

2016

STATISTICS

[Honours]

PAPER – I

Full Marks : 90

Time : 4 hours

*The figures in the right hand margin indicate marks
Candidates are required to give their answers in their
own words as far as practicable*

Illustrate the answers wherever necessary

[NEW SYLLABUS]

GROUP – A

(*Descriptive Statistics*)

1. Answer any five questions : 5 × 5

(a) What is percentile ? How do you obtain percentile for a grouped frequency distribution ?

(Turn Over)

- (b) What do you mean by frequency curve ? Describe different types of frequency curve.
- (c) What is Sheppard's correction for moments ? Give the conditions for which these conditions are valid.
- (d) Show that, the value of the correlation coefficient is not affected by the change of origin and scale of the variables.
- (e) Define Chi square measure of association. What are its defects ? How do you remove these defects ?
- (f) What are the different relative measures of dispersion ? Discuss their uses.
- (g) What do you mean by Kurtosis of a frequency distribution ? Show that, $b_2 \geq 1$, where b_2 is the moment measure of kurtosis.
- (h) Show that, numerically the correlation coefficient is the proportion of the total variability of the response variable which is accounted for by its linear regression on an explanatory variable.

(i) What is scatter diagram? Draw scatter diagram for the following cases :

(I) $r = 0$

(II) $r = 1$

(III) $r = -1$

(j) What is outlier? How do you determine outlier in a dataset using quartile?

2. Answer any *two* questions : 10 × 2

(a) What is rank correlation? Deduce Spearman's rank correlation coefficient for both tied and non-tied case.

(b) Distinguish between the following pairs with examples :

(i) Primary data vs. Secondary data

(ii) Ordinal data vs. Nominal data

(iii) Discrete variable vs. Continuous variable.

(c) (i) What is regression coefficient? Deduce

the relationship between regression coefficient and correlation coefficient. How do you conclude about the type of correlation from the sign of the regression coefficients ? Justify.

- (ii) For the variables X, Y, U, V , if $U = aX + bY$ and $V = bX + aY$, show that U and V are uncorrelated if

$$\frac{ab}{a^2 - b^2} = \frac{rs_x s_y}{s_x^2 - s_y^2}$$

- (d) (i) Show that, the numerical value of correlation coefficient is equal to the correlation between actual value and predicted value (from linear regression) of the response variable.
- (ii) Suppose there are two variables X and Y such that $Y = a + bX$ where a and b are unknown constants. Given a pair of n values of X and Y , how do you determine the values of a and b so that the total

square difference between actual values and functional values of Y is minimum ?

GROUP – B

(*Matrix Algebra*)

3. Answer any two questions :

5 × 2

(a) Show that if S is a skew symmetric matrix and $(I + S)$ is non-singular, then $(I + S)^{-1}(I - S)$ is orthogonal.

(b) What is a positive definite matrix ? Show that if $A = (a_{ij})$ is a positive definite matrix of order p , then

$$|A| \leq \prod_{i=1}^p a_{ij}.$$

(c) Show that

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A| \cdot |D - CA^{-1}B|,$$

if A is a non-singular matrix.

(d) Show that if λ is a characteristic root of an orthogonal matrix, then $\frac{1}{\lambda}$ is also a characteristic root of the matrix.

4. Answer any *one* question : 10 × 1

(a) (i) Suppose A and B be two matrices of order $p \times q$ and $q \times m$ respectively. Show that

$$\text{Rank}(AB) \geq \text{Rank}(A) + \text{Rank}(B) - q.$$

(ii) Suppose A is a square matrix of order $p \times p$ with $\text{Rank}(A) = p - 1$. Find $\text{Rank}(\text{adj } A)$.

(b) (i) Show that any set of linearly independent vectors in a finite dimensional vector space is either a basis or it can be extended to a basis.

(ii) State and prove Gram-Schmidt orthogonalisation process.

GROUP – C

(*Mathematical Analysis*)

5. Answer any *one* question : 10 × 1

(a) Define Cauchy sequence. Prove that every convergent sequence is Cauchy sequence. Show that the sequence

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

is not a Cauchy sequence.

(b) Prove that monotonically increasing sequence bounded above is convergent. Show that

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e.$$

6. Answer any *three* questions : 5 × 3

(a) Show that

$$f(x) = \sin(x), x \in R$$

is continuous everywhere and

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

is everywhere discontinuous.

(b) Let $[x]$ denote the largest integer not exceeding x and $f(x) = x - [x]$. Determine discontinuities of f .

(c) Find a and b such that

$$\lim_{x \rightarrow 0} \frac{a \sin 2x - b \sin 3x}{5x^3} = 1.$$

(d) Show that the integral

$$\int_{-\infty}^{\infty} \frac{x}{1+x^2}$$

is not convergent.

(e) Show that every absolutely convergent series of real numbers is convergent. Is the converse true? Justify your answer.

(f) Let $f(x)$ be a function such that

$$\forall x \in (a, b) \quad f'(x) = 0.$$

Show that $f(x)$ must be constant on (a, b) .

[Internal Assessment : 10 Marks]