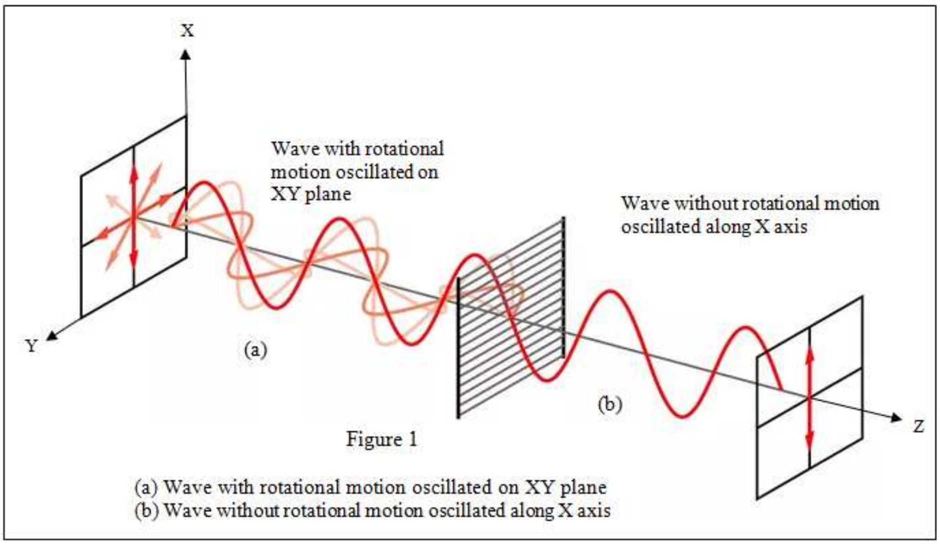

Figure 1:

Now, for this composition, we may consider six reference frames S, S_1, S_2, S_3, S_4, and S_5 where, S is the frame of observer and S contains S_1, S_1 contains S_2, S_2 contains S_3, S_3 contains S_4, and S_4 contains S_5.

Also, S and S_1 have both their Z axes parallel and S_1 is rotating with an angular velocity ω about Z axis as observed by S. Next, S_1 and S_2 have both their co-ordinate axes aligned parallel and S_2 is oscillating along X_1 axis with angular frequency $\bar{\omega}$. Next, Y axis of S_2 and S_3 are parallel and S_3 has a rotation through an angle $\theta = 90^O$ in X_2 Z_2 plane

Again, S_3 and S_4 have both their Z axes parallel and S_4 is rotating with an angular velocity ω about Z_3 axis as observed by S_3. Next, S_4 and S_5 have both their co-ordinate axes parallel and S_4 is oscillating along X_4 axis with angular frequency $\bar{\omega}$.

Now, we may transfer any coordinate from S_5 to S which is effected due to composition of four velocities. In our first order approximation, this type of incidence is performed in an electron. But, we have to show that

a) It supports the first experimental result i.e. frequency of composed system will be double that of concerned photons.
b) Composed system has one kind of spherical harmonic oscillation so that, it is existed or energy is conserved without linear motion.

Now, coordinate transformation from S_5 to S is

$$x^i = A_{ij} O_{jk} R_{kl} A_{lm} O_{mn} \overline{x}^n \tag{1}$$

Accordingly,

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega t & -\sin\omega t & 0 \\ 0 & \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sin\overline{\omega}t & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega t & -\sin\omega t & 0 \\ 0 & \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sin\overline{\omega}t & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \\ \overline{t} \end{pmatrix}$$

…(2)

From (2) we obtain $x = -\overline{x}\sin^2\omega t \sin\overline{\omega}t - \overline{y}\sin\omega t\cos\omega t - \overline{z}\sin\overline{\omega}t\cos\omega t$,
$y = \overline{x}\sin\omega t\cos\omega t\sin\overline{\omega}t + \overline{y}(\cos\omega t)^2 - \overline{z}\sin\omega t\sin\overline{\omega}t$,
$z = \overline{x}\sin\overline{\omega}t\cos\omega t - \overline{y}\sin\omega t$ and $t = \overline{t}$.

