UG/III/STAT/H/VI/16

2016

STATISTICS

[Honours]

PAPER – VI

Full Marks : 100

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

Answer Q. Nos. 1 or 2 and Q. Nos. 3 or 4 and Q. No. 5

1. (*a*) State Fisher-Neyman factorisation theorem.

   (*b*) Show that if T is a sufficient statistic for θ and $\psi(T)$ is a one to one function of T then $\psi(T)$ is also sufficient statistic for θ.

(*Turn Over*)

(*c*) Suppose $X_1, X_2, ..., X_n$ is a random sample from the distribution having pmf

$$f(x;\theta_1,\theta_2) = \begin{cases} \theta_1(1-\theta_1)^{x-\theta_2} & ;\ x = \theta_2, \theta_2+1, \ldots.. \\ 0 & ;\ \text{otherwise} \end{cases}$$

where $\theta = (\theta_1, \theta_2)$ and $0 < \theta_1 < 1$. Find sufficient statistic for $\theta = (\theta_1, \theta_2)$. 15

2. (*a*) Define consistent estimator.

(*b*) State and prove a sufficient condition for estimator being sufficient for $\gamma(\theta)$.

(*c*) Let $X_1, X_2, ..., X_n$ is a random sample from uniform $(0, \theta)$; $\theta > 0$. Examine whether the geometric mean $\left(\prod_{i=1}^{n} X_i\right)^{1/n}$ is a consistent for θe^{-1} or not. 15

3. (*a*) What is the principle of Likelihood Ratio (LR) test ?

(*b*) Suppose $X_1, X_2, \ldots, X_m$ is a random sample from $N(\mu_1, \sigma^2)$ and $y_1, y_2, \ldots, y_n$ is a random sample from $N(\mu_2, \sigma^2)$, where μ_1, μ_2, σ^2 are unknown. Derive LR test for testing $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$. Show that this test can be performed using t-test. 15

4. (*a*) Describe sequential probability ratio test (SPRT) procedure.

(*b*) Derive SPRT for testing $H_0 : \theta = 1$ against $H_1 : \theta = -1$ when the population is $N(\theta, 1)$. Find the OC and ASN functions of the above SPRT. 15

5. Answer any *three* of the following questions : 8×3

(*a*) Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of a random sample of size 5 from uniform $(0, \theta)$; $\theta > 0$. Find K such that KY_3 is an unbiased estimator of θ.

(*b*) Show that for a random sample of size n from Cauchy distribution

$$f(x;\theta)=\begin{cases}\dfrac{1}{\pi[1+(x-\theta)^2]} & ; -\infty<x<\infty \\ 0 & ; \text{otherwise},\end{cases}$$

where $\theta>0$, sample mean is not a consistent estimator of θ.

(*c*) Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution having pdf

$$f(x;\theta)=\begin{cases}\dfrac{1}{2}e^{-|x-\theta|} & ; -\infty<x<\infty \\ 0 & ; \text{otherwise}.\end{cases}$$

Find MLE of θ.

(*d*) Let X be a random variable for which pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f_{H_0}(x)$	0·01	0·01	0·01	0·01	0·01	0·01	0·94
$f_{H_1}(x)$	0·06	0·05	0·04	0·03	0·02	0·01	0·79

Find MP test for testing H_0 against H_1 with size $\alpha = 0{\cdot}04$. Compute the probability of Type-II error for this test.

(*e*) Discuss a suitable non-parametric test for testing the equality of location parameters of two populations.

GROUP – B

6. Answer any *three* of the following questions : 8 × 3

(*a*) Suppose a SRSWOR of size n is drawn from a population of N units. Let a_i be a random variable which takes the value '1' if the ith unit of the population be included in the sample and the value '0' otherwise. The sample mean $\bar{x}$ may then be written as

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} a_i x_i .$$

Find $\text{var}(\bar{x})$.

(*b*) Assume a standard cost function, find the optimum value of $\{n_k\}$ in stratified random sampling for a fixed variance so that the cost is minimum. Hence find the variance of the estimator of the population mean under Neyman's allocation.

(*c*) Compare the systematic random sampling with simple and stratified random samplings for some specified population with linear trend.

(*d*) Derive the expression of standard error of p (sample proportion of members possessing some characteristic) in SRSWR and SRSWOR case.

(*e*) For two stage sampling, where the first stage units are of equal size, obtained an estimator of the population mean. Also obtained the expression for the variance of the estimator.

7. Answer any *two* of the following questions : 6× 2

(*a*) Write a short note on any *two* of the following:

(*i*) Advantage of sample survey over complete enumeration (*ii*) *Cluster sampling method*

(*iii*) Non-sampling error.

(*b*) Discuss the use of random sampling numbers in drawing simple random sample with and without replacement from a finite population.

(*c*) Define linear systematic sampling. Suggest an unbiased estimator for the population total in linear systematic sampling and find its variance.

(*d*) Prove that in large samples, with simple random sampling, the ratio estimate $\hat{Y}_R$ has a smaller variance than the estimate $\hat{Y} = N\overline{Y}$ if

$$\rho > \frac{1}{2}\left(\frac{S_x}{\overline{\overline{X}}}\right) \bigg/ \left(\frac{S_y}{\overline{\overline{Y}}}\right).$$

[*Internal Assessment* : 10 Marks]
