2015

STATISTICS

[Honours]

PAPER – II (New)

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP—A

1. Answer any *five* questions : 5×5

(*i*) Establish a recursion relation between central moments of a normal (μ, σ^2)-distribution. Hence find a moment measure of kurtosis of the distribution and add an appropriate comment. 5

(*Turn Over*)

(*ii*) Show that for n events $A_1, A_2, \ldots, A_n$

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$ 5

(*iii*) If two fair dice are rolled once, show that the two events 5

A : The sum of the two dice is 7

B : Two dice have the same number are not statistically independent.

(*iv*) Each of two people tosses a fair coin n times. Find the probability that they score the same number of heads. 5

(*v*) Give the Axiomatic definition of probability due to Kolmogorov. Use this definition to prove that

$$P(A) + P(B) \geq P(A \cup B),$$

where A and B are any two events of the associated sigma field. 2 + 3 (*vi*) The joint density function of X and Y is given by

$$f(x, y) = 2e^{-x-2y}, \quad 0 < x, y < \infty$$
$$= 0 \quad , \text{ otherwise}$$

Compute $P(X < Y)$. 5

(*vii*) For mutually exclusive events $B_1, B_2, \ldots, B_n$ with $P(B_i) > 0$ let $B = \bigcup_{i=1}^{n} B_i$ and $P(A \mid B_i) = p$ for $i = 1, 2, \ldots, n$. Show that $P(A|B) = p$. 5

(*viii*) Find the moment generating function (m.g.f.) of a random variable X following a binomial (m, p)-distribution and use it to find the expectation of $X(X-1)$. 5

(*ix*) If the cumulative distribution function (c.d.f.) $F(.)$ of a random variable X is given by

$$F(x) = \begin{cases} 0 & , \text{ if } x < 1 \\ \dfrac{k(k+1)}{42} & , \text{ if } k \le x < k+1, \text{ for } x = 1, 2, \ldots, 5, \\ 1 & , \text{ if } x \ge 6 \end{cases}$$

find the probability mass function (p.m.f.) and the expectation of X. 3 + 2

2. Answer any *two* questions : 10 × 2

(*a*) For the events A and B show that

$$\max\{0, P(A)+P(B)-1\} \le P(A \cap B) \le$$
$$\min\{P(A), P(B)\} \le \max\{P(A), P(B)\} \le$$
$$P(A \cup B) \le \min\{P(A)+P(B), 1\}$$ 10

(*b*) Write down the probability mass function of a negative binomial distribution. Show that geometric distribution is a special case of negative binomial distribution. Also show that for negative binomial distribution with parameters 'k' and 'p' 10

$$\mu_{r+1} = q\left[\frac{\partial \mu_r}{\partial q} + \frac{rk}{p^2}\mu_{r-1}\right].$$

(*c*) (*i*) Give an example of a continuous distribution having 'lack of memory' property and establish this property for this distribution. 5

(*ii*) Derive the moment generation function (m.g.f.) of a bivariate normal distribution. 5

(*d*) A question paper containing five questions has been answered by 600 students. The answer papers are checked and the number of students answering a question correctly has been obtained for all the five questions. Describe, in details, a procedure of finding the difficulty values of the five questions on the basis of these data. 10

GROUP—B

3. Answer any *two* questions : 5×2

(*a*) Show that *n* th order finite difference of a polynomial of degree *n* is constant.

(*b*) Obtain the convergence criterion of iteration method to obtain numerical solution of a equation in one unknown.

(*c*) Show that

$$u_{2n} - \binom{n}{1} 2u_{2n-1} + \binom{n}{2} 2^2 u_{2n-2} - \dots + (-2)^n u_n = (-1)^n (c - 2an)$$

where $u_x = ax^2 + bx + c$.

(*d*) Derive Simpson's $\frac{1}{3}$ rule for numerical integration.

4. Answer any *one* question : 10 × 1

(*a*) Describe Newton-Raphson method to obtain numerical solution of transcendental equations. Give the geometric interpretation of this method. Obtain the formula for square root of an integer by using the above method.

(*b*) Derive Lagrange's interpolation formula. Hence show that Lagrange's interpolation formula is a weighted average of the entries. Discuss the important uses of this formula.

GROUP—C

5. Answer any *three* questions : 5 × 3

(*i*) What are the different methods for determining trend ? Discuss the merits and demerits of each method.

(*ii*) Prove that Laspeyres' price index tends to be greater than Paasche's price index.

(*iii*) Describe the exponential smoothing technique of time-series data.

(*iv*) State the main functions of CSO and mention one of its important publications.

(*v*) What is a Lorenz curve ? How is it used to indicate the income inequality ?

(*vi*) What are the different types of errors that may creep in during the construction of index numbers ?

(*vii*) What is seasonal variation ? What are different causes for seasonal variation ?

6. Answer any *one* question : 10×1

(*a*) State the different components of a time series with appropriate examples of each. Explain the ratio to trend method to determine the seasonal indices in a time-series. 10

(*b*) Distinguish between a 'fixed-base index number' and a 'chain base index number'. Describe the constructions of chain-base index number with its advantages and disadvantages. 10

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