Therefore, we get

$$x^2 + y^2 + z^2 = (-\overline{x}\sin^2\omega t\sin\overline{\omega}t - \overline{y}\sin\omega t\cos\omega t - \overline{z}\sin\overline{\omega}t\cos\omega t)^2$$
$$+(\overline{x}\sin\omega t\cos\omega t\sin\overline{\omega}t + \overline{y}(\cos\omega t)^2 - \overline{z}\sin\omega t\sin\overline{\omega}t)^2 + (\overline{x}\sin\overline{\omega}t\cos\omega t - \overline{y}\sin\omega t)^2 \tag{3}$$

Now, taking $x^2 + y^2 + z^2 = R^2$ and after some calculations one can write from (3)

$$R^2 = \overline{x}^2\sin^2\omega t + \overline{y}^2 + \overline{z}^2\sin^2\omega t = (\overline{x}^2 + \overline{z}^2)\sin^2\omega t + \overline{y}^2 \tag{4}$$

From, equation (4) we get table-1 and figure-1

Table-1. Change of radial distance of a point with time period in the system

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	$\frac{8T}{8}$
R^2	$\bar{y}^2$	$\frac{\bar{x}^2+2\bar{y}^2+\bar{z}^2}{2}$	R^2_{max}	$\frac{\bar{x}^2+2\bar{y}^2+\bar{z}^2}{2}$	$\bar{y}^2$	$\frac{\bar{x}^2+2\bar{y}^2+\bar{z}^2}{2}$	R^2_{max}	$\frac{\bar{x}^2+2\bar{y}^2+\bar{z}^2}{2}$	$\bar{y}^2$

where, $R^2_{\max} = \bar{x}^2 + \bar{y}^2 + \bar{z}^2$.

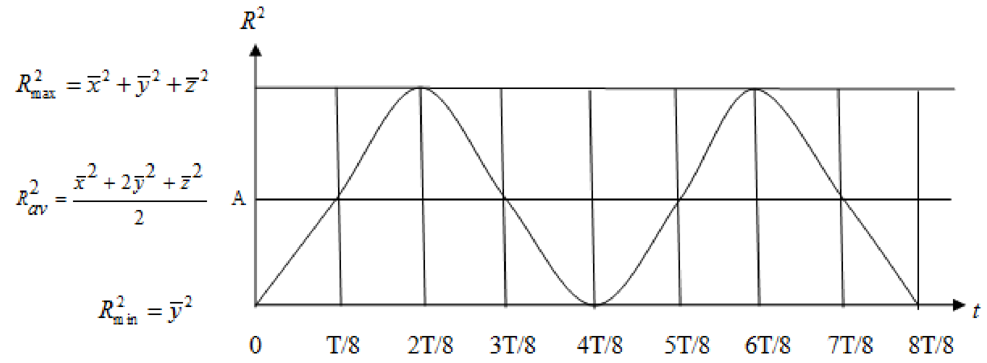

Figure 2. Change of radial distance of a point with time period in the system

Now, from figure-2 it is clear that

i) The composed system is oscillating along radial direction. This means that within a certain period of time, the system becomes compressed and then expanded.
ii) This oscillation is a kind of simple harmonic motion along radial direction.
iii) The frequency of this oscillation is double that of two interacting photons or any point in the system about the equilibrium position A shown in figure 2.

Therefore, with some approximations, equation for displacement of any point in the system of electron is

$$r = r_0 \sin \omega_s t \qquad (5)$$

where, $r_0^2 = (\bar{x}^2 + \bar{y}^2 + \bar{z}^2) - (\bar{x}^2+2\bar{y}^2+\bar{z}^2)/2 = (\bar{x}^2+\bar{z}^2)/2$, so, $r_0 = \sqrt{(\bar{x}^2+\bar{z}^2)/2}$ and $\omega_s = 2\omega$. Since (5) represents a simple harmonic oscillation so, this work is agreed with experimental results shown in Case-I and Case-II due to the fact that energy of the system would be conserved and frequency of oscillation of the system would be double that of interacting photons. It is also pointed out that in equation (1) O_{jk} and O_{pq} are two matrices with same rotational directions. So, helicity of two interacting photons are same (say +1). Now, to get the composition of two photons of opposite helicity we have to put $-\omega t$ instead ωt in one of the rotational matrices (say O_{pq}).

