UG/II/STAT/H/III/16 (Old)

# 2016

## STATISTICS

## [ Honours ]

## PAPER — III

*Full Marks* : 90

*Time* : 4 hours

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

*Illustrate the answers wherever necessary*

## [ OLD SYLLABUS ]

## GROUP — A

Answer any **three** questions : 18 × 3

1. (*a*) Let $\underset{\sim}{X}_{PX1} = (X_1 \ldots X_p)'$ have the distribution $N_p(\mu, \Sigma)$. Show that $C_1X_1 + \cdots + C_pX_p$ as a normal random variable. 6

( Turn Over )

(*b*) If $\underset{\sim}{X}_{P \times 1} = (X_1 \ldots X_P)'$ follow $P$-variate normal distribution and $\underset{\sim}{Y} = A\underset{\sim}{X}$ with $A$ non-singular, show that $\underset{\sim}{Y}$ is also normal random vector. 6

(*c*) Obtain the MGF of $\underset{\sim}{X}$, where $\underset{\sim}{X} \sim N_P(\mu, \Sigma)$. 6

2\. (*a*) Suppose $\underset{\sim}{X} = (X_1, \ldots X_x)'$ follow multinomial distribution with parameters $(m, p_1, \ldots p_x)$ where $p_i > 0$, $i = 1, 2 \ldots x$.

$$\sum_{i=1}^{x} P_i = 1 \text{ and } \sum_{i=1}^{x} X_i = m.$$

Obtain the variance-covariance matrix for $\underset{\sim}{X} = (X_1, \ldots, X_x)$. Indicate the condition for non-singularity of the above mentioned matrix. 8

(*b*) Explain the idea of concentration ellipsoid for univariate random variable. Extend it for bivariate distributions. 10

3\. (*a*) Define chi-square distribution. Show that if $X_1 \sim \chi^2_{(n_1)}$ and $X_2 \sim \chi^2_{(n_2)}$ where $X_1$ and $X_2$ are independent, then $X_1 + X_2 \sim \chi^2_{(n_1 + n_2)}$. 8

(b) Let $X_1, X_2, \ldots, X_n$ be independent random variables, and $X_i \sim N(0, 1)$, $i = 1, 2, \ldots, n$. Obtain the conditional distribution of $X_1^2 + \ldots + X_n^2$ under $m$-restrictions

$$\begin{array}{l} a_{11}X_1 + a_{12}X_2 + \cdots + a_{1n}X_n = 0 \\ \vdots \\ a_{m1}X_1 + \cdots + a_{mn}X_n = 0 \end{array}$$ 10

4. (a) Let $X_1$, $X_2$, $X_3$ be independent $N(0, 1)$ random variables. Find the distribution of

$$\frac{(X_1 + X_2 + X_3)/\sqrt{3}}{(X_1 - 2X_2 + X_3)/\sqrt{6}}.$$ 6

(b) If $X_1$ and $X_2$ are independently distributed random variables each in the form $U(0, 1)$. Show that

$$Y_1 = \sqrt{-2\ln X_1}\cos 2A\, X_2$$

$$\text{and } Y_2 = \sqrt{-2\ln X_1}\sin 2A\, X_2$$

are independent. 6

(*c*) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, 1)$. Find the distribution of their geometric mean. 6

5. (*a*) Let $X_1, X_2 \sim U(0, 1)$. Find the distributions of $X_1 + X_2$ and $X_1 - X_2$. 8

(*b*) Let $X_i \sim N(0, 1)$, $i = 1, 2 \ldots, n$. Establish the independence of

$$Y_1 = \frac{X_1}{\sqrt{\sum_{i=2}^{n} X_i^2}}, Y_2 = \frac{X_2}{\sqrt{\sum_{i=3}^{n} X_i^2}} \cdots Y_{n-1} = \frac{X_{n-1}}{|X_n|}, Y_n = \sum_{i=1}^{n} X_i^2.$$

Determine the distribution of each $Y_i$. 10

## GROUP – B

Answer any one question : 18 × 1

6. (*a*) Explain 3-sigma limits and probability limits for control charts. 6

(*b*) Describe control charts for number of defectives and control charts for number of defects. 12

7. (*a*) Explain Consumer's risk and Producer's risk in the context for sampling inspection plan. 6

(*b*) Explain single sampling inspection plan. Obtain expression of OC and ASN functions. 12

## GROUP – C

Answer any **one** question : 18 × 1

8. (*a*) Write an algorithm to calculate AM, GM and HM of ungrouped data. 9

(*b*) Write an algorithm to find factorial of a positive integer. 9

9. (*a*) Write a C-program to generate a random sample of size '*n*' from exponential distribution. 9

(*b*) Write a C-program to calculate correlation coefficient from a ungrouped data. 9

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