2015

STATISTICS

[Honours]

PAPER — VI

Full Marks : 100

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

Answer **Q. No. 1 or 2 and Q. No. 3 or 4 & 5**

1. (*a*) Under regularity assumptions, stated by you, state and prove Rao-Cramer inequality.

(*b*) When does the equality in Rao-Cramer inequality hold ?

(*c*) Examine, if Rao-Cramer inequality holds, for a random sample of size n from Rec$(0, \theta)$ distribution. 15

2. (*a*) Define unbiased estimator, estimable parametric function, asymptotically unbiased estimator.

(*b*) Show that if T is an unbiased estimator of θ, then T^2 is a biased estimator of θ^2.

(*c*) Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from $N(\mu, \sigma^2)$, all parameters unknown. Show that

(*i*) Sample mean $\overline{X}$ and sample variance

$$s^2\left(=\frac{1}{n-1}\Sigma\left(X_i-\overline{X}\right)^2\right)$$

are the unbiased estimators of μ and σ^2 respectively.

(*ii*) Sample standard deviation is not an unbiased but asymptotically unbiased estimator of σ. 15

3. (*a*) State and prove Neyman-Pearson fundamental lemma.

(*b*) Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from the population having pdf

$$f(x,\theta) = \begin{cases} \theta x^{\theta - 1} ; 0 < x < 1 \\ 0 \qquad ; \text{otherwise} \end{cases}$$

where $0 < \theta < \infty$.

Derive UMP critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$.

Show that the above testing can be done using a chi-square statistic. 15

4. (*a*) Define power, level of significance and size of a non-randomized test.

(*b*) Given a random sample of size n from $N(\mu, \sigma^2)$ with all parameters unknown, derive the likelihood ratio test for $H_0 : \mu = 0$ against all possible alternatives. Show that the test may be based on a t-statistic. 15

5. Answer any *three* from the following : 8 × 3

(*a*) Define sufficient statistic. Show that $X_1 + X_2$ is sufficient for θ if X_1 and X_2 constitute a random sample from Bernoulli (θ).

(*b*) Suppose $X_1, X_2, ..., X_n$ is a random sample of size n from Rec$(0, \theta)$. Show that $X_{(n)} = \max_{1 \le i \le n} \{X_i\}$ is a consistent estimator of θ.

Is $Y_n = \frac{2}{n}\sum_{i=1}^{n} X_i$

Consistent for θ ?

(*c*) What do you mean by minimum variance bound (MVB) estimator ? Show that the sample mean, based on a random sample of size n from

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & ; 0 < x < \infty \\ 0 & ; \text{otherwise} \end{cases}$$

where $0 < \theta < \infty$, is the MVB estimator of θ and has variance $\frac{\theta^2}{n}$.

(*d*) On the basis of a random sample from Poisson distribution with parameter λ, discuss, how you construct the MP Test of exact size α $(0 < \alpha < 1)$ for testing $H_0 : \lambda = 3$ against $H_1 : \lambda = \lambda_1 (> 3)$.

(*e*) Define confidence interval. On the basis of a random sample of size n from $N(\mu, \sigma^2)$, σ^2 known obtain $100(1-\alpha)\%$ confidence interval for the parameter μ.

GROUP – B

6. Answer any *three* of the following : 8 × 3

(*a*) Write short notes on any *two* of the following :

(*i*) Pilot survey

(*ii*) Non-sampling error

(*iii*) Random sampling numbers.

(*b*) A sample of size 3 is drawn from a population of size N by simple random sampling with replacement (SRSWS). Show that the probabilities that the sample contains 1, 2 and 3 different units are

$$P_1 = \frac{1}{N^2},\ P_2 = \frac{3(N-1)}{N^2},\ P_3 = \frac{(N-1)(N-2)}{N^2}.$$

As an estimate of the population mean $(\bar{Y})$, we take $\bar{y}'$, the unweighted mean over the different units in the sample. Show that the average variance of $\bar{y}'$ is

$$\operatorname{var}(\bar{y}') = \frac{(2N-1)(N-1)S^2}{6N^2},$$

$$\text{where } S^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2$$

(c) In simple random sampling without replacement, show that the value of the sample size n which minimize $Q(n) = V(n) + C(n)$ is given by

$$n = \sqrt{\frac{S^2}{C_1}},$$

where the cost function $C(n)$ is given by $C_0 + C_1 n$ and the $V(n)$ is the variance of the sample mean.

(*d*) Give an unbiased estimator of population mean and derive the expression of its variance in SRSWOR.

(*e*) Prove that, with usual notation, for estimating the population mean under SRSWOR

$$V_{\text{opt}} \leq V_{\text{prop}} \leq V_{\text{rand}}.$$

7. Answer any *two* of the following : 6 × 2

(*a*) Let X_{ij} be the value X for the jth selected individual from the i th stratum in stratified random sampling. Show that

$$T = \sum_{i} \sum_{j} a_{ij} X_{ij}$$

be the BLUE of population mean, where a_{ij}'s are suitable constants.

(*b*) Describe systematic sampling procedure. Discuss the advantages and disadvantages of this sampling technique.

(*c*) What is stratified sampling ? Discuss the advantages of the stratified random sampling over simple random sampling.

(*d*) What is meant by ratio estimator of a population mean? Derive approximate formulae for its expectation and variance.

[*Internal Assessment* : 10 Marks]