3. Fundamental tensor in the system of electron

Our assumption that two exactly similar photonic systems interacting perpendicularly create a system (assumed spherical) oscillating in radial direction which is an electron. Since there is no sharp boundary of photon due to its electric wave form so, electron also, has no sharp boundary. Therefore, one kind of energy-momentum tensor exists in the space surrounding an electron. Now space-time coordinate in this system will be as shown in equation (1) which is clearly specified in (2). One may go to spherical coordinate system taking $\bar{x} = r\sin\theta\cos\phi$, $\bar{y} = r\sin\theta\sin\phi$, $\bar{z} = r\cos\theta$ and $\bar{t} = \bar{t}$ in (2). Therefore, we obtain

$$\begin{aligned}
x &= -r(\sin\theta\cos\phi\sin^2\omega t\sin\bar{\omega}t + \sin\theta\sin\phi\sin\omega t\cos\omega t + \cos\theta\sin\bar{\omega}t\cos\omega t) \\
y &= r(\sin\theta\cos\phi\sin\omega t\cos\omega t\sin\bar{\omega}t + \sin\theta\sin\phi\cos^2\omega t - \cos\theta\sin\omega t\sin\bar{\omega}t) \\
z &= r(\sin\theta\cos\phi\sin\bar{\omega}t\cos\omega t - \sin\theta\sin\phi\sin\omega t) \\
t &= \bar{t}
\end{aligned} \qquad (6).$$

But, we have line element in 4-dimentional space time

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \qquad (7)$$

which gives

$$g_{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -c^2 \end{pmatrix}$$

(8)

Now, using (6) and (8) one may get the fundamental tensor in the system of electron as given below

$$\bar{g}_{ij} = \frac{\partial x^a}{\partial \bar{x}^i}\frac{\partial x^b}{\partial \bar{x}^j} g_{ab} \qquad (9)$$

where, $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^4 = t$ and $\bar{x}^1 = r$, $\bar{x}^2 = \theta$, $\bar{x}^3 = \phi$, $x^4 = \bar{t}$ which are specified in (6)

Since, in equation (8), $g_{ab} = 0$ for $a \neq b$, $g_{11} = g_{22} = g_{33} = 1$ and $g_{44} = -c^2$.
Therefore, equation (6), (8) and (9) give us

$$\bar{g}_{11} = (\frac{\partial x}{\partial r})^2 + (\frac{\partial y}{\partial r})^2 + (\frac{\partial z}{\partial r})^2, \qquad \bar{g}_{12} = \frac{\partial x}{\partial r}\frac{\partial x}{\partial \theta} + \frac{\partial y}{\partial r}\frac{\partial y}{\partial \theta} + \frac{\partial z}{\partial r}\frac{\partial z}{\partial \theta},$$

$$\bar{g}_{13} = \frac{\partial x}{\partial r}\frac{\partial x}{\partial \phi} + \frac{\partial y}{\partial r}\frac{\partial y}{\partial \phi} + \frac{\partial z}{\partial r}\frac{\partial z}{\partial \phi}, \; \bar{g}_{14} = \frac{\partial x}{\partial r}\frac{\partial x}{\partial t} + \frac{\partial y}{\partial r}\frac{\partial y}{\partial t} + \frac{\partial z}{\partial r}\frac{\partial z}{\partial t}, \quad \bar{g}_{12} = \bar{g}_{21}, \; \bar{g}_{13} = \bar{g}_{31},$$

$$\bar{g}_{14} = \bar{g}_{41}, \; \bar{g}_{22} = (\frac{\partial x}{\partial \theta})^2 + (\frac{\partial y}{\partial \theta})^2 + (\frac{\partial z}{\partial \theta})^2 \quad \bar{g}_{23} = \frac{\partial x}{\partial \theta}\frac{\partial x}{\partial \phi} + \frac{\partial y}{\partial \theta}\frac{\partial y}{\partial \phi} + \frac{\partial z}{\partial \theta}\frac{\partial z}{\partial \phi},$$

$$\bar{g}_{24} = \frac{\partial x}{\partial \theta}\frac{\partial x}{\partial t} + \frac{\partial y}{\partial \theta}\frac{\partial y}{\partial t} + \frac{\partial z}{\partial \theta}\frac{\partial z}{\partial t}, \quad \bar{g}_{32} = \bar{g}_{23}, \quad \bar{g}_{42} = \bar{g}_{24},$$

$$\bar{g}_{33} = (\frac{\partial x}{\partial \phi})^2 + (\frac{\partial y}{\partial \phi})^2 + (\frac{\partial z}{\partial \phi})^2, \quad \bar{g}_{34} = \frac{\partial x}{\partial \phi}\frac{\partial x}{\partial t} + \frac{\partial y}{\partial \phi}\frac{\partial y}{\partial t} + \frac{\partial z}{\partial \phi}\frac{\partial z}{\partial t}, \quad \bar{g}_{43} = \bar{g}_{34},$$

$$\bar{g}_{44} = (\frac{\partial x}{\partial t})^2 + (\frac{\partial y}{\partial t})^2 + (\frac{\partial z}{\partial t})^2 - c^2$$

Hence, we obtain the metric in the system of electron to be

$$ds^2 = \bar{g}_{\alpha\beta} d\bar{x}^{\alpha} d\bar{x}^{\beta} \tag{10}$$

where, $\alpha, \beta = 1, 2, 3, 4$ and $d\bar{x}^1 = dr$, $d\bar{x}^2 = d\theta$, $d\bar{x}^3 = d\phi$, $d\bar{x}^4 = dt$.

