2015

OLD

Part I 3-Tier

STATISTICS

PAPER—I

(Honours)

Full Marks : 90

Time : 4 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group—A

1. Answer *either (a)* or *both (b) and (c).*

(a) Define intra class correlation. Distinguish it from inter class correlation. Derive the formula for intra class correlation coefficient when the variable x is observed for p families, each consisting of K members. Also derive the limits of intra class correlation coefficient and discuss the marginal cases.

15

(b) Discuss the following terms with examples, if necessary :

Correlation Ratio and Correlation Index, Cross-sectional data and Time series data.

8

(c) Suppose in a grouped frequency distribution of k classes, the ratio of upper and lower boundaries is a constant say r. If m_1 is the mid value of the first class and f_i is the frequency of the i-th class such that N is the total frequency, show that :

$$\log GM = \log m_1 + \frac{\log r}{N} \sum_{i=2}^{k} (i-1) f_1,$$

where GM denotes the geometric mean of the grouped frequency distribution. 7

2. Answer *either (a)* or *both (b) and (c).*

(a) Derive the regression line of y on x. Explain why two regression lines are needed. When do they coincide?

Also derive three fundamental properties related to regression lines. 15

(b) In the context of paired data on (y, x) if r_{xy} and e_{yx} denote correlation coefficient between x, y and correlation ratio of y on x respectively, show that :

$$0 \le r_{xy}^2 \le e_{yx}^2 \le 1$$

Also interpret the cases $r_{xy}^2 = e_{yx}^2$ and $e_{yx}^2 \le 1$. 8

(c) Prove that $\frac{R^2}{2n} \le S^2 \le \frac{R^2}{4}$ where R and S denote the range and the standard deviation respectively of n observations. Also discuss the cases of equality in the above inequality. 7

3. Answer any *three* questions : 3×5

(a) Define Pearson's measure of skewness and derive its range.

(b) Discuss merits and demerits of different methods of collecting primary data.

(c) Derive a formula for median of a grouped frequency distribution from its ogive.

(d) Describe Stem-Leaf display with its possible merit(s) and demerit(s), if any.

(e) What is Likert Scaling? Discuss its uses.

Group—B

4. Answer any *three* questions : 3×6

(a) Show that basis of a vector space is not unique.

(b) Derive the eigen values of an Idempotent matrix and those of an Orthogonal matrix. Is it possible that a matrix is Idempotent as well as Orthogonal? Justify your answer.

(c) What is orthogonal matrix? Show that if α is a characteristics root of an orthogonal matrix, then $\frac{1}{\alpha}$ is also a characteristics root of the matrix.

(d) Let a square matrix $A_{n\times n}$ be partitioned as

$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ where A_{11} is of order $m\times m$ $(m<n)$ and

A_{22} is non-singular. Show that :

$$|A| = |A_{22}| \left| A_{11} - A_{12} A_{22}^{-1} A_{21} \right|.$$

(e) If A and P be both $n \times n$ matrices and P be non-singular, prove that A and $P^{-1}AP$ have the same eigen values.

Group—C

5. Answer any *five* questions : 5×5

(a) Prove that the sequence $\{u_n\}$ defined by $u_1 = 2$ and $u_{n+1} = \sqrt{2u_n}$ $\forall\, n \geq 1$ converges to 2.

(b) State and prove Sandwich theorem.

(c) State and prove the sequential criterion of limits.

(d) Define uniform continuity of a function. Prove that a uniformly continuous function is always continuous but the converse is not true.

(e) Evaluate : $\iiint_{x^2+y^2+z^2 \leq c^2} (xy + yz)\, dx\, dy\, dz$

(f) If $f''(x) \geq 0$ on [a, b], prove that

$$f\left(\frac{x_1 + x_2}{2}\right) \leq \frac{1}{2}\left[f(x_1) + f(x_2)\right]$$

for any two points $x_1, x_2 \in [a, b]$.

(g) What do you mean by a power series? Check the convergence of the power series :

$$x + \frac{2!^2}{4!}x^2 + \frac{3!^2}{6!}x^3 + \cdots \cdots$$

(h) State and prove Lagrange's Mean Value Theorem.

6. Answer any *one* question : 2×1

(a) Continuity of f at a point c does not ensure differentiability of f at c — justify.

(b) Check the convergence of the series : $\sum_{n=1}^{\infty} \frac{n}{n+1}$.