2015

STATISTICS

[Honours]

PAPER – III

Full Marks : 100

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

Answer any **three** questions

1. (*a*) Suppose $(X_1, X_2, ..., X_k)$ or multinomial $(m, p_1, p_2, ..., p_k)$ where

$$p_i > 0 \;\; V_i = 1, 2, ..., k, \; \sum_{i=1}^{k} p_i = 1 \text{ and } \sum_{i=1}^{k} X_i = m.$$

(*Turn Over*)

Show that the correlation coefficient between X_i and X_j for any $i \neq j$, $i, j = 1, 2, ..., k$ is negative. Show that the regression of any one variable on the others is linear. Also show that the variance of the conditional distribution is linear. Derive the expression for $\rho_{12.34}$.

(*b*) The vector variable $\underset{\sim}{X}_{p \times 1}$ follows $N_p(\underline{\mu}, \underline{\Sigma})$. Derive the distribution of $(\underline{X} - \underline{\mu})' \Sigma^{-1} (\underline{X} - \underline{\mu})$.

12 + 6

2. (*a*) Let $\underset{\sim}{X}_{p \times 1} = (X_1, X_2, ..., X_p)'$ have the distribution $N_p(\underline{\mu}, \Sigma)$. Find the moment generating function of $\underset{\sim}{Y} = \underline{B}\underset{\sim}{X}$, Where $\underline{B}$ is a $q \times p$ matrix of rank $q \,(\leq p)$.

(*b*) Let $\underset{\sim}{X} = (X_1, X_2, ..., X_p)'$ follow a p-variate normal distribution. Find the conditional distribution of X_1 when $X_2 = x_2, ..., X_p = x_p$. Hence show that the regression of X_1 on $X_2, X_3, ..., X_p$ is linear and the conditional variance is independent of $x_2, x_3, ..., x_p$.

8 + 10

3. Define multiple correlation coefficient. Derive the expression for the multiple correlation coefficient $r_{1.23...p}$ based on observed data x_{ij}, $i = 1(1)p$, $j = 1(1)n$. Show that $r_{1.23...p}$ is numerically at least as high as any total or partial correlation coefficient involving x_1.

2 + 8 + 8

4. (a) Suppose two independent random variables X_1 and X_2 follow the exponential distribution with p.d.f.

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda > 0$. Find the distribution of $X_1 - X_2$.

(b) Let X and Y be two independent random variable, each distributed in the form $N(0, 1)$. Show that $Z = X / |Y|$ has Cauchy distribution.

(c) Let $F_i(x)$ be the cumulative distribution function of the random variable X_i, $i = 1, 2, \ldots, n$. If the random variables are absolutely continuous and independent, find the distribution of $\left[\prod_{i=1}^{n} F_i(x)\right]^{1/n}$.

6 + 6 + 6

5. (a) The random variables X_i $(i = 1, 2, \ldots, n)$ are independently distributed, respectively, as

$N(\mu_i, \sigma_i^2)$. Let $\overline{X}_w = \sum_{i=1}^{n} w_i X_i \Big/ \sum_{i=1}^{n} w_i$,

where $w_i = \dfrac{1}{\sigma_i^2}$.

Show that the $\overline{X}_w$ is independent of

$$S_w^2 = \sum_{i=1}^{n} w_i (X_i - \overline{X}_w)^2$$ and S_w^2 is

distributed as a chi-square with $(n-1)$ d.f.

(*b*) Deriye the sampling distribution of the smallest among n sample observations from the exponential population with density function

$$f(x)=\frac{1}{\theta}\ e^{-x/\theta},\ x\geq 0.$$ 12 + 6

GROUP – B

Answer any **one** question

6. (*a*) What are control charts ? Write down their uses. Describe the construction of s.d. chart.

(*b*) Distinguish between (*i*) process control and product control (*ii*) assignable causes and chance causes. (3 + 3+ 6) + (3 + 3)

7. Describe a double sampling inspection plan for an attribute and determine the constants involved in the plan. 18

GROUP – C

Answer any **one** question

8. (*a*) Write an algorithm to calculate quartile deviation from a set of observations.

   (*b*) Write an algorithm to obtain real root of an equation using Newton-Raphson method. 9 + 9

9. (*a*) Write a C program to obtain matrix B if matrix A is given where $AB = I$ (I is the identity matrix of suitable order).

   (*b*) Write a C program to calculate coefficient of variation from a data set. 10 + 8

[*Internal Assessment* – 10 Marks]