Due to complex motion in the system of electron space-time becomes curved. Now, using fundamental tensor specified in (10) we get Christoffel tensor, Ricci's tensor and curvature invariant respectively as given below

$$\Gamma^{l}_{\alpha\beta} = \frac{1}{2}\bar{g}^{lk}(\frac{\partial \bar{g}_{\alpha k}}{\partial \bar{x}^{\beta}} + \frac{\partial \bar{g}_{\beta k}}{\partial \bar{x}^{\alpha}} - \frac{\partial \bar{g}_{\alpha\beta}}{\partial \bar{x}^{k}}), \quad R_{\alpha\beta} = \frac{\partial \Gamma^{k}_{\alpha k}}{\partial \bar{x}^{\beta}} - \frac{\partial \Gamma^{k}_{\alpha\beta}}{\partial \bar{x}^{k}} + \Gamma^{l}_{\alpha k}\Gamma^{k}_{l\beta} - \Gamma^{l}_{\alpha\beta}\Gamma^{k}_{lk},$$

$$R = \bar{g}^{\alpha\beta} R_{\alpha\beta}$$

Again, Einstein's field equation for gravity is

$$R_{\alpha\beta} - \frac{1}{2}\bar{g}_{\alpha\beta} R = kT_{\alpha\beta} \tag{11}$$

where, k, $\bar{g}_{\alpha\beta}$ and $T_{\alpha\beta}$ respectively are the coupling constant, the fundamental tensor and the energy momentum tensor in case of gravitational field. This equation also has been extended to Einstein-Maxwell's equation for electromagnetic field omitting the cosmological constant. Therefore, we may assume, the relation in (11) to be the general equation for electromagnetic field.

Now, using Christoffel tensor, Ricci's tensor and curvature invariant as specified above, we may get energy-momentum tensor of electric field in the space surrounding an electron as given below

$$T_{\alpha\beta} = C(R_{\alpha\beta} - \frac{1}{2}\bar{g}_{\alpha\beta}R) \tag{12}$$

where C is the exact coupling constant for this system.

However, before ending the work we want to propose a process by which we can find out the energy- momentum tensor of electromagnetic field of a linearly moving electron.

Now, let an electron in a frame S' be moving with velocity u with respect to *S* along positive Z axis where, co-ordinate axes of both frames are parallel. Then space- time nature surrounding the electron will be changed. This space-time nature will be adjusted if we take a Lorentz transformation from S' to S. So, space-time co-ordinate transformation in the system of moving electron will be

$$x^i = A_{ij}O_{jk}R_{kl}A_{lm}O_{mn}L_{np}\bar{x}^p \tag{13}$$

where, A_{ij}, O_{jk}, R_{kl}, A_{lm}, O_{mn} are specified in (2) and L_{np} is Lorentz matrix. Now, equation (13) gives us fundamental tensor $\bar{g}_{\alpha\beta} = \frac{\partial x^a}{\partial \bar{x}^\alpha}\frac{\partial x^b}{\partial \bar{x}^\beta} g_{ab}$ where, g_{ab} is specified in (8).

Now, using this tensor one can find out energy-momentum tensor from equation (13). That tensor should be the energy-momentum tensor of electromagnetic field of a linearly moving electron.

4. Conclusion

Following conclusions may be drawn from this work:-

1. Due to superposition of two exactly similar photons perpendicularly, an electron is created.
2. Electron is a sphere like particle, it has radial oscillation i.e. the sphere is simultaneously compressed and expanded within a certain period of time. This oscillation is simple harmonic one. But, it has no sharp boundary.
3. Due to this oscillation energy in the system is conserved and it is stable. Due to resultant motion of the system, energy momentum tensor obtained is the representation of field and this system is the fundamental particle responsible for the concerned field.
4. But, above energy momentum tensor is obtained by using fundamental tensor in microscopic world as discussed in the text.
5. Therefore, general relativity and particle physics are strongly related in microscopic world.